# LED FAR-FIELD PATTERN APPROXIMATION FUNCTIONS STUDY

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### Abstract

*The approximation functions of the far-field pattern (FFP) of the light emitting diodes (LED) used in LED video displays have been investigated. The simplicity of an approximation function and ease of analytical handling have been targeted. Four candidate approximation functions were identified and the approximation performance evaluation criteria were analyzed. The relative intensity approximation root mean square (RMS) error and relative half power beam angle error have been selected. The influence of the angles range used in approximation was analyzed. The final performance evaluation is done on eight batches of LEDs sample. These LED samples have been chosen to represent the variety of the FFP shapes, the main colors and the range of the most popular viewing angles were used in LED displays design. The approximation by Gaussian function with DC offset performed the best.*

## 1. INTRODUCTION

The light emitting diode (LED) application in video displays has proven a reasonable alternative when a large area of a high brightness imaging is required [1]. LED directional properties among the other LED parameters define the image quality at various viewing angles [2, 3]. The far-field pattern (FFP) [4] is used to determine the spatial directivity properties. The measurement is usually performed by measuring the intensity *I* distribution over the observation angles $\Theta$, say $I(\Theta)$. A numerical parameters, such as: the peak emission direction $\Theta_{peak}$ and a half power beam angle $2\Theta_{0.5}$, where the source's relative intensity is dropping to the half of the peak emission can be obtained from the FFP using the measured $I(\Theta)$.

The FFP of the real LED can be treated as a sum of an ideal LED FFP and a clutter created by sidewall emissions, reflection distortions, tinting, etc. The LEDs used in video displays have wide $2\Theta_{0.5}$ angles. Here the main portion of FFP is created by focusing lens. In addition, these LEDs usually are tinted. Then such LED FFP can be approximated by a simple function [4]. The intention is to use this expression in LED directivity *in-situ* measurement system [5] without dismantling the LEDs from the tile. The goal of this paper was to establish a numerical approximation performance evaluation criterion, to provide the comparison and to indicate the best candidate function for FFP approximation.

## 2. THE APPROXIMATION FUNCTIONS

The polynomial fit could be the first candidate for any approximation. The second order polynomial fit for FFP approximation was suggested. The LED intensity at some angle $\Theta$ is a parabolic function of a form

$$I(\Theta) = a_0 + a_1\Theta + a_2(\Theta)^2, \qquad (1)$$

where $a_0$, $a_1$, $a_2$ are polynomial coefficients. Obtaining the equation for $I(\Theta_{peak})$ and solving for a half of it, the viewing angle is

$$2\Theta_{0.5} = -\frac{\sqrt{2a_1{}^2 - 8a_0a_2}}{2a_2}. \qquad (2)$$

The publication [4] presents the *cos* in power (*g*-1) function as a candidate for LED FFP approximation. A point light source is usually assumed in luminous intensity measurements. The intensity *I* angular distribution can be approximated as:

$$I(\Theta) = I_{\max}\cos(\Theta - \Theta_{peak})^{g-1}, \qquad (3)$$

where *g* is a coefficient, proportional to viewing angle $2\Theta_{0.5}$. Solving (3) for $2\Theta_{0.5}$ it can be obtained

$$2\Theta_{0.5} = 2\arccos(e^{\left(\frac{\ln(2)}{1-g}\right)}). \qquad (4)$$

Gaussian approximation is most often used in RF antenna pattern approximation [6] as an idealized pattern of an antenna having a smooth main-lobe with no sidelobes:

$$I(\Theta) = I_{\max} \cdot e^{\left(-\ln(2)\frac{(\Theta-\Theta_{peak})^2}{\Theta_{0.5}^2}\right)}. \qquad (5)$$

Ambient light during the measurement process under some circumstances can not be completely removed and the DC offset occurs. Then the Gaussian with DC offset $I_{Off}$ can be used for approximation:

$$I(\Theta) = I_{Off} + I_{\max} \cdot e^{\left(-\ln(2)\frac{(\Theta-\Theta_{peak})^2}{\Theta_{0.5R}^2}\right)}. \qquad (6)$$

The half power beam angle in (6) is evaluated after removing the DC component of the FFP. This property is useful if DC offset occurs due to the ambient light. But in case the offset is a property of LED then the half power angle will have a large systematic error. The half power angle has a notation with the index R ($2\Theta_{0.5R}$) to distinguish from the conventional result. Then the corrected half power angle is:

$$2\Theta_{0.5} = -\frac{2\Theta_{0.5R}\sqrt{\ln\left(\frac{I_{\max}-I_{off}}{I_{\max}}\right)-\ln(2)}}{\ln(2)}. \qquad (7)$$

The candidate search was limited by four functions mentioned above.

The LEDs used in video screens usually have elliptical directivity diagram, i.e. vertical and horizontal directivity differ. The amount of possible horizontal and vertical angles combinations is large. The analysis can be greatly simplified if only the one dimensional approximation is used. The analyzed approximation functions can be easily converted into two dimensional by multiplying the horizontal $I_H(\Theta_H)$ and vertical $I_V(\Theta_V)$ diagram approximation functions:

$$I_{2D}(\Theta_H,\Theta_V) = I_H(\Theta_H)\cdot I_V(\Theta_V). \qquad (8)$$

Then analysis was concentrated on 1D approximation functions.

## 3. EVALUATION CRITERIA

The relative value of the intensity approximation error root mean square (RMS) is suggested as maximum likeness criterion

$$\delta_{RMS} = \frac{\sqrt{\int_{\Theta_1}^{\Theta_2}\left(I(\Theta)-\tilde{I}(\Theta)\right)^2 d\Theta}}{\sqrt{\int_{\Theta_1}^{\Theta_2}\left(I(\Theta)\right)^2 d\Theta}}\cdot 100\%, \qquad (9)$$

where $I(\Theta)$ is the original FFP, $\tilde{I}(\Theta)$ is the approximating function, $\Theta_1$ and $\Theta_2$ are the boundary values of the approximation range. The batch of 40 blue LEDs with specified 70 degrees horizontal angle has been used for analysis. Results obtained by applying this criterion are presented in Figure 1. Relative intensity approximation error is presented as a box-and-whisker plot. The box encloses 50% of the data (the interquartile range, IQR), a line in the box represents the median; mean is shown as a square. The whiskers are of 1.5 IQR and the stars represent the minima and maxima of the data.

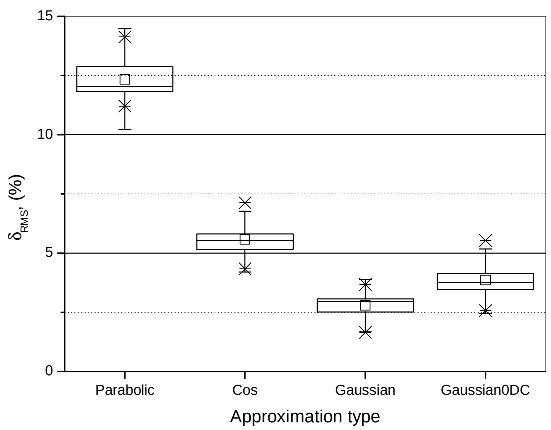


Fig. 1. Intensity approximation error $\delta_{RMS}$ vs. approximation functions

The performance of various approximation functions is clearly distinguishable. Application of the Gaussian function possesses the lowest intensity approximation error reaching 2.5%. The parabolic function approximation has the worst performance (12%).

It would be good to have some numerical performance values that are related to the parameters of LED directivity. Therefore, the analysis of $\Theta_{peak}$ and $2\Theta_{0.5}$ values obtained from approximation was suggested.

The initial investigation has been carried out on $\Theta_{peak}$ values before the approximation and after the approximation. The same LEDs' batch as in Fig.2 has been used for the experiments. It is interesting to point out that peak emission direction obtained from the approximation function is following the values obtained from the original FFP. The results indicated that mean values of $\Theta_{peak}$ have a similar variance for all approximation types. The explanation is that LEDs dedicated for professional LED display have been used. Those LEDs are tinted, so FFP is quite smooth and it makes no sense to analyze the $\Theta_{peak}$ values.

The same analysis can be applied on half power angle of $2\Theta_{0.5}$. The absolute error of $2\Theta_{0.5}$ was calculated as a difference between $2\Theta_{0.5A}$ angles obtained from the approximation and $2\Theta_{0.5O}$ of original FFP:

$$\varepsilon_{2\Theta 0.5} = 2\Theta_{0.5A} - 2\Theta_{0.5O} \,. \tag{10}$$

The results when $2\Theta_{0.5}$ error analysis criterion is applied on the same batch as above are presented in Figure 2.

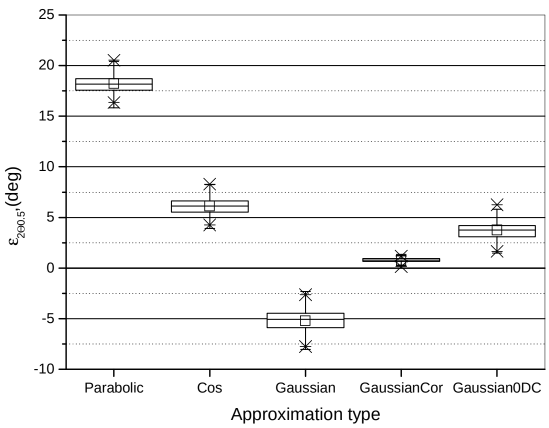


Fig. 2. Half power angle error $\varepsilon_{2\Theta 0.5}$ for various approximation functions

There is a difference in angle $2\Theta_{0.5}$ errors obtained for Gaussian approximation using (7) - $2\Theta_{0.5R}$ (labeled as "Gaussian") and corrected according (8) - $2\Theta_{0.5}$ (labeled as "GaussianCor"). The analysis of initial approximation performance study has indicated that the parabolic function has the worst accuracy. The parabolic function is not able to bend up (Figure1) at the lower end of the FFP curve. The result can be different if other approximation range is used. In order to verify whether this assumption is correct the approximation performance for various approximation ranges was investigated.

## 4. RANGE INFLUENCE

The same batch of blue LEDs has been used for the analysis. The relative intensity approximation error $\bar{\delta}_{RMS}$ mean has been calculated at every approximation range. The approximation range has been varied from 75° (slightly above $2\Theta_{0.5}$) up to reasonable maximum of 175° (slightly below 180°). Graphs in (Figure 3) indicate that the intensity approximation error is increasing when range is increased. The increase is moderated only for both Gaussian approximations.

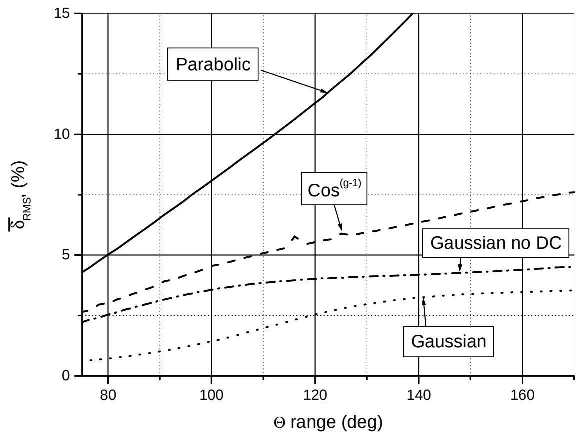


Fig. 3. Intensity approximation error $\bar{\delta}_{RMS}$ mean versus range

The maximum error of $\Theta_{peak}$ has been investigated varying the approximation range. The $\Theta_{peak}$ error has quite negligible decrease with the range (maximum 0.6° change for the worst case of parabolic approximation). The Gaussian approximation is exhibiting the lowest $\Theta_{peak}$ error variation with the range.

The similar analysis was done for the maximum error of $2\Theta_{0.5}$ (12). The same approximation range has been used. The results are presented in Fig. 4.

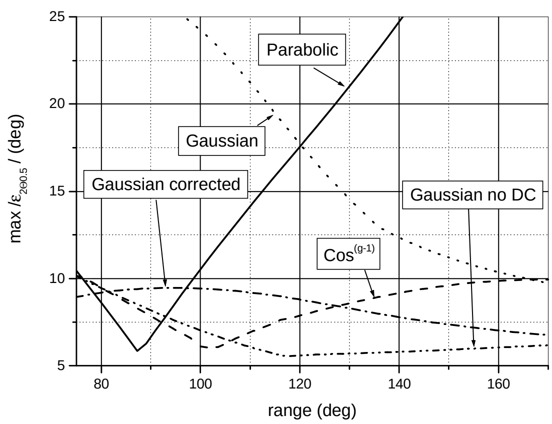


Fig. 4. Max error $\varepsilon_{2\Theta 0.5}$ vs. range

Again, only the Gaussian approximations are able to maintain the performance within a moderate range. The results of Figure5 indicate that uncorrected $2\Theta_{0.5R}$ for Gaussian approximation is approaching but not reaching the corrected value $2\Theta_{0.5}$ when the range is wide. Gaussian approximation angle $2\Theta_{0.5R}$ has a different physical meaning. Therefore, it was decided to use the corrected angle $2\Theta_{0.5}$ (obtained using (8) for further approximation performance analysis of Gaussian with offset. Almost all the curves in Figure5 have a minimum. The reason can be that this minimum is the point where the low intensity area approximation is performing with a maximum of efficiency. It was concluded that it makes no sense to use the approximation range much wider than $2\Theta_{0.5}$ since there is no essential data beyond this range. For further analysis it was decided to use the range of 170% of $2\Theta_{0.5}$ (the last minimum position in Figure 4).

## 5. FINAL EVALUATION

We are aware that presented above analysis has covered only one type of LED. The LEDs of various FFP shape, color, and $2\Theta_{0.5}$ value should be investigated for more extensive final approximation performance evaluation. The LEDs (Table 1) have been chosen to represent the different FFP shapes, the main colors, and the range of most popular angles.

Table 1. LEDs used in investigation

| Notation | Specified $2\Theta_{0.0,5}$, deg | Color | Batch size |
|---|---|---|---|
| BrGH | 110 | green | 20 |
| SBORH | 110 | red | 20 |
| Z2BH | 70 | blue | 37 |
| GrbGH | 70 | green | 20 |
| BrGV | 45 | green | 20 |
| Z2BV | 40 | blue | 37 |
| GrbGV | 40 | green | 20 |
| SBORV | 45 | red | 22 |

The representative batches have been approximated by all the candidate functions. The obtained approximations have been analyzed using the relative intensity approximation RMS error mean $\overline{\delta}_{RMS}$ and a half power angle error mean $\overline{\delta}_{2\Theta0.5}$. All LEDs have been measured with goniometer by 0.9° angular step in ±90° range with resulting 200 data points. Further data processing has been done using MATLAB. To make the decision on the approximation range, the $2\Theta_{0.5}$ angle has been measured on original FFP the first. Then this $2\Theta_{0.5}$ angle was used for approximation range decision. Every FFP has produced the relative intensity approximation error $\delta_{RMS}$ and the relative viewing angle approximation error $\delta_{2\Theta0.5}$. The mean values for these errors have been calculated after processing the whole batch. The results of obtained means of relative approximation error $\delta_{RMS}$ and viewing angle approximation error $\delta_{2\Theta0.5}$ for all Table 1 LEDs are presented in Figure 5 and Figure 6.

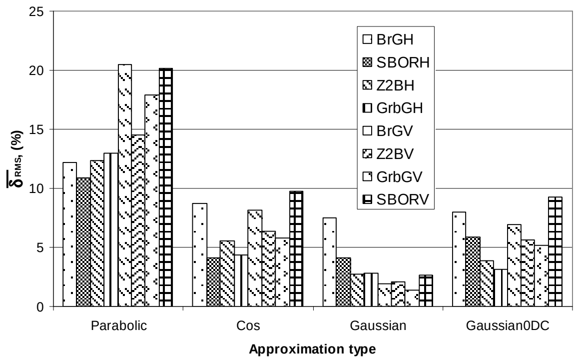


Fig. 5. Relative intensity approximation error $\overline{\delta}_{RMS}$

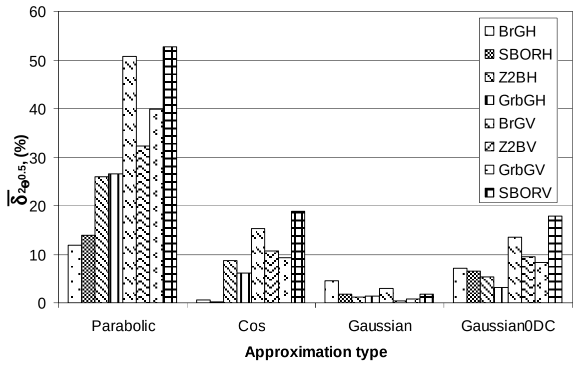


Fig. 6. Error $\overline{\delta}_{2\Theta0.5}$ , % of $2\Theta_{0.5}$

Results indicate that Gaussian approximation with DC offset has the best performance: majority of errors for Gaussian $\delta_{RMS}$ results are below 5%, and of $2\Theta_{0.5}$ error $\delta_{2\Theta0.5}$ is well below 5% limit. It is interesting to point out that the *cos* in power (*g*-1) function presented better results for the large angle (>90°) LEDs. Nevertheless, the individual FFP approximation analysis indicates that *cos* in power (*g*-1) function is getting unstable and the results start to vary significantly at large angles: coefficient *g* is close to 1 at a large angle so floating point accuracy influence increases.

The research presented in [7] investigated the performance of approximation when FFP original data is corrupted by the noise. The original FFP was added with an additive white Gaussian noise (AWGN) and then the approximation has been applied. The resulting intensity approximation error was calculated. The average error graph is presented in Figure 7.

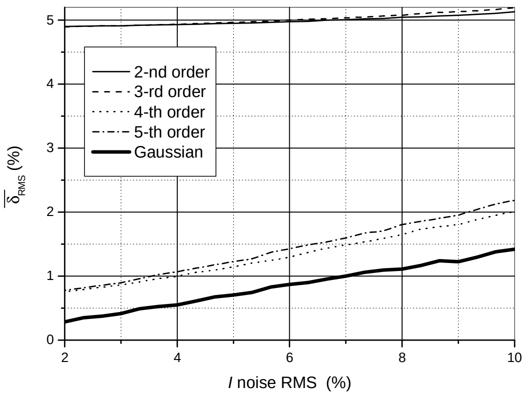


Fig. 7. Relative intensity approximation error $\overline{\delta}_{2\Theta0.5}$ vs. noise

It is interesting to point out that even high polynomial orders possess the approximation error higher that Gaussian approximation. Experiments also indicate that only second order of polynomial has lower than Gaussian standard deviation of obtained $2\Theta_{0.5}$ angles. We think that increasing the polynomial order also increases the sensitivity for noise.

## 6. CONCLUSIONS

The relative intensity approximation RMS error $\delta_{RMS}$ and viewing angle $2\Theta_{0.5}$ error $\varepsilon_{2\Theta0.5}$ have been assigned as the approximation performance evaluation criteria. The approximation using the Gaussian with DC offset function performed the best among four candidate functions evaluated. The intensity approximation error $\overline{\delta}_{RMS}$ and $2\Theta_{0.5}$ error $\overline{\delta}_{2\Theta0.5}$ are below 5% limit. Such precision we consider as sufficient. Therefore we indicate the Gaussian with DC offset function as the best candidate for LED FFP approximation if simple analytical form is needed.

## References


[1] F. Nguyen, "Challenges in the design of a RGB LED display for indoor applications", Synthetic Metals, vol.122, 2001, pp. 215-219.

[2] B.Wendler, "LED Viewing Angles vs. LED Video Display Viewing Angles", Technical Note 40, Daktronics Inc., USA, 2002.

[3] L. Svilainis, V. Dumbrava, "LED goniometry system", Electronics and Electrical Engineering, vol. 72 (8), 2006, pp. 69-74.

[4] "The radiometry of Light Emitting Diodes", Labsphere Inc., UK, 2001.

[5] L. Svilainis, "LED directivity measurement in-situ", Measurement, 2008, vol. 41(6), pp. 647-654.

[6] D. Barton, "Radar Technology Encyclopedia", Artech House inc., 1998.

[7] L. Svilainis, V. Dumbrava, "LED Far Field Pattern Approximation Performance Study", in: Proceedings of 29th IEEE ITI'07 conference, Croatia, 2007, pp.645-649.