NON-RELATIVISTIC MOTION OF IONS IN LIVE TISSUES

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Abstract

In this paper certain aspects are considered of the motion in live tissues of ions under influence of an electro-magnetic field. Some results of computer simulation of space configuration of low frequency magnetic field around the human body is presented, also.

Introduction

The problem that presents itself is that of solving the equations of motions given in relativistic form, namely

$$\frac{d}{dt}\left[\frac{m_0\vec{v}(t)}{\sqrt{1-\frac{|\vec{v}(t)|^2}{|\vec{c}|^2}}}\right]=e[\vec{E}(t)+\vec{v}x\vec{B}(t)] \qquad (1)$$

The problem becomes, in principle, straightforward if it be assumed that $\vec{E}$, $\vec{B}$ appearing in the right-hand side of (1) is a given field; the difficulties are then of purely technical kind that can also arise in traditional particle mechanical forces. The assumption is often a legitimate approximation, and the most of the work presented here is in this context. In fact, however, the charge itself also contributes to the electromagnetic field, and its contribution depends on its motion. The inclusion of this “self-force” presents quite fundamental difficulties because of the infinities associated with the concept of a point charge. Attempts to describe the motion of an electron can perhaps be classified in terms of the following alternatives (a) abandon the idea that the electron is a point charge, and give it some intrnal structure; (b) introduse into classical mathematical framework some formalism that succeeds in discarding the infinites; (c) abandon the idea that the electron can be described in any fundamental way by classical theory, and insist that appeal be made to quantum laws. The procedure (b) has been given a good deal of attention, and has led to an equation of motion which at least in certain contexts is acceptable theoretically and is in general agreement with practice. A derivation of this equation is given later in paper. Any discussion of alternatives(a) and (c) is outside the scope of this paper.

1. Equation of motion

In attempting to solve (1) it may helpful to make use of the associated energy equation

$$\frac{d}{dt}\left(\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)=e\vec{E}.\vec{v} \qquad (2)$$

If the electric field is purely static, so that $\vec{E}=-grad\phi$, this gives

$$\frac{m_0c^2}{\sqrt{\left(1-v^2/c^2\right)}}+e\phi=cons\tan t. \qquad (3)$$

In the important special case $\vec{E} = 0 \Rightarrow \vec{v} = const$. Since a time-varying magnetic field cannot exist without an associated electric field. When

v = const $\dot{\vec{v}}$ is along the principal normal to the trajectory and of magnitude

v^2/p_p, where p_p is the principal radius of curvature; hence

$$p_p = \frac{m_0 v}{eB\sin\theta\sqrt{\left(1-v^2/c^2\right)}}, \qquad (4)$$

where θ is the angle between $\vec{B}$ and $\vec{v}$,commonly called the pitch angle.

Often it is not necessary to take account of relativistic effects, and the equations of motions and energy equation can then taken as

$$m\frac{d\vec{v}(t)}{dt} = e[\vec{E}(t) + \vec{v}(t)x\vec{B}(t)] \qquad (5)$$

$$\frac{d}{dt}\left(\frac{1}{2}mv(t)^2\right) = e\vec{E}.\vec{v} \qquad (6)$$

It may be noted that in the case $\vec{E} = 0$,

Where the field is purely magnetostatic, the relativistic equations are the same as the non-relativistic, except only that *m* in the latter is replaced by the constant

$$\frac{m_0}{\sqrt{\left(1-v^2/c^2\right)}}.$$

It is well known that the equations of motion can be cast into two alternative forms, familiar in classical mechanics as Lagrangian and Hamiltonian forms. In the present calculations no extensive use is made of these forms, but it is convenient to state theme here. In the non-relativistic treatment the usual form of Lagrange`s equations is equivalent to (5) if the Lagrangian is taken as

$$L = \frac{1}{2}mv^2 + e\vec{v}.\vec{A} - e\phi, \qquad (7)$$

where

ϕ is scalar potentials of the electromagnetic field

$\vec{A}$ is vector potentials of the electromagnetic fields.

Likewise the usual form of Hamilton's equations is equivalent to (5) if the Hamiltonian is taken as

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + e\phi, \qquad (8)$$

where

$\vec{p} = m\vec{v} + e\vec{A}$

is generalized momentum.

To recapture the relativistic equation (1) the Llagrangian has to be

$$L = -m_0c^2\sqrt{\left(1-\frac{v^2}{c^2}\right)} + e\vec{v}.\vec{A} - e\phi. \qquad (9)$$

The corresponding relativistic Hamiltonian is

$$H = c\sqrt{\left\{m_0{}^2c^2 + \left(\vec{p} - e\vec{A})^2\right)\right\}}. + e\phi,$$

whit

$$\vec{p} = \frac{m_0\vec{v}}{\sqrt{\left(1-v^2/c^2\right)}} + e\vec{A}. \qquad (10)$$

in what follows consideration is given first to exact solution of (5) for some simple special cases, starting with uniform and constant

$$\vec{E}(x,y,z,t) = const \wedge \vec{B}(x,y,z,t) = const$$

2. Non-relativistic motion in simple in live tissues

2.1. Uniform static fields

When both $\vec{E}$ and $\vec{B}$ constant in time and uniform in space, that is, each vector always and everywhere has the same magnitude and same direction, the full solution in the non-relativistic approximation is easily obtained.

First it is noted that in a uniform electrostatic field $\vec{E}$, with no magnetic field, the particle simply travels in a straight line with constant acceleration $e\vec{E}/m$.

Next, motion in a uniform magnetostatic field $\vec{B}$,with no electric field, is considered. The speed v is then constant; so is $v_{\parallel}$,since the force is at right angles to $\vec{B}$,and so also therefore is the magnitude $v_{\perp}$ of $\vec{v}_{\perp}$,where suffices $\parallel$ and $\perp$ are used to denote components parallel and perpendicular to $\vec{B}$.Now for motion perpendicular to $\vec{B}$ the force is $ev_{\perp}B$ at right angles to

$\vec{v}$, since the acceleration at right angles to $\vec{v}$ is $v_{\perp}^2/\rho$,where ρ is radius of curvature of the projection of the path on to a plane perpendicular to B .it follows that

$$p = \frac{mv_{\perp}}{eB}, \qquad (11)$$

which is constant. Of corse,(11) is just a special case of the non-relativistic form of (4), with $p_p \sin^2\theta = p, v\sin\theta = v_{\perp}$.

The general motion is therefore an arbitrary constant velocity along $\vec{B}$, superposed on circular motion perpendicular to $\vec{B}$ described with constant angular velocity

$$\Omega = eB/m \ . \qquad (12)$$

It is readily checked that a poitively charged particle spirals in the sense of a left-handed screw about the magnetic field direction, a negatively charged particle in a right-handed sense.

The expression Ω and ρ are commonly called the (angular) gyro (or Larmor) frequency and radius. In this non-relativistic approximation Ω is independent of the energy of the particle; for an electron in the earth's magnetic field of, say, B=0.05mT it is about 8.8 $\times 10^{-6} rad\,\sec^{-1}$.But p is proportional to the square root of the energy; for a 10 eV electron in a field of B=0.05mT it is about 0.21 m.

If $\vec{E}$ and $\vec{B}$ are both present, and each is constant and uniform, the motion can be obtained in following way. Along $\vec{B}$ there is con stant acceleration $eE_{\parallel}/m$.Perpendicular to $\vec{B}$, the equation of motion (5) is

$$\vec{E}_{\perp} + \vec{u} \times \vec{B} = 0 \qquad (13)$$

giving, for motion perpendicular to $\vec{B}$,

$$\vec{u} = \frac{\vec{E} \times \vec{B}}{B^2} \qquad (14)$$

In (13) it is convenient to single out the velocity $\vec{u}$ explicitly by writing

$$\vec{v}_{\perp} = \vec{u} + \vec{v}_{\perp}^{'} \qquad (15)$$

say. The substitution of (16) into (13) then gives

$$m\dot{\vec{v}}_{\perp}^{'} = e\vec{v}_{\perp}^{'} \times \vec{B} \qquad (16)$$

and this equation for $\vec{v}_{\perp}^{'}$ is precisely what would obtain for $\vec{v}_{\perp}$ in the case $\vec{E} = 0$.

3. Sourses of low frequency magnetic field in medical therapy

Usually the above description of ion's movement is useful in medical therapy for visualization and optimization of parameters of medical apparatus. The low frequency magnetic field is provided by coils. A girdle coil is applied usually (fig. 1).

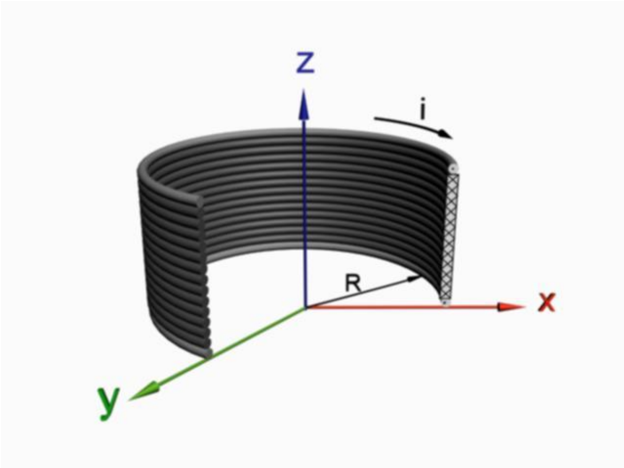

Fig. 1

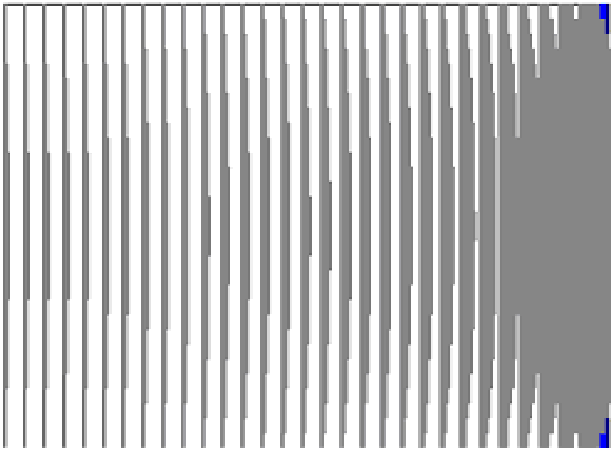

Fig. 2

The human body is in the girdle coil during the procedure. The space distribution of the module of magnetic induction in the coil can be seen on the fig. 2.

Conclusion

The general motion can therefore be visualized simply by observing that the additional motion arising from the electric field $\vec{E}$ is a constant acceleration eE_{II}/m along $\vec{B}$, and a constant velocity perpendicular to both $\vec{E}$ and $\vec{B}$, in the sense of a right-handle screw from $\vec{E}$ to $\vec{B}$, of magnitude $E_{\perp}/B$.

References

[1]. [1.] D.Tz. Dimitrov, Computer Simulation of Space Configuration of Low Frequency Magnetic Field in Magnetotherapy, p.28-32,Journal " Electronics and Electrical Engineering",Litvania, ISSN 1392-9631, 2005, -No.3(59).

[2]. [2]. D.Tz. Dimitrov, Iproving the Perfrmance of Program Package for 3D Simulation of Low Frequency Magnetic Field in Medical Therapy, p.69-72, Journal "Electronics and Electrical Engineering",Litvania, ISSN 1392-9631, 2007, No.1(73)

[3]. [3]. D.Dimitrov, Visualization of a Low Frequency Magnetic Field, generated by Girdle Coil in Magnetotherapy, p.57-60,Journal"Electronics and Electrical Engineering",Litvania, ISSN 1392-1215, 2007, -No.6(78), Fig.1.