MOVEMENT OF IONS IN ALIVE TISSUES UNDER NONUNIFORM MAGFNETIC FIELDS CREATED BY APPARATUS FOR MAGNETOTHERAPY

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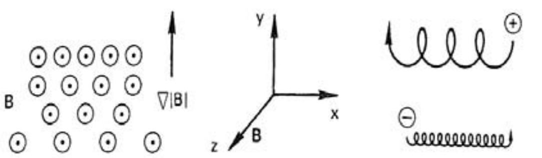
Abstract

It's well known that in the case of real application of electromagnetic field in physiotherapy, the electromagnetic field in alive tissues usually is non-uniformed. Therefore it's difficult to provide a precise mathematical descriptions, but it's possible to solve the problem in same cases as movement of ions under magnetic field with curved lines and invariant value of magnetic induction B (curvative drift) or movement of ions under magnetic field of magnetic mirrors. A theoretical investigations on movement of ions in alive tissues in these two cases is described in the paper

1. INTRODUCTION

Now the concept of a guiding center drift is firmly established. It's possible to discuss the motion of particles in inhomogeneous field – E and B fields, which vary in space or time. For uniform fields can be obtained exact expressions for the guiding center drifts. In the case of inhomogeneity, the problem becomes too complicated to solve exactly.



The drift of a gyrating particle in a nonuniform magnetic field. FIGURE 2

Fig. 1

To get an approximate answer, it is customary to expand in the small ratio r_L/L, where L is the scale length of the inhomogeneity. This type of theory, called orbit theory, can become extremely involved. It would be possible to examine only the simplest cases, where only one inhomogeneity occurs at a time $\nabla B \perp B$.

Here the lines of force are straight, but their density increases, say in the y direction (Fig. 1). It would be possible to anticipate the result by using our simple physical picture. The gradient in |B| causes the Larmor radius to be larger at the bottom of the orbit than at the top, and this should lead a drift , in opposite directions for ions and electrons, perpendicular to both B and ∇B. The drift velocity should obviously be proportional to r_L / L and to $v_\perp$. Consider of the Lorenz force $\mathrm{F} = \mathrm{qv} \times \mathrm{B}$ averaged over a gyration. Clearly, $\overline{F}_x = 0$, since the particle spends as much time moving up as down. It's possible to calculate $\overline{F}_y$, in an approximate fashion, by using the undisturbed orbit of the particle to find the average. IIn this case:

$$F_y = -qv_x B_z(y) = \\ = -qv_\perp (\cos\omega_c t)\left[B_0 \pm r_L(\cos\omega_c t)\frac{\partial B}{\partial y}\right] \quad (1)$$

It possible to made a Taylor expansion of B field about the point $x_0 = 0$, $y_0 = 0$:

$$B = B_0 + (r.\nabla)B + ... \\ B_z = B_0 + y(\partial B_z / \partial y) + ... \quad (2)$$

The magnetic field lines are often called "lines of force". They are not lines force. The misnomer is perpetuated here to prepare the student for the treacheries of his profession.This expansion of course requires $r_L/L<<1$, where L is the scale length of $\partial B_z / \partial y$ The first term of equation (1) averages to zero in a gyration, and the average of $\cos^2 \omega_c t$ is 1/2 so that.

$$\overline{F}_y = \mp qv_\perp r_L \frac{1}{2}(\partial B / \partial y) \quad (3)$$

The guiding center drift velocity is then:

$$v_{gc} = \frac{1}{q}\frac{F \times B}{B^2} = \frac{1}{q}\frac{\overline{F}_y}{|B|}\hat{x} = \\ = \mp\frac{v_\perp r_L}{B}\frac{1}{2}\frac{\partial B}{\partial y}\hat{x} \quad (4)$$

Since the choice of the y axis was arbitrary, this can be generalized to:

$$v_{\nabla B} = \pm\frac{1}{2}v_\perp r_L \frac{B \times \nabla B}{B^2} \quad (5)$$

This has all the dependences, which can be expected from the physical picture; only the factor ½ (arising from the averaging) was not predicted. Note that the $\pm$ stands for the sign of the charge, and the lightface B stands for |B|. the quantity is called the grad-B drift; it is in opposite directions for ions and electrons and causes a current transverse to B. An exact calculation of $v_{\nabla B}$ would require using the exact orbit, including the drift, in the averaging process.

2. MOVEMENT OF IONS UNDER MAGNETIC FIELD WITH CURVED LINES AND INVARIANT VALUE OF MAGNETIC INDUCTION B (CURVATIVE DRIFT)

In this case the lines of force to be curved with a constant radius of curvature R_c and

|B| =const (Fig. 2). Such a field does not obey Maxwell's equations in a vacuum, so in practicle the grad- B drift will always be added to the effect derived here. A guiding center drifts aries from the centrifugal force felt by the particles as they move along the field lines in their thermal motion. If $v_\parallel^2$ denotes the average square of the component of random velocity along B, the average centrifugal force is :

$$F_{cf} = \frac{mv_\parallel^2}{R_c}\hat{r} = mv_\parallel^2\frac{R_c}{R_c^2} \quad (6)$$

This gives rise to a drift :

$$v_R = \frac{1}{q}\frac{F_{cf} \times B}{B^2} = \frac{mv_\parallel^2}{qB^2}\frac{R_c \times B}{R_c^2} \quad (7)$$

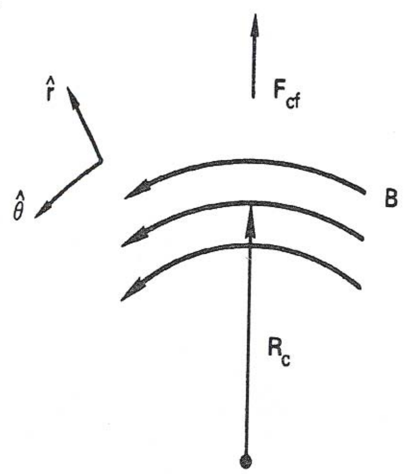


Fig. 2. A curved magnetic field

The drift v_R is called the curvature drift.

It's necessary to complete the grad-B drift which accompanies this when the decrease of |B| with radius is taken into account. In a vacuum $\nabla \times B = 0$.In the cylindrical coordninates of Fig. 2 has only a z component, since B only a θ component and ∇B only an r component. Therefore:

$$(\nabla \times B)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_0) = 0 \\ B_\theta \propto \frac{1}{r} \quad (8)$$

Thus

$$|B| = \propto \frac{1}{R_c} \qquad \frac{\nabla|B|}{|B|} = -\frac{R_c}{R_c^2} \quad (9)$$

Using equation (5):

$$v_{\nabla B} = \mp\frac{1}{2}\frac{v_\perp r_L}{B^2}B \times |B|\frac{R_c}{R_c^2} = \\ = \pm\frac{1}{2}\frac{v_\perp^2}{\omega_c}\frac{R_c \times B}{R_c^2 B} = \frac{1}{2}\frac{m}{q}v_\perp^2\frac{R_c \times B}{R_c^2 B^2} \quad (10)$$

Adding this to v_r, it's possible to obtain the total drift in a curved vacuum field:

$$v_r + v_{\nabla B} = \frac{m}{q}\frac{R_c \times B}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right) \quad (11)$$

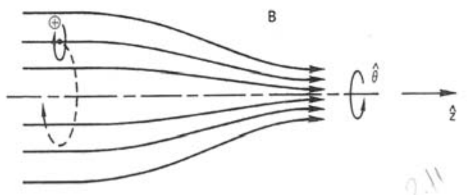
It is unfortunate that these drifts add. For a Maxwellian distributional, indicate that $\overline{v}_{\parallel}^2$ and $\frac{1}{2}\overline{v}_{\perp}^2$ are each equal to KT/m since v involves two degrees of freedom. Therefore:

$$\overline{v}_R + \nabla B = \pm \frac{v_{th}^2}{R_c \omega_c}\hat{y} = \pm \frac{\overline{r}_L}{R_c} v_{th}\hat{y} \qquad (12)$$

Where: $\hat{y}$ here is the direction $R_c \times B$.

This shows that $\overline{v}_R + \nabla B$ depends on the charge of the species but not on its mass.

3. MOVEMENT OF IONS UNDER MAGNETIC FIELD B||B: MAGNETIC MIRRORS



Drift of a particle in a magnetic mirror field.

Fig. 3. Drift of a particle in a magnetic mirror field

This is a magnetic field which is pointed primarily in the z direction and whose magnitude varies in the z direction. Let the field be axisymmetric, with $B_\theta = 0$ and $\partial / \partial\theta = 0$. Since the lines of force converge and diverge, there is necessarily a component B_r (Fig. 3). We wish to show that this gives rise to a force, which can trap a particle in a magnetic field. It's possible to obtain B_r from $\nabla . B = 0$:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \qquad (13)$$

If $\partial B_z / \partial_z$ is given at r=0 and does not vary much with r, it's possible to have approximately

$$rB_r = -\int_0^r r\frac{\partial B_z}{\partial z}dr \cong -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \qquad (14)$$

The various of |B| with r causes a grad- B drift of guiding centers about the axis of symmetry, but there is no radial grad-B drift, because $\partial B / \partial\theta = 0$. The components of the Lorenz force are:

$$\begin{aligned} F_r &= q(v_\theta B_z - v_z B_\theta) \\ F_\theta &= q(-v_r B_z + v_z B_r) \\ F_z &= q(v_r B_\theta - v_\theta B_r) \end{aligned} \qquad (15)$$

Two terms vanish if $B_\theta = 0$ and terms 1 and 2 give rise to the usual Larmor gyration. Term 3 vanishes on the axis; when it does not vanish, the azimuthal force causes a drift in the radial direction. This drift merely makes the guiding centers follow the lines of force. Term 4 is the one we are interested in. Using equation (14), it's possible to obtain:

$$F_z = \frac{1}{2}qv_\theta r(\partial B_z / \partial z) \qquad (16)$$

It's necessary to average over one gyration. For simplicity, consider a particle whose guiding center lies on the axis. Then v_θ is a constant during a gyration; depending on the sign of q, v_θ is $\mp v_\perp$

Since $r = r_L$ the average force is

$$\begin{aligned} \overline{F}_z &= \mp\frac{1}{2}qv_\perp r_L\frac{\partial B_z}{\partial z} = \\ &= \mp\frac{1}{2}q\frac{v_\perp^2}{\omega_c}\frac{\partial B_z}{\partial z} = -\frac{1}{2}\frac{mv_\perp^2}{B}\frac{\partial B_z}{\partial z} \end{aligned} \qquad (17)$$

The magnetic moment of the gyrating particle to be:

$$\mu \equiv \frac{1}{2}mv_\perp^2 / B \qquad (18)$$

CONCLUSION

1. A mathematical analysis of movement of ions in alive tissues under non uniform magnetic field created by apparatus for magnetotherapy is described in the paper.
2. The trajectories of movement of ions have been obtained.

3. An analysis of connection between parameters of ion's trajectories and masse of ions, electrical charge of ions, magnetic induction have been obtained.

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