CALCULATION RESULTS OF SCATTERING OF ELECTROMAGNETIC WAVES FROM RECTANGULAR PERFECTLY CONDUCTING PLATE USING AN EXTENDED THREE DIMENSIONAL STATIONARY PHASE METHOD WHICH IS BASED ON FRESNEL FUNCTIONS (SPM-F)

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Abstract

Radiocoverage simulation tools require multiple scattering radio propagation calculations in urban outdoor areas. The simulation results, if performed numerically, are usually time consuming due to the multiple numerical calculations included in their core scattering formulas. The main goal of this paper is to present the calculation results of an extended three dimensional Stationary Phase Method which is based on Fresnel functions (SPM-F). In previous conferences we presented a novel three dimensional high frequency analytical method for the calculation of the scattered electromagnetic (EM) field from a Perfect Electric Conductor (PEC) plate, which is based on the Physical Optics (PO) approximation and the Stationary Phase Method (SPM) approximation. This method (SPM) showed fast and accurate results, nevertheless its accuracy around the area of the main scattering lobe could be further enhanced by utilizing higher order approximation terms, based on the use of Fresnel functions (SPM-F). Then, proximity of the stationary point to the edges of the scatterer does not constitute a problem anymore, since the troublesome vanishing denominators have now been removed.

1. INTRODUCTION

Radio coverage prediction models [1] require several calculations of scattering of electromagnetic waves. In references [2]-[3] we have developed and presented an enhanced three dimensional analytical formulation for the calculation of the vector potential and eventually the scattered electric field of a geometry presented in Fig. 1. The method is based on the Stationary Phase Method (SPM) and its novelty lies on the inclusion of the edge contribution to the resulting asymptotic expressions, a fact which was not documented in the literature for the case of a double integral. In this paper we present a higher order approximation term for the edge contribution, which is based again on the Stationary Phase Method, but also includes Fresnel functions (SPM-F). Vector potential and scattered electric field calculations appear in section 2, below, while the general idea of this new approach is presented in section 3 of this paper. The Fresnel functions in SPM analytical results have not been documented either in literature. In section 4, below, simulation results are compared to results obtained with standard numerical integration and SPM results that do not include the Fresnel functions, for the case of single and double integrals and for small/large scatterers in the areas of Near field, Fresnel Zone and Far Field. SPM-F method presented here, is found to be equally fast to SPM, but more accurate in the area of the main scattering lobe, where proximity of the stationary point to the edges and vanishing denominators in edge contribution equations was causing some inaccuracies to the earlier derived SPM results.

The generalization of the method leads to a three dimensional model which works properly, if both transmitter and receiver are located below rooftop level and well above the ground. Then the three dimensional scattering could be modeled through the prerequisite problem of an electromagnetic (EM) wave which is assumed to be incident on a perfectly conducting rectangular plate of finite dimensions which in real life happens to be the road-side of a three dimensional building.

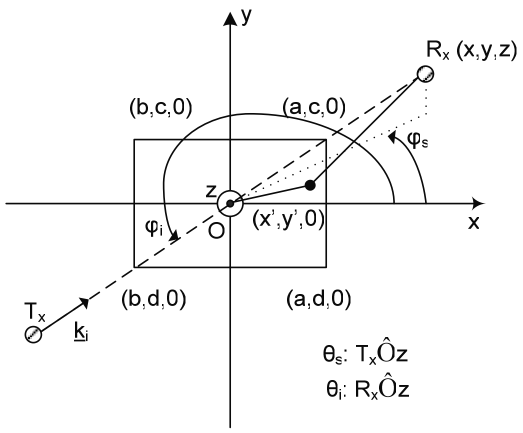

Figure 1 - 3D geometry xy plane projection

Let us consider the observation point $R_x(x,y,z)$ in a propagation problem layout of an electromagnetic (EM) wave with wavevector k_i, which is assumed to be incident on a perfectly conducting rectangular plate of finite dimensions (Fig.1).

This three dimensional scatterer is the necessary prerequisite in order to model propagation in an urban outdoor environment, which consists of three dimensional walls and obstacles that often pertain to modern high frequency communication wireless networks, such as GSM, UMTS, Wi-Fi and Wi-Max technologies. Should the rectangular plate lies on the xy plane, and since the operating frequency is considered to be 1GHz or higher, the following calculations regarding vector potential and scattered electric field hold.

2. VECTOR POTENTIAL $\vec{A}$ AND SCATTERED ELECTRIC FIELD $\vec{E}$

According to the layout in Fig. 1 the induced current density on the surface of the plate after applying P.O. theory yields the following expression:

$$\underline{J}_s^{P.O.}(x',y') = \frac{-2e^{jkh}}{\eta}[\hat{x}(E_{0\theta}\cos\phi_i - \mathrm{E}_{0\phi}\cos\theta_i\sin\phi_i) + \hat{y}(E_{0\theta}\sin\phi_i + E_{0\phi}\cos\theta_i\cos\phi_i)] \quad (1)$$

By utilising the three dimensional Green's function given below:

$$G(r,r') = \frac{\exp(-j\cdot k\cdot|r-r'|)}{4\pi\cdot|r-r'|} \quad (2)$$

the vector potential $\vec{A}$ at the observation point $R_x(x,y,z)$ is given by:

$$\underline{A}(x,y,z) = \mu_0\cdot\int_{y'=d}^{c}\int_{x'=b}^{a}\left[\bar{J}_s^{P.O.}(x',y')\cdot G(r,r')\right]\cdot dx'dy' \quad (3)$$

Substitution of total current density yields the following expression for the vector potential:

$$\underline{A}(x,y,z) = \frac{-\mu_0}{2\pi\cdot\eta}\cdot [\hat{x}\cdot(E_{0\theta}\cdot\cos\phi_i - \mathrm{E}_{0\phi}\cdot\cos\theta_i\cdot\sin\phi_i) + \hat{y}\cdot(E_{0\theta}\cdot\sin\phi_i + E_{0\phi}\cdot\cos\theta_i\cdot\cos\phi_i)]\cdot \int_{y'=d}^{c}\int_{x'=b}^{a}\frac{\exp\left\{j\cdot k\left[\begin{array}{l}(x'\cdot K + y'\cdot L) - \\ -\sqrt{(x-x')^2+(y-y')^2+z^2}\end{array}\right]\right\}}{\sqrt{(x-x')^2+(y-y')^2+z^2}}dx'dy' \quad (4)$$

since it is easily obtained from Fig. 1 that

$$|\underline{r}-\underline{r'}| = \sqrt{(x-x')^2+(y-y')^2+z^2} \quad (5)$$

and K, L constants which depend on the incident angles θ_i and φ_i satisfying Eqs. 6-7:

$$K = \sin\theta_i \cdot \cos\phi_i \quad (6)$$

$$L = \sin\theta_i \cdot \sin\phi_i \quad (7)$$

Finally, the scattered electric field is calculated from the formula:

$$\underline{E}_s(x,y,z) = = -j \cdot \omega \cdot \underline{A} - j \cdot \frac{\omega}{k^2} \cdot grad\left(div\left(\underline{A}\right)\right) \quad (8)$$

3. SPM-F METHOD

Operating at the frequency of 1GHz or higher, scatterers that appear in the above networks are considered to be electrically large, and current density may be calculated with good accuracy using the physical optics (P.O.) approximation. Modifying appropriately Eq. 4 we can apply SPM approximations which result in calculating the vector potential and eventually the total scattered electric field at the observation point $R_x(x,y,z)$. The amplitude and phase functions we use are derived in Eqs. 9-10 respectively:

$$F(x',y') = \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \quad (9)$$

$$f(x',y') = x'K + y'L - -\sqrt{(x-x')^2 + (y-y')^2 + z^2} \quad (10)$$

As described in [2]-[3], the inner integral terms of the three dimensional modified Stationary Phase Method which perform the calculation of the diffraction from the edges of the PEC plate (a, b) and (c ,d), should be replaced [4] by higher order approximation terms that imply the use of Fresnel functions, i.e. the substitution for term I_a should be:

$$\tilde{I}'_a = -\frac{1}{jk} \frac{F(a,y')}{\left.\frac{\partial f}{\partial x'}\right|_{x'=a,y'=y'}} \exp\{jkf(a,y')\} + O\left(k^{-2}\right) \leftrightarrow$$

$$\tilde{I}'_a = \varepsilon_1 \cdot F(a,y') \exp\left\{jkf(a,y') \mp ju_a^2\right\} \times \times \sqrt{\frac{2}{k\left|\left.\frac{\partial^2 f}{\partial x'^2}\right|_{x'=a,y'=y'}\right|}} F_\pm[u(a,y')] \quad (11)$$

where, in the above replacement, the new term represents a higher order approximation that includes a Fresnel function

$F_\pm(u)$ [$F_\pm(u) = \int_u^\infty \exp\left(\pm jt^2\right) dt$] and

$$\varepsilon_1 = \operatorname{sgn}(a - x_o) \quad (12)$$

$$u_a = \sqrt{\frac{k}{2\left|\left.\frac{\partial^2 f}{\partial x'^2}\right|_{x'=a,y'=y'}\right|}} \left|\left.\frac{\partial f}{\partial x'}\right|_{x'=a,y'=y'}\right| \quad (13)$$

The replacement for term I_b is given by:

$$\tilde{I}'_b = -\frac{1}{jk} \frac{F(b,y')}{\left.\frac{\partial f}{\partial x'}\right|_{x'=b,y'=y'}} \exp\{jkf(b,y')\} + O\left(k^{-2}\right) \leftrightarrow$$

$$\tilde{I}'_b = \varepsilon_2 \cdot F(b,y') \exp\left\{jkf(b,y') \mp ju_b^2\right\} \times \times \sqrt{\frac{2}{k\left|\left.\frac{\partial^2 f}{\partial x'^2}\right|_{x'=b,y'=y'}\right|}} F_\pm[u(b,y')] \quad (14)$$

$$\varepsilon_1 = \operatorname{sgn}(b - x_o) \quad (15)$$

$$u_b = \sqrt{\frac{k}{2\left|\left.\frac{\partial^2 f}{\partial x'^2}\right|_{x'=b,y'=y'}\right|}} \left|\left.\frac{\partial f}{\partial x'}\right|_{x'=b,y'=y'}\right| \quad (16)$$

In the case of scattering from a rectangular plate in three dimensions, the above formulas will escalate accordingly to all eight terms of the modified Stationary Phase Method as described in [2],[3].

The idea of our calculation approach has been presented in [2], [3]. The initial integral is extended to infinite for both x' and y' variables. This extension primarily enables us to utilize the standard SPM formulas, well known for an infinite double integral. Since the plate is finite, additional correction terms should be included in the final SPM result. Those terms are obtained by subtracting the remaining areas around the finite plate. For those terms, the results are computed by utilizing a combination of SPM and edge contribution side formulas, that include the Fresnel functions and which have only been documented for a single integral. By applying such calculations twice, we produce new amplitude and phase functions on which we re-apply SPM approximations, and finally result in analytical solutions for a double integral using Fresnel functions for the edge contribution.

4. SIMULATION RESULTS

The accuracy of our Fresnel asymptotic calculations can be evaluated by using, standard (e.g. Gaussian) numerical integration. We present here a set of results based on three different methods. SPM-F results are drawn in cyan line and include the Fresnel functions in edge contribution, SPM results drawn in red line (also presented in [2], [3]), and numerical integration results drawn in blue line. Due to the complexity of the functions on which SPM is applied, the calculations were carried out using MATLAB's symbolic toolbox. Analytical results derived from MATLAB calculations constitute an aggregation of complicated formulas. The total scattered electric field was calculated for distances in the Far Field, Fresnel Region and Near Field area. In Figs. 2–13 we compare SPM-F, SPM and numerical integration for the frequency of 1GHz and for the case of single and double integrals. Line segments (single integrals) and rectangular plates (double integrals) with side length equal from 20λ to 80λ were examined in our simulation results. Elevation and azimuth angles of incidence were assumed equal to 45 degrees.

4.1. Single Integral

4.1.1. Small Scatterer

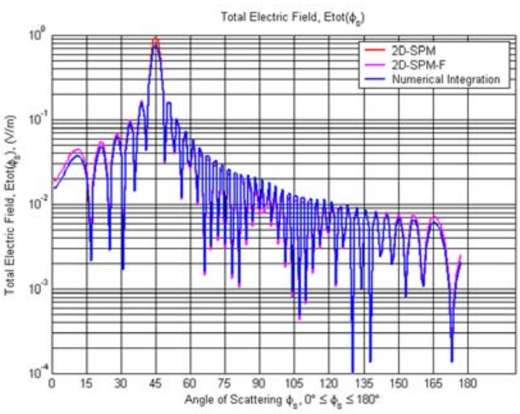

Figure 2. 20λ line segment, Far Field area

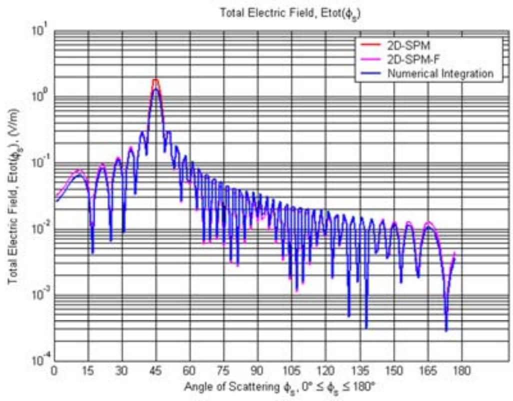

Figure 3. 20λ line segment, Fresnel area

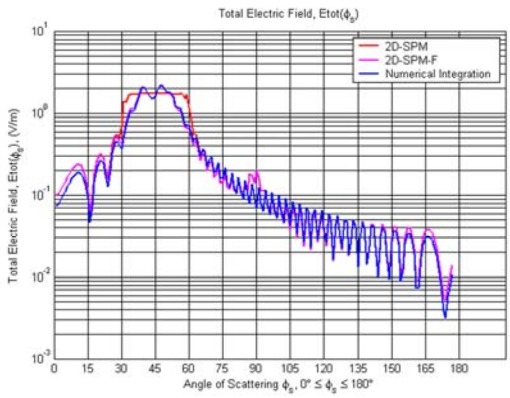

Figure 4. 20λ line segment, Near Field area

In Figs. 2–4 above, numerical results are shown for a rectangular line segment of dimension 20λ in the Far Field area (r=300m), the Fresnel area (r=100m) and the Near Field area (r=10m).

4.1.2.. Large Scatterer

In Figs. 5–7, we provide numerical results for the case of a larger scatterer of dimension 80λ in the Far Field area (r=4500m), the Fresnel area (r=1000m) and the Near Field area (r=100m).

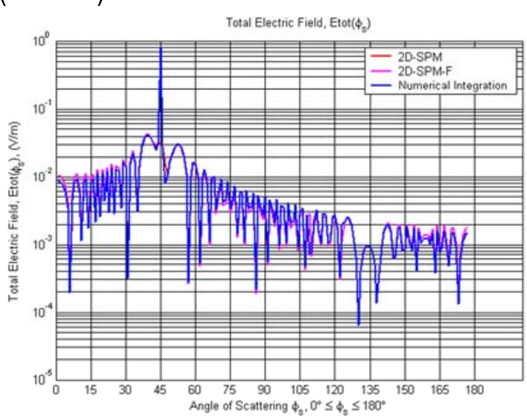

Figure 5. 80λ line segment, Far Field area

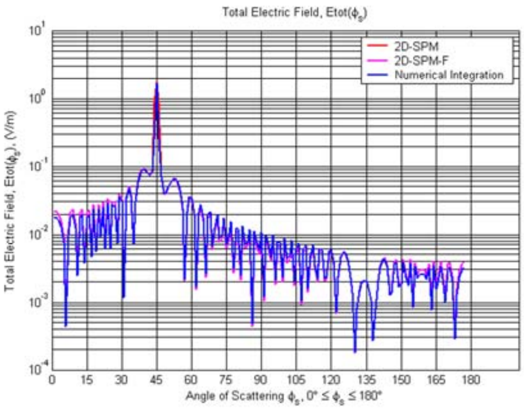

Figure 6. 80λ line segment, Fresnel area

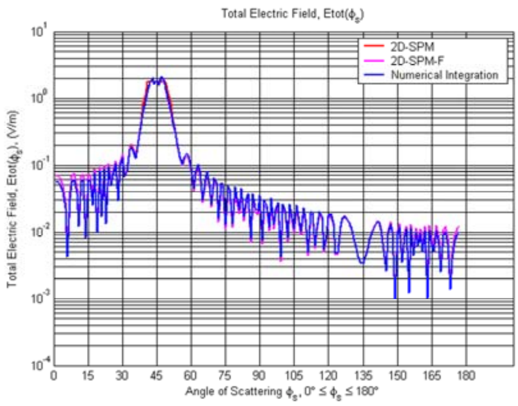

Figure 7. 80λ line segment, Near Field area

4.2. Double Integral

4.2.1. Small Scatterer

Scatterer of dimension 20λ in the Far Field area (r=600m), the Fresnel area (r=100m) and the Near Field area (r=25m) (Figs 8-10).

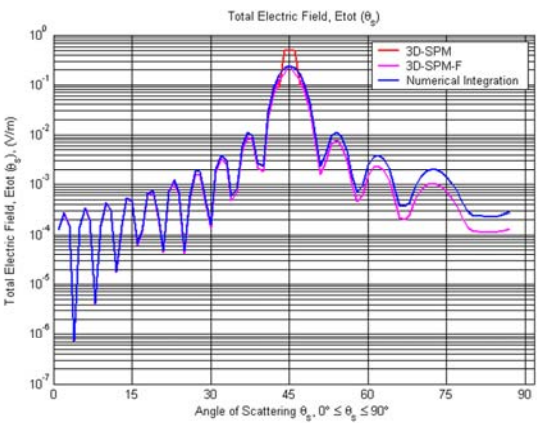

Figure 8. 20λ plate side, Far Field area

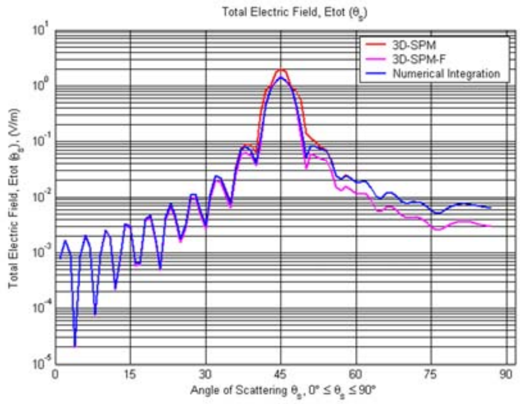

Figure 9. 20λ plate side, Fresnel area

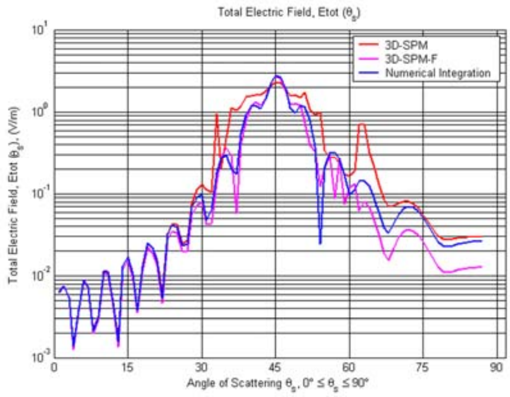

Figure 10. 20λ plate side, Near Field area

4.2.2.. Large Scatterer

Scatterer of dimension 80λ in the Far Field area (r=9000m), the Fresnel area (r=500m) and the Near Field area (r=200m) (Figs. 11-13).

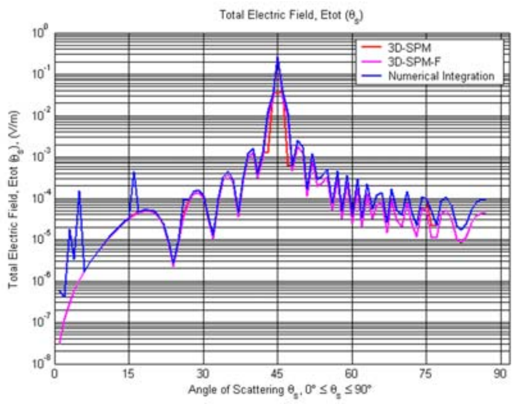

Figure 11. 80λ plate side, Far Field area

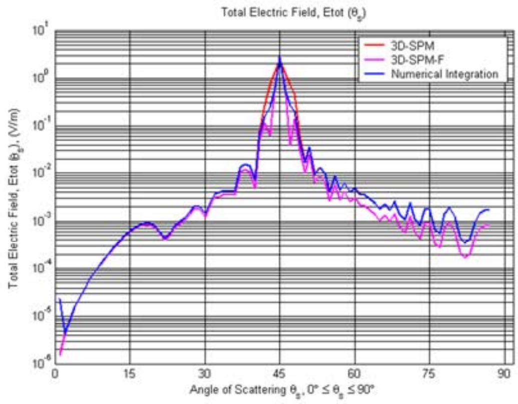

Figure 12. 80λ plate side, Fresnel area

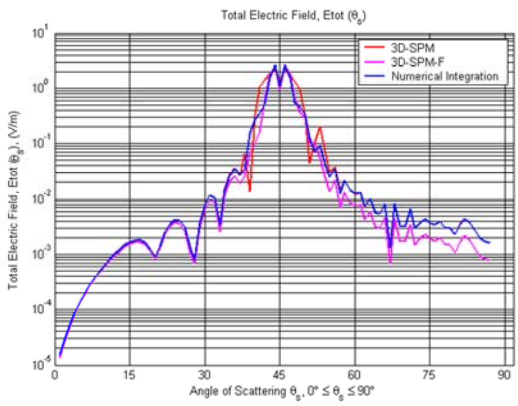

Figure 13. 80λ plate side, Near Field area

Comparison graphs in Figs. 2 – 13 above indicate the improved behaviour around the area of the main scattering lobe between the asymptotic results of the SPM and SPM-F methods. Note here that for large angles of scattering θ_s (θ_s>60°) parameters u_a, u_b in Eqns. (13),(16) are larger than 3.0 (u_a,u_b>3.0) thus the replacement formulas in Eqns. (11),(14) are not uniform [4], and equations without Fresnel functions [2],[3] must be used for best accuracy (i.e. combination formulas, which are currently developed by our research group).

5. CONCLUSION

Even though rather complicated mathematical formulas are involved in the proposed SPM-F method, it is much faster than the numerical integration method (3-150 times faster). This is very important for a simulated propagation problem in an urban outdoor environment, in which case many scatterers (walls) and multiple reflection phenomena are present. Furthermore, SPM-F method presented here was found also to be very accurate for the calculation of vector potential $\vec{A}$ and electric field $\vec{E}$ in various radio propagation simulation tools.

References

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