MOVEMENT OF CHARGED PARTICLE IN ALIVE TISSUES UNDER EXTERNAL ELECTRICAL FIELD

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Abstract

This article treats the motion of charged particles, mainly electrons or protons, composed of a uniform axial component, in an external electric field. These equations can be the basis of computer programs to calculate particle trajectories in the general case, a valuable resource, but such calculations do not give a general understanding of particle motion. This article is intended to foster such an understanding as an aid to reasoning about natural phenomena.

INTRODUCTION

In this article certain aspects are considered of the motion in a vacuum of a point charge under the influence of on electric field.

MAIN TEXT

The classical motion of the particle in such a field is determined by Newton's equation of motion, where the force is given by:

$$a = \frac{F}{m} = \frac{q}{m}(E + vXB) \qquad (1)$$

The problem that presents itself is that of solving the equations of motion given in relativistic from

$$\frac{d}{dt}\left\{\frac{m_o v}{\sqrt{(1-v^2/c^2}}\right\} = e(E + v\mathrm{X}B) \quad (2)$$

The problem becomes, in principle, straightforward if it be assumed that E,B appearing in the right-hand side of (2) is a given field; the difficulties are then of the purely technical kind that can also arise in traditional particle mechanics with mechanical forces. The assumption is often a legitimate approximation, and most of the work presented here is in this context. In fact, however, the charge itself also contributes to the electromagnetic field, and its contribution depends on its motion. The inclusion of this “self-force” presents quite fundamental difficulties because of the infinities associated with the concept of a point charge. Attempts to describe the motion of an electron can perhaps be classified in terms of the following alternatives: (I) abandon the idea that the electron is a point charge, and give it some internal structure; (II) introduce into the classical mathematical framework some formalism that succeeds in discarding the infinities. The (II) has led to an equation of motion which at least in certain contexts is acceptable theoretically an is in general agreement with practice.

In attempting to solve (1) it may be helpful to make use of the associated energy equation

$$\frac{d}{dt}\left\{\frac{m_o c^2}{\sqrt{(1-v^2/c^2}}\right\} = e(E.v) \qquad (3)$$

If the electric field is purely static, so that E =-grad φ, this gives

$$\frac{m_o c^2}{\sqrt{(1-v^2/c^2}} + e\phi = const \qquad (4)$$

If we have an uniform electrostatic field (E,0,0), If the motion is confined to the x-axis the equation of motion integrates at once to

$$\frac{m_o v}{\sqrt{(1-v^2/c^2}} = eEt \qquad (5)$$

with appropriate choice of time zero. The equation can easily be solved for v in terms of t and integrated again. But it is even quicker to use also the energy equation (4), which here is

$$\frac{m_o c^2}{\sqrt{(1-v^2/c^2}} = eEx \qquad (6)$$

with appropriate choice of origin. For the elimination of v between (5) and (6) gives immediately

$$x = c\sqrt{\left(\frac{m_0{}^2 c^2}{e^2E^2} + t^2\right)} \qquad (7)$$

In the motion specified by (7) the speed tends to the limit c as t->±∞. As t comes up zero the particle moves toward the origin along the positive x –axis with a deceleration which brings it to rest at t=0; it then accelerates back on the return journey with a speed that approaches c asymptotically. The motion is, in fact, that sometimes called *hyperbolic,* in which the space part of the relativistic four-acceleration is constant.

The general motion in the electrostatic field (E,0,0) can be found in the such same way. Choose the x-axis in the direction of the electric field, and the y-axis so that the xy-plane contains both the electric field and the initial velocity. Since the acceleration normal to this plane is zero, and the initial velocity normal to it is zero, the motion remains in this plane, say z=0. Then, with appropriate choice of time zero, the x- and y-components of the equation of motion integrate to

$$\frac{m_o x}{\sqrt{(1-v^2/c^2}} = eEt \qquad (8)$$

$$\frac{m_o y}{\sqrt{(1-v^2/c^2}} = p_0 \qquad (9)$$

where the constant p0 is the initial value of the relativistic momentum, and $v^2 = x^2 + y^2$. By squaring and adding

$$\frac{m^2{}_o v^2}{1-v^2/c^2} = e^2E^2t^2 + p_0{}^2 \qquad (10)$$

and the elimination of v^2 between (10) and the energy equation (6) gives

$$x = c\sqrt{\left(\frac{p^2 + m_0{}^2c^2}{e^2E^2} + t^2\right)} \qquad (11)$$

Also from (9) an (6)

$$y = \frac{c^2 p_0}{eE} x$$

And substitution for x from (11) followed by integration results in

$$y = \frac{cp_0}{eE}\sinh^{-1}\left\{\frac{eEt}{\sqrt{(p_0{}^2 + m_0{}^2c^2)}}\right\} \qquad (12)$$

The equation of the path is obtained by elimination t between (11) and (12). It can be written

$$x = \frac{c\sqrt{(p_0{}^2 + m_0{}^2c^2\ )}}{eE}\cosh\left(\frac{eEy}{cp_0}\right) \qquad (13)$$

The solution of this equation, subject appropriate initial conditions, give the path of particle resulting from action of the electric forces. It should be remarked that the motion (11) and (12) is not hyperbolic. General hyperbolic motion, an a plane rather than along a line, could be derived from (7) by a Lorenz transformation; and such motion would be that particle was acted on by the field twat was the transformation of (E,0,0), a filed which would, in fact, be partly magnetic.

CONCLUSION

A solution is proposed of tracing the path of a charged particle resulting from action of the electric forces.

References

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