DECIMATIVE SPECTRUM ESTIMATION METHOD FOR HIGH-RESOLUTION RADAR PARAMETER ESTIMATES

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Abstract

Scattering center extraction for radar targets is strongly related to high-resolution parameter estimation.

In this paper, we propose the use of a decimative spectrum estimation method (DESED) for the estimation of the parameters of a synthetic radar signal. DESED is compared to two variants of the widely applied root-MUSIC algorithm, in terms of range and amplitude estimation errors.

The proposed method outperforms root-MUSIC with modified spatial smoothing pre-processing, for both well-separated and closely spaced point scatterers. Additionally, it appears to be slightly more accurate than root-MUSIC with decimation, especially for low signal-to-noise ratio and in the case of well-separated point scatterers.

1. Introduction

Parameter estimation is a fundamental field of signal processing. Two main approaches are followed: Bayes estimation which is based on a priori knowledge about the examined signal (probability distribution functions of parameters), and maximum likelihood (ML) estimation which maximizes a likelihood function that depends on signal parameters. Algorithms based on singular value decomposition (SVD) have been proposed to solve the parameter estimation problem [1]. With the recent advance of digital signal processors (DSP) and personal computers, moderately sized SVD analysis (square matrix order of 50-70) has become feasible and computationally efficient software implementations are available. For this reason, parameter estimation methods that embody SVD appear to be appealing, resulting in estimation accuracy at the cost of additional computational burden.

High-resolution radar imaging is performed with the use of spectrum estimation methods [2], [3]. Parametric methods are of special interest, since they employ a parametric model to accurately describe the signal segment under spectral analysis. A parameter vector has to be estimated before the computation of the signal's spectral content. Thus, parameter estimation is highly related to radar imaging, and is a primary step of the whole process.

Decimative spectrum estimation constitutes a very interesting field of signal processing research [4], [5], [6], compared to the classical methods that were proposed a few decades ago. Decimation improves resolution capability of a frequency estimation method. In [5], two superresolution subspace methods, namely MUSIC and ESPRIT, are shown to provide more accurate frequency estimates when data decimation is applied. Furthermore, in [4] it is pointed out that these methods impose a constraint on the model order with respect to the decimation factor, resulting in reduced efficiency in case of an overdetermined model.

In this paper, we propose the use of a decimative spectrum estimation method, namely DESED [4], for estimating the parameters of a radar signal model, which is based on the geometrical theory of diffraction (GTD). The adopted model has been proved to perform accurate modeling of the backscattered field for a wide range of scatterers [7]. Therefore, using GTD-based model for the description of the received frequency-domain samples of a stepped frequency (SF) radar waveform is reasonable.

DESED method makes use of decimation and SVD, exploits all data available, and does not impose any constraint on the decimation factor and the model order. Frequencies, dam-ping factors and complex amplitudes of the damped exponential (DE) signal model are estimated by DESED. In case of radar scattering data, radial locations, geometry parameters and scattering amplitudes can be respe-ctively derived through mathematical expressions relating DE and GTD models.

The remainder of the paper is organized as follows. Section 2 briefly delineates the GTD-based parametric model for radar scattering, while Section 3 provides the theoretical background of the DESED method. Section 4 presents the simulation results obtained by DESED method and two superresolution techniques, with respect to the estimation of the radar signal parameters. Namely two variants of the root-MUSIC algorithm are compared to DESED method, and useful conclusions are drawn in Section 5.

2. Scattering Model

Radar targets can be adequately characterized by a small number of scattering centers, in the high-frequency limit. The GTD-based model [7], which is used in the present study, provides a parametric description of the measured scattering behavior of a radar target. Its main advantage is that it embodies a parameter that characterizes the geometry of each scattering center. Its accuracy is attributed to its close relation with the physics of electromagnetic scattering. As stated in [7], the GTD-based model describes the frequency dependence of the scattering data more accurately than the DE model. Especially for large relative radar bandwidths, the DE model fails to satisfactorily represent canonical scattering mechanisms and results in worse scattering center resolution than the GTD model.

The GTD model equation for the backscattered field of a perfectly conducting target is:

$$E(f) = \sum_{i=1}^{M} A_i \left(\frac{f}{f_c}\right)^{\alpha_i} e^{-j4\pi f r_i / c} \qquad (1)$$

where f denotes the radar frequency, M is the number of scattering centers, A_i, α_i and r_i symbolize the scattering amplitude, the geometry parameter and the range position of the *i*th scattering center, f_c is the center frequency, and c is the speed of light. Table 1 summarizes the geometry parameters for a number of canonical scattering geometries.

In case of SF waveform, the N frequencies spanning the utilized radar bandwidth are given by:

$$f_n = f_c + n \cdot \Delta f \,, \quad n = -\frac{(N-1)}{2}, \ldots, \frac{(N-1)}{2} \qquad (2)$$

where N is assumed to be odd in order to provide symmetry around the center frequency, and Δf is the chosen frequency step.

The GTD model can precisely describe various scattering mechanisms. In [7], edge and corner diffractions, as well as reflection mechanisms from three canonical surfaces (sphere, cylinder at broadside and flat plate), are shown to be in accordance to the GTD model. Thus, it stands to reason that a radar target can be considered as a collection of several different scattering geometries, as those cited in Table 1.

Table 1. Geometry parameters for canonical scattering geometries

Example scattering geometries	Geometry parameter value
corner diffraction	– 1
edge diffraction	– ½
ideal point scatterer; doubly curved surface reflection; straight edge specular	0
singly curved surface reflection	½
flat plate at broadside; dihedral	1

3. Proposed Method

Decimative spectrum estimation by a factor of D (DESED) has been introduced in [4]. Frequencies and damping factors are estimated both in least squares (LS) and total least squares (TLS) sense. Amplitude estimation is accomplished in LS sense, by substituting frequency and damping factor estimates in the model equation for the noiseless signal.

DESED assumes that the spectrally analyzed signal is represented by the generalized sinusoidal (also known as damped exponential) model:

$$s(n) = \sum_{i=1}^{p} \left(a_i e^{j\phi_i} \right) \cdot e^{(-d_i + j2\pi f_i)n}$$

$$= \sum_{i=1}^{p} g_i \cdot z_i^n , \quad n = 0,1,\ldots,N-1$$

(3)

where p is the model order, a_i, ϕ_i, d_i and f_i denote the amplitude, phase, damping factor and frequency of the *i*th sinusoid, and N is the number of data samples. Equation (3) describes the noiseless signal under spectral analysis. Nonetheless, noisy data measurements are encountered in practical applications, and the noise is usually assumed to be additive white Gaussian (AWGN).

The algorithmic steps of the LS-DESED method are the following:

Step 1: Computation of the Hankel matrix S from the N data points of the examined signal $s(n)$, using equation (3). The Hankel matrix is formulated as follows:

$$S = \begin{pmatrix} s(0) & s(1) & \ldots & s(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ s(L-1) & s(L) & \cdots & s(N-1) \end{pmatrix} \quad (4)$$

Step 2: Derivation of the decimated versions of the Hankel matrix, $S_{\downarrow D}$ by deleting the top D rows of S, and $S_{\uparrow D}$ by deleting the bottom D rows of S.

Step 3: SVD decomposition of matrix $S_{\uparrow D}$, resulting in $S_{\uparrow D} = U_{\uparrow D} \Sigma_{\uparrow D} V_{\uparrow D}^H$, and truncation to order p, by retaining only the p largest singular values in matrix $\Sigma_{\uparrow D}$ and only the first p columns of matrices $U_{\uparrow D}$ and $V_{\uparrow D}$. The resulting matrix is given by:

$$S_{\uparrow D}^{trunc} = U_{\uparrow D}^{trunc} \Sigma_{\uparrow D}^{trunc} \left(V_{\uparrow D}^{trunc} \right)^H \quad (5)$$

Step 4: Computation of the truncated SVD solution of equation $X \cdot S_{\uparrow D} = S_{\downarrow D}$ in LS sense, where X is an $(L-D)$-order matrix:

$$X = S_{\downarrow D} \cdot \left(S_{\uparrow D}^{trunc} \right)^{\dagger} \qquad (6)$$

where $A^{\dagger}$ denotes the pseudo-inverse of a matrix A.

Step 5: Estimation of frequencies f_i and damping factors d_i ($i = 1, \ldots, p$) through the p largest eigenvalues of matrix X, which are proven in [4] to be equal to the decimated signal pole estimates z_i^D. Note from equation (3) that the signal poles are given by $z_i = e^{(-d_i + j2\pi f_i)}$.

Step 6: Estimation of amplitudes a_i and phases ϕ_i ($i = 1, \ldots, p$) by finding a LS solution to equation (3), with z_i replaced by the respective estimates of the previous step and $s(n)$ being the noisy measured data.

The only constraints set by DESED concern the dimensions of the Hankel matrix and its decimated versions. These are:

$$p \le L - D \le M \qquad (7)$$

$$L + M - 1 \equiv N \qquad (8)$$

It worths mentioning that the best results of DESED come from decimated Hankel matrices as square as possible ($L - D \approx M$).

4. Simulation Results

In our simulations, we assume that the radar target consists of ideal point scatterers, and for this reason we simulate the GTD model with zero-valued geometry parameters.

LS-DESED method and two variants of the root-MUSIC algorithm, with modified spatial smoothing pre-processing (MSSP) [8], and with decimation [5], have been tested for two simulation scenarios. The first simulation scenario involves well-separated point scatterers (five distinct radial positions), whereas the second scenario includes two scatterers separated in range by $1/3$ of the Fourier bin δr. All scattering amplitudes are set to unity.

Choosing the radar bandwidth $B = 400\,MHz$ and the center frequency $f_c = 9\,GHz$, the scatterers' separation is calculated: $\Delta r = \left(c/2B \right)/3 = 0.125\,m$. The relative radar bandwidth takes the value of $\gamma = B/f_c \approx 0.044$, which is small enough to justify the assumption that the DE model approximates the GTD model. The radar frequency step is set to $\Delta f = 2\,MHz$, resulting in a total of $N = 201$ frequency-domain data samples.

In view of equations (1) and (3), taking into account the assumption of small relative bandwidth, we can easily deduce the mathematical relationship between the frequency estimates of the DE model and the range estimates of the GTD model:

$$r_i = -\frac{c\,f_i}{2\,\Delta f} \qquad (9)$$

Average RMS values of the range and amplitude estimation errors have been derived for the LS-DESED and the two root-MUSIC variants, for a total of 100 Monte-Carlo trials. The data snapshot length for the root-MUSIC techniques is selected to be $m = 40$, providing a satisfactory tradeoff between frequency resolution and accuracy in the covariance matrix estimate. Decimation factor D takes two values, 2 and 3, for the two techniques that use decimation.

Figs. 1 and 2 graphically depict simulation results in the case of well-separated scatterers, for signal-to-noise ratio (SNR) ranging from 5 to 40 dB. For the second simulation scenario, we present the respective graphs for range estimates in Fig. 3. Table 2 displays the average range estimation errors for the forth and the fifth point scatterer, which are separated by $\delta r/3$. Only the two decimative methods are compared, for decimation factor of 3.

As we can observe from Figs. 1 and 3, the DESED method outperforms root-MUSIC with MSSP, in terms of range estimates. In the case of closely spaced scatterers, the proposed method breaks down at an average SNR value of 15 dB, whereas the root-MUSIC with MSSP is already deteriorated at 25 dB. All three figures indicate that the two decimative methods exhibit similar performance in terms of range and amplitude estimates. DESED appears to be slightly better than root-MUSIC with decimation, and both techniques provide increased resolution compared to the root-MUSIC with MSSP. Furthermore, it is obvious from these results that negligible estimation accuracy can be gained by increasing the decimation factor from 2 to 3. This can be attributed to the relatively short data length.

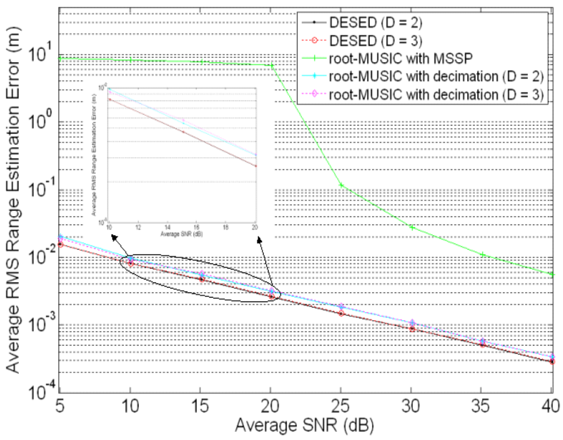

Figure 1. Average RMS range estimation error versus average SNR for well-separated point scatterers

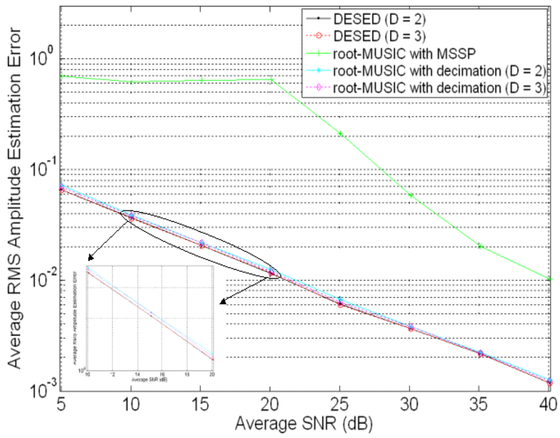

Figure 2. Average RMS amplitude estimation error versus average SNR for well-separated point scatterers

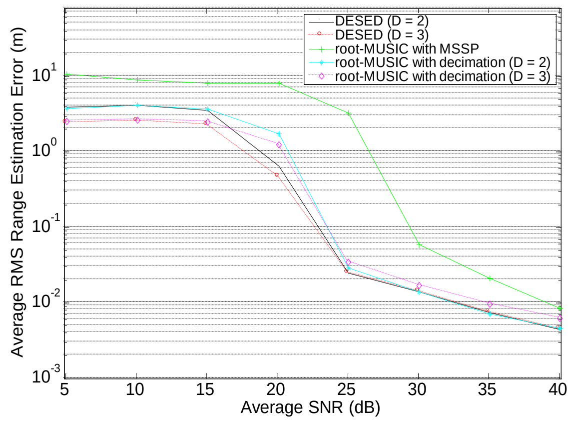

Figure 3. Average RMS range estimation error versus average SNR for closely spaced point scatterers

Table 2. average range estimation errors for two closely spaced point scatterers

SNR (dB)	Average range estimation error (m)			
	Forth scatterer		Fifth scatterer	
	DESED (D = 3)	root-MUSIC (D = 3)	DESED (D = 3)	root-MUSIC (D = 3)
5	1.0930	1.0468	1.6454	1.7395
10	1.1503	1.0103	1.5067	1.9683
15	0.9613	0.9118	1.6680	1.8995
20	0.2492	0.7518	0.3966	0.8590
25	0.0331	0.0601	0.0388	0.0393
30	0.0187	0.0232	0.0210	0.0260
35	0.0107	0.0114	0.0105	0.0137
40	0.0065	0.0088	0.0063	0.0087

5. Conclusions

In this paper, we have proposed the use of a decimative spectrum estimation method, named DESED, for the estimation of the parameters of a synthetic radar signal. The proposed method is much more accurate than the classical root-MUSIC algorithm, in terms of range and amplitude estimates. Simulation results for both well-separated and closely spaced point scatterers reveal slightly better performance than the root-MUSIC method that uses decimation.

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