A MOTION OF IONS IN LIVE TISSUES UNDER INFLUENCE OF PERMANENT OR LOW FREQUENCY MAGNETIC FIELD

Boncho Angelov Bonchev

Technical Univesrsity of Sofia, Bulgaria

boncho71@abv.bg

Abstract

In this paper the case is presented in which there is no electric field,and in wich the magnetic field is constant in time and everywhere has the same direction but no necessarily the same magnitude. A motion of ions in live tissues is observed..

1. Introduction

Vector of electric field $\vec{E}=0$,in rectangular Cartesian coordinates

$$\vec{B}=[0,0,B(x,y)] \qquad (1)$$

say, where the requirement div$\vec{B}$ ensures that B cannot depend on z. Unless $\vec{B}$ is uniform curl $\vec{B}$ does vanish and there is a current density associated with the field.

2. Motion of ions in uni-dirctional magnetic field

Clearly now both v and v_z and so therefore is

$$v_\perp=\sqrt{\left(v_x^2+v_y^2\right)}\ . \qquad (2)$$

Furthermore, the projection of the path of the particle on the x y-plane has radius of curvature

$$p(x,y)=\frac{mv_\perp}{eB}. \qquad (3)$$

The curvature of the projected path is therefore a constant multiple of the magnetic induction. This fact can lead to a general appreciation of the nature of the path ,and could be used for accurate computation.

To investigate the possibility of an analytic solution it is noted that the equation of motion are

$$\dot{v}_x=\Omega v_y,\ \dot{v}_y=-\Omega v_x, \qquad (4)$$

where the local giro-frequency

$$\Omega=eB/m \qquad (5)$$

is now a function of x and y ,and only motion in the x y-plane need be considered, with $v_\perp=v$.

Since equations (6) can be written

$$\frac{dv_x}{dy}=\Omega,\frac{dv_y}{dx}=-\Omega. \qquad (6)$$

it is clear that a solution by quadratures is possible if Ω depends on only one of x or y. For if it be supposed that B is independent of y, say, the second equation of (5) gives

$$v_y=-\int\Omega(x)dx, \qquad (7)$$

so that $v_x=\sqrt{\left(v^2-v_y^2\right)}$ is a known function of x, and t is given in terms of x by a further integration. Alternatively, the path can be determined directly from the fact that

$$\frac{dy}{dx}=\frac{v_y}{v_x}=\frac{v_y}{\sqrt{\left(v^2-v_y^2\right)}} \qquad (7)$$

is a known function of x.

The same results follow readily from the Lagrangian formulation. The vector potential

$\vec{A}$ = [0,A(x),0]

corresponds to magnetic induction

$\vec{B}$ = (0,0,$\frac{dA}{dx}$);

and Lagrangian formulation reads

$$L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+e\dot{y}A(x),$$

the y-coordinate is ignorable and corresponding Lagrange equation has first integral

$$\frac{\partial L}{\partial v_y}=0,$$

that is

$$mv_y+eA=consyant\ , \qquad (8)$$

which is the same as (6).

The example leading to the most elementary analytic details seems to be

$$\Omega=\frac{cons\tan t}{x^2}\ . \qquad (9)$$

For if $mu=const \wedge ux_0=const$ are chosen as the respective constants in (8) and (9), then these equations give

$$v_y=-u\left(1-x_0/x\right), \qquad (10)$$

form which (7) can evidently be integrated in terms of elementary functions. Without pursuing the details, the topology of the path can be ascertained from the fact that v_y^2 cannot exceed the constant v^2. Stated algebraically

$$\left(v^2-u^2\right)\left(x+\frac{ux_0}{v-u}\right)\left(x-\frac{ux_0}{v+u}\right)\geq 0\ . \qquad (11)$$

Since (9) is symmetric in x, and the particle cannot cross the plane x=0, there is no loss of generality in confining attention to positive values of x. let it also be assumed that ux_0>0 corresponding to a magnetic field in the positive z-direction. Then if $|u|<v$, (11) shows that the path goes to infinity and that

$$x>\frac{ux_0}{v+u}; \qquad (19)$$

if also $u>0$(implayng $x_0>0$) then (19) is less than x_0, the value of x for which v_y vanishes. If $|u|>v$,u must be positive, and the particle is confined to the range

$$\frac{ux_0}{u+v}<x<\frac{ux_0}{u-v}; \qquad (20)$$

this range contains x=x_0, where v_y vanishes.

Another comparatively simple example of some interest is that in which the magnetic field varies linearly with x. If, say.

$$\Omega=\frac{2|u|}{x_0^2}x\ , \qquad (21)$$

Then (6) can be written

$$v_y=u-\frac{|u|}{x_0^2}x^2\ , \qquad (15)$$

where u is an arbitrary constant and the constant x_0 can be taken as positive. The solution of (7) can be expressed in terms of elliptic integrals, but again the general picture is most easily obtained from the fact that v_y^2 cannot exceed v^2. For this now reads

$$\left(v^2-u^2\right)\left[1+\frac{|u|x^2}{(v-u)x_0^2}\right]\left[1-\frac{|u|x^2}{(v+u)x_0^2}\right]\geq 0, \quad (16)$$

which shows that three cases can be distinguished as follows. If $|u|<v$, then

$$x^2 \leq \frac{v+u}{|u|}x_0^2 \quad (17)$$

and the particle is confined between the two planes x=$\pm\sqrt{\left[(v+u)/|u|\right]}x_0$; if, also, u>0, the range of x includes $\pm x_0$, the values of x for which v_y vanishes.

Suppose, now, that instead of being a function of x only the field is a function only of $r=\sqrt{\left(x^2+y^2\right)}$, that is, there is cylindrical symmetry about the z-axis. To conclude this sub-section it is noted that this is a second case where solution by quadratures is possible, and a specific example is considered.

In cylindrical polar coordinates r,θ,z, with

$$\vec{B}=\left[0,0,B(r)\right], \quad (22)$$

the equation of motion in the θ-direction is

$$\frac{1}{r}\frac{d}{dt}(r^3\dot{\theta})=-\Omega\dot{r},$$

giving

$$v_\theta = r\dot{\theta}=-\frac{1}{r}\int r\Omega(r)dr. \quad (23)$$

Then

$$\dot{r}^2=v_r^2=v^2-v_\theta^2$$

is a known function of r, and t is given in terms of r by a further integration. Or again, the equation of the path is given directly by integrating

$$r\frac{d\theta}{dr}=\frac{v_0}{\sqrt{v^2-v_\theta^2}}. \quad (24)$$

For the Lagrangian formulation the vector potential in cylindrical polar coordinates can be taken as

$$\vec{A}=[0,A(r),0], \quad (25)$$

where

$$B=\frac{1}{r}\frac{d}{dr}(rA). \quad (26)$$

Then

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)+eA(r)r\dot{\theta},$$

so that θ is ignorable and

$$\frac{\partial L}{\partial\theta}=const,$$

which is readily seen to be identical with (23).

An example for which the analytic details are elementary is A = constant. If the constant is written as $\vec{A}=\frac{mu}{e}=const$, then

$$\Omega=u/r, \quad (27)$$

and (23) gives

$$v_0=-u\left(1-r_0/r\right), \quad (28)$$

where u can be taken positive, but r_0 may be positive or negative. The analysis is evidently virtually the same as that in the first example considered in this sub-section, which was based on equation (10).The requirement that v_θ^2 cannot exceed the constant v^2 shows that there are three types of path.

3. Application of low frequency magnetic field in medicine

The motions of ions under permanent or low frequency magnetic field is used often in medicine for treatment of knee. (fig. 1). The low frequency magnetic field in this case is provided by two inductors.

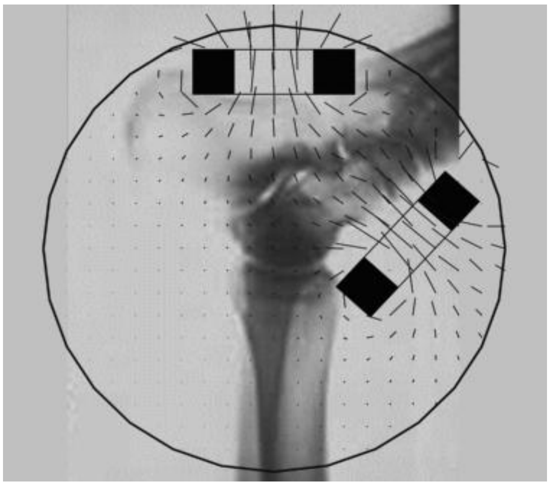

Fig. 1

Conclusion

The conclusion exemplifies the main results and the fundamental ideas presented in the paper. It should allow to fully recognize the goals of the presentation. In the conclusion, papers concerning educational activities should include an opening to the education community.

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