MOVEMENT OF IONS IN ALIVE TISSUES UNDER UNIFORM MAGFNETIC FIELD OF APPARATUS FOR MAGNETOTHERAPY AND UNIFORM ELECTRICAL FIELDS

Boncho A. Bonchev

Technical University of Sofia, Kl. Ohridsky str.8, 1000 Sofia

Abstract

Usually in physiotherapy application of magnetic or electrical field is the main reason for movement of ions in alive tissues. The trajectories of movement of ions depend to the space-temporal configuration of electromagnetic field. Therefore the first step in the process of investigation of movement of ions should be determination of space-temporal configuration of electromagnetic field. This is one complicated mathematical task, but it can be solved more easy in the case of uniform electrical and magnetic field. This method can be used for mathematical description of space-temporal configuration of electromagnetic field for one enough small volume as part of alive tissues.

1. MOVEMENTS OF IONS IN UNIFORM MAGNETIC FIELD

$$E = 0 \wedge B(x, y, z) = const \qquad (1)$$

Because of that, a charged particle has a simple cyclotron gyration. The equation of motion is:

$$m\frac{dv}{dt} = qv \times B \qquad (2)$$

Where:
m is the masse of charged particle;
v is the velocity charged particle;
q is the electrical charge of particle;
B is the magnetic induction in the point were is situated the charged particle;

Taking $\hat{z}$ to be the direction of B (B=B $\hat{z}$):

$$m\dot{v}_x = qBv_y\,;\quad m\dot{v}_y = -qBv_x$$
$$m\dot{v}_z = 0$$

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x \qquad (3)$$
$$\ddot{v}_y = \frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

This describes a simple harmonic oscillator at the cyclotron frequency ω_c.

Where:

$$\omega_c \equiv \frac{|q|B}{m} \qquad (4)$$

By the convention ω_c can be always non-negative. Then the solution of equations (3) is:

$$v_{x,y} = v_\perp \exp\left(\pm i\omega_c t + i\delta_{x,y}\right) \qquad (5)$$

the $\pm$ denoting the sign of electrical charge q. The phase δ can be choose, so that:

$$v_x = v_\perp e^{i\omega_c t} = \dot{x} \qquad (6)$$

Where:
$v_\perp$ is a positive constant denoting the speed in the plane perpendicular to the vector of magnetic induction B.
Then:

$$v_y = \frac{m}{qB}\dot{v}_x = \pm\frac{1}{\omega_c}\dot{v}_x = \pm i v_\perp e^{i\omega_c t} = \dot{y} \qquad (7)$$

After integration of equation (6) can be obtained:

$$x - x_0 = -i\frac{v_\perp}{\omega_c}e^{i\omega_c t} \qquad (8)$$

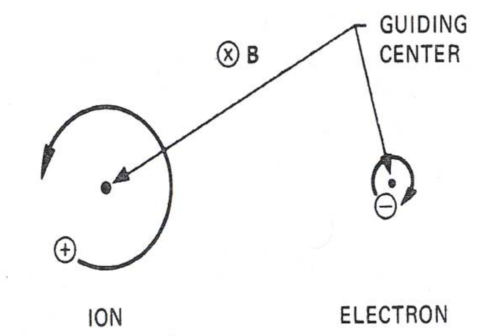
$$y - y_0 = \pm \frac{v_\perp}{\omega_c} e^{i\omega_c t}$$

The radius of rotation of charged particle (Larmor radius on fig.1) is:

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B} \qquad (9)$$

Taking the real part of equation (7) can be obtained:

$$x - x_0 = r_L \sin \omega_c t$$
$$y - y_0 = \pm r_L \cos \omega_c t \qquad (10)$$



Larmor orbits in a magnetic field.

Fig. 1

It's clear that the equation (8) describes a circular orbit a guiding center (x_0,y_0) which is fixed (Fig.1).The direction of the gyration is always such that the magnetic field generated by the charged particle is opposite to the externally imposed field. The particles in alive tissue, therefore, tend to reuse the magnetic field, and alive tissues are diamagnetic. In addition to this motion, there is an arbitrary velocity v_z along the direction of the vector of magnetic induction B, which is not affected by B. The trajectory of charged particle in alive tissue is, in general, a helix.

2. MOVEMENTS OF IONS IN UNIFORM MAGNETIC FIELD AND UNIFORM ELECTRICAL FIELD

$$E(x,y,z) = const \wedge B(x,y,z) = const \qquad (11)$$

In this case, the motion of charged particle will be found to be the sum of two motions: the usual circular Larmor gyration plus a drift of the guiding center. The vector of intensity of electrical field E can be choose to lie in the x-z plane so that E_y=0. As before, the z component of velocity is unrelated to the transverse components and can be treated separately. The equation of motion of charged particle is:

$$m \frac{dv}{dt} = q(E + v \times B) \qquad (12)$$

Whose z component of the velocity is:

$$\frac{dv_z}{dt} = \frac{q}{m} E_z$$

or (13) $v_z = \frac{qE_z}{m} t + v_{z0}$

This is straightforward acceleration along magnetic induction B. The transverse components of equation (12) are:

$$\frac{dv_x}{dt} = \frac{q}{m} E_x \pm \omega_c v_y$$
$$\frac{dv_y}{dt} = 0 \mp \omega_c v_x \qquad (14)$$

If $E(x,y,z,t) = const$:

$$\ddot{v}_x = -\omega_c^2 v_x \qquad (15)$$

$$\ddot{v}_y = \mp \omega_c \left(\frac{q}{m} E_x \pm \omega_c v_y \right) = -\omega_c^2 \left(\frac{E_x}{B} + v_y \right)$$

It can be write as:

$$\frac{d^2}{dt^2} \left(v_y + \frac{E_x}{B} \right) = -\omega_c^2 \left(v_y + \frac{E_x}{B} \right) \qquad (16)$$

Therefore, that equations (15) is reduced to the previous case if replacement of, v_y by $v_y + (E_x / B)$.

Equations (5) and (6) is therefore replaced by:

$$v_x = v_\perp e^{i\omega_c t}$$
$$v_y = \pm i v_\perp e^{i\omega_c t} - \frac{E_x}{B} \qquad (17)$$

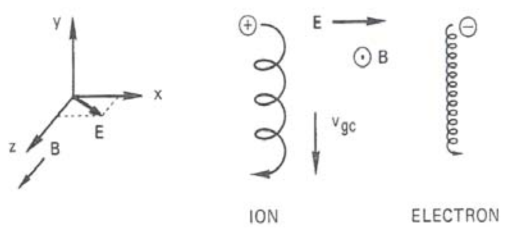


Fig.2. Particle drifts in crossed electric and magnetic fields

The Larmor motion is the same as in the case where $E = 0$, but there is superimposed a drift v_{gc} of the guiding center in the –y direction (for E_x>0), Fig.2

The equation (12) should be solved for obtaining a general formula for the velocity Vgc. The equation (12) can be solved in vector form. It's possible to omit the m dv/dt term in equation (12), since this term gives only the circular motion at wc., which we already know about. Then equation (12) becomes:

$$E + v \times B = 0 \quad (16)$$

Taking the cross product with magnetic induction B:

$$E \times B = B \times (v \times B) = vB^2 - B(v.B) \quad (17)$$

The transverse components of this equation are.

$$v_{\perp gc} = E \times B / B^2 \equiv v_E \quad (18)$$

Where:
v_E is the electric field drift of the guiding center.
In magnitude, this drift is:

$$v_E = \frac{E(V/m)}{B(tesla)} = \left[\frac{m}{\sec}\right] \quad (19)$$

It is important to note that velocity v_E is independent of electrical charge q, masse m, and velocity $v_\perp$. The reason is obvious from the following physical picture. In the first half cycle of the ion's orbit in Fig. (2), it gains energy from the electric field and increases in velocity $v_\perp$ and hence in radius r_L. In the second half-cycle it losses energy and decreases in radius r_L. This difference in r_L on the left and right sides of the orbit causes the drift with velocity v_E. A negative ion gyrates in the opposite direction but also gains energy in the opposite direction. For particles of the same velocity but different mass, the lighter one will have smaller radius r_L and hence drift less per cycle. However, its gyration frequency is also larger, and the two effects exactly cancel. Two particles of the same mass but different energy would have the same cyclotron frequency ω_c. The slower one will have smaller radius r_L and hence gain less energy from E in a half-cycle. However, for less energetic particles, the fractional change of radius r_L for a given change in energy is larger, and these two effects cancel.

CONCLUSION

1. A mathematical analysis of movement of ions in alive tissues under uniform electrical and magnetic field is described in the paper.
2. The trajectories of movement of ions have been obtained.
3. An analysis of connection between parameters of ion's trajectories and masse of ions, electrical charge of ions, magnetic induction and intensity of electrical field have been obtained.

References

[1] Sabin, M.A. (1985) Contouring - the State of the Art, in *Fundamental Algorithms for Computer Graphics*, R.A. Earnshaw (ed.), Springer Verlag, pp. 411-482

[2] Levoy, M. (1988) Display of Surfaces from Volume Data, *IEEE Computer Graphics and Applications* 8(3), pp. 29-37

[3] Lorensen, W.E., H.E. Cline (1987) Marching Cubes: a High Resolution 3D Surface Construction Algorithm, Proceedings Siggraph '87, *Computer Graphics* 21(4), pp. 163-169

[4] R.M. Kirby, H. Marmanis, and D. Laidlaw, Visualizing multivalues data from 2D incopressible flows using concepts from painting, In *Visualization 99*, pp. 333-340, October 1999.

[5] U. Adamy, J. Giesen, and M. John, New techniques for topolgically correct surface reconstruction, In *Visualization 2000*, pp. 373-380, October 2000.

[6] D.M. Weinstein, G.L. Kindlmann, and E.C. Lundberg, Tensorlines: Advection-diffusion based propagation through diffusion tensor fields, In *Visualization 99*, pp. 249-250, October 1999.

[7] H. Garcke, T. Preußer, M. Rumpf, A. Telea, U. Weikrad, and K. Van Wijk. A continuous clustering method for vector fields, In *Visualization 2000*, pp. 351-358, Oct 2000.