RADIATION OF ELECTRIC AND MAGNETIC DIPOLES IN UNIAXIAL MAGNETIC CRYSTAL MEDIA

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Abstract

The problem of radiation of arbitrarily distributed currents in boundless uniaxial magnetic crystal media is considered through the method of generalized solutions of the system of Maxwell's equations in an exact form. The solution resolves into two independent solutions. The first corresponds to the isotropic solution for currents directed along the crystal axis, while the second corresponds to the anisotropic solution when the currents are perpendicular to the axis. Through the use of the expressions for current density of the point magnetic and electric dipoles using delta-function representations, the formulae for the radiated electromagnetic waves, as well as the corresponding radiation patterns, are derived. The obtained solution in the anisotropic case yields the well – known solutions for the isotropic case as a limiting case. Furthermore, the numerical calculation of the solution of Maxwell's equations shows that it satisfies the energy conservation law, i.e. the time average value of energy flux through the surface of a sphere with a point dipole placed at its center remains independent of the radius of the sphere. The obtained generalized solutions of the Maxwell's equations are valid for any values of the elements of the permeability tensor, as well as for sources of the electromagnetic waves described by discontinuous and singular functions.

1. INTRODUCTION

Anisotropic materials have found wide application in the microcircuits working on ultrahigh frequencies. Thin films from monocrystals are effectively used as waveguide's systems.

The problem of radiation of an elementary electric dipole in uniaxial infinite crystal was considered in Ref. [1] with the help of the theory of the generalised functions. The present work is continuation for a case of a magnetic dipole. There are the electromagnetic field and the directivity diagrams of the point magnetic dipole is considered.

2. SOLUTION OF MAXWELL'S EQUATIONS FOR UNIAXIAL ANISOTROPIC MEDIUM

Maxwell's equations for uniaxial anisotropic electromagnetic media of stationary processes are:

$$\begin{cases} rot\, E - i\omega \cdot B = \mathbf{0}, \\ rot\, H + i\omega \cdot D = j, \end{cases} \qquad (1)$$

which is possible to be presented in matrix form:

$$\mathbf{MU} = \mathbf{J}\,, \qquad (2)$$

where

$$\mathbf{M} = \begin{pmatrix} -i\omega\varepsilon\varepsilon_0\mathbf{I} & \mathbf{G}_0 \\ \mathbf{G}_0 & i\omega\hat{\mu}\mu_0 \end{pmatrix}, \ \mathbf{U} = \begin{pmatrix} E \\ H \end{pmatrix},$$

$$\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \ \boldsymbol{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \ \mathbf{J} = \begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{0} \end{pmatrix},$$

$$\boldsymbol{j} = \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}, \ \boldsymbol{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \ \hat{\boldsymbol{\mu}} = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_1 \end{pmatrix},$$

where ω is the constant frequency of electromagnetic field, $\mathbf{M}$ is Maxwell's operator, $\mathbf{I}$ is a identity matrix 3x3, $\boldsymbol{E}$, $\boldsymbol{H}$ are the intensity of electric and magnetic fields, $\mathbf{J}$ is vector of current density.

In magnetically anisotropic media the relation between induction and intensity of the magnetic field is:

$$\boldsymbol{B} = \mu_0 \hat{\boldsymbol{\mu}} \boldsymbol{H}$$

and vector of electric induction:

$$\boldsymbol{D} = \varepsilon\varepsilon_0 \boldsymbol{E}\,.$$

3. SOLUTION OF THE PROBLEM

A method based on the theory of the generalized function of the Fourier transformation is used for solving the matrix equation (2) [1]:

$$\widetilde{\boldsymbol{E}}(\boldsymbol{k}) = \mathrm{F}[\boldsymbol{E}(\boldsymbol{r})] = \int_{\mathrm{R}^3} \boldsymbol{E}(\boldsymbol{r}) \exp(-i\boldsymbol{kr}) dV$$

$$\mathrm{F}^{-1}[\widetilde{\boldsymbol{E}}(\boldsymbol{k})] = \frac{1}{(2\pi)^3} \int_{\mathrm{R}^3} \widetilde{\boldsymbol{E}}(\boldsymbol{k}) \exp(i\boldsymbol{kr}) d^3k$$

where

$$d^3k = dk_x dk_y dk_z .$$

By means of direct Fourier transformation we write down the system of equations (1) or (2) in matrix form:

$$\widetilde{\mathbf{M}}\widetilde{\mathbf{U}} = \widetilde{\mathbf{J}} . \quad (3)$$

The solution of this problem is reduced to the solution of the system of the linear algebraic equations (4), where $\widetilde{\mathbf{U}}$ is defined by means of inverse matrix $\widetilde{\mathbf{M}}^{-1}$:

$$\widetilde{\mathbf{U}} = \widetilde{\mathbf{M}}^{-1}\widetilde{\mathbf{J}} . \quad (4)$$

By introducing new functions according to

$$\widetilde{\Psi}_0 \overset{\text{def}}{=} \frac{1}{k_0^2 - k_x^2 - k_y^2 - k_z^2} ,$$
$$\widetilde{\Psi}_1^m \overset{\text{def}}{=} \frac{1}{k_n^2 - k_x^2 - k_y^2 - \dfrac{\mu_1}{\mu} k_z^2} , \quad (5)$$
$$\widetilde{\Psi}_2^m \overset{\text{def}}{=} \left(\frac{\mu_1}{\mu} - 1\right) \widetilde{\Psi}_1^m \widetilde{\Psi}_0$$

the components of the electromagnetic field after transformations in image space can be written as follows:

$$\widetilde{\boldsymbol{E}} = -\frac{i}{\varepsilon\varepsilon_0\omega}(k_0^2\left\{\widetilde{\boldsymbol{j}}\widetilde{\Psi}_0 + \left[\boldsymbol{k}, \mathbf{e}_z[\boldsymbol{k}, \widetilde{\boldsymbol{j}}_\perp]_z\right]\Psi_2^m\right\} - \boldsymbol{k}(\boldsymbol{k}\widetilde{\boldsymbol{j}})\widetilde{\Psi}_0, \quad (6)$$

$$\widetilde{\boldsymbol{H}} = i(\boldsymbol{k}_z - \boldsymbol{k})k_z[\boldsymbol{k}, \widetilde{\boldsymbol{j}}_\perp]_z \widetilde{\Psi}_2^m + i\mathbf{e}_z[\boldsymbol{k}, \widetilde{\boldsymbol{j}}_\perp]_z (\widetilde{\Psi}_0 - \widetilde{\Psi}_1^m) - i[\boldsymbol{k}, \widetilde{\boldsymbol{j}}]\widetilde{\Psi}_0, \quad (7)$$

where

$$\widetilde{\boldsymbol{j}}_0 = (0,0,\widetilde{j}_z) , \ \widetilde{\boldsymbol{j}}_\perp = \left(\widetilde{j}_x, \widetilde{j}_y, 0\right),$$
$$k_0^2 = \omega^2\varepsilon_0\varepsilon\mu\mu_0 , \ k_n^2 = k_0^2\mu_1 / \mu .$$

After the inverse Fourier transformations from (6) and (7) we obtain:

$$\boldsymbol{E} = -\frac{i}{\varepsilon_0\varepsilon\omega}\Big((k_0^2 + \mathrm{graddiv})\boldsymbol{j} * \Psi_0 - k_0^2 \, \mathrm{rot}(\mathbf{e}_z(\mathbf{e}_z \mathrm{rot}\boldsymbol{j}_\perp)) * \Psi_2^m\Big), \quad (8)$$

$$\boldsymbol{H} = (k_0^2\mathbf{e}_z + \frac{\partial}{\partial z}\mathrm{grad})\mathrm{rot}_z \boldsymbol{j}_\perp * \Psi_2^m - \mathrm{rot}\boldsymbol{j} * \Psi_0. \quad (9)$$

This solution can be written in the form of the sum of two solutions:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 , \ \boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 .$$

It should be noted that the first of them is the 'isotropic' solution. It is defined by Green's function Ψ_0 and the density of the current $\boldsymbol{j}_0$ directed endwise the axis *z* (of the anisotropy):

$$\begin{cases} \boldsymbol{E}_1 = -\dfrac{i}{\varepsilon_0\varepsilon\omega}(\mathrm{graddiv} + k_0^2)(\Psi_0 * \boldsymbol{j}_0), \\ \boldsymbol{H}_1 = -\mathrm{rot}(\Psi_0 * \boldsymbol{j}_0), \end{cases} \quad (10)$$

where the Green's function Ψ_0 can be defined from (5) by inverse Fourier transformation [1]

$$\Psi_0 = \mathrm{F}^{-1}[\widetilde{\Psi}_0] = -\frac{1}{4\pi}\frac{\exp(ik_0 r)}{r} , \quad (11)$$
$$r = \sqrt{x^2 + y^2 + z^2} .$$

The second solution can be written by using the component of the density of the current $\boldsymbol{j}_\perp$ perpendicular to axis *z* and the Green's functions Ψ_0 and Ψ_2^m:

$$\begin{cases} \boldsymbol{E}_2 = -\dfrac{i}{\varepsilon_0\varepsilon\omega}\Big((k_0^2 + \mathrm{graddiv})\boldsymbol{j}_\perp * \Psi_0 - k_0^2 \mathrm{rot}(\mathbf{e}_z \mathrm{rot}_z \boldsymbol{j}_\perp) * \Psi_2^m\Big), \\ \boldsymbol{H}_2 = (k_0^2\mathbf{e}_z + \dfrac{\partial}{\partial z}\mathrm{grad})\mathrm{rot}_z \boldsymbol{j}_\perp * \Psi_2^m - \mathrm{rot}\boldsymbol{j}_\perp * \Psi_0. \end{cases} \quad (12)$$

$$\Psi_1^m = -\frac{1}{4\pi}\sqrt{\frac{\mu}{\mu_1}}\frac{\exp(ik_n r')}{r'} , \quad (13)$$

$$r' = \sqrt{x^2 + y^2 + \frac{\mu}{\mu_1} z^2} \ ,$$

$$\Psi_2^m = (\mu_1 / \mu - 1)\Psi_0 * \Psi_1^m \qquad (14)$$

or

$$\Psi_2^m = \frac{1}{i8\pi k_0}[e^{ik_0 z}(\mathrm{Ci}(k_0(r-z)) + iS\,\mathrm{i}(k_0(r-z))) +$$
$$e^{-ik_0 z}(\mathrm{Ci}(k_0(r+z)) + iS\,\mathrm{i}(k_0(r+z))) -$$
$$e^{ik_0 z}(\mathrm{Ci}(k_n r' - k_0 z) + iS\,\mathrm{i}(k_n r' - k_0 z)) -$$
$$e^{-ik_0 z}(\mathrm{Ci}(k_n r' + k_0 z) + iS\,\mathrm{i}(k_n r' + k_0 z))],$$

where integral cosine and sine are defined by formulae's:

$$\mathrm{Ci}(z) = \gamma + \ln z + \int_0^z \frac{\cos t - 1}{t} dt \, ,$$

$$\mathrm{Si}(z) = \int_0^z \frac{\sin t}{t} dt - \frac{\pi}{2}$$

and Euler's constant $\gamma = 0{,}5772$.

4. RADIATION PATTERNS OF HERTZIAN RADIATOR

The moment of point electric dipole is given by

$$\boldsymbol{p} = \boldsymbol{n} p e^{-i\omega t} \ \ (\boldsymbol{p} = \boldsymbol{p}_0 + \boldsymbol{p}_\perp), \qquad (15)$$

where *p* is a constant. It corresponds to the current density defined by means of the Dirac delta-function:

$$\boldsymbol{j} = -i\omega \cdot \boldsymbol{p}\delta(\boldsymbol{r}) \, . \qquad (16)$$

The expression of the electromagnetic field for electric radiator will take the following form as for isotropic medium, when the direction of the dipole moment is parallel to the axis *z* ($\boldsymbol{p} = \boldsymbol{p}_0$) (Fig.1):

$$\begin{cases} \boldsymbol{E}_1 = -(\varepsilon_0 \varepsilon)^{-1} (\mathrm{graddiv} + k_0^2)(\Psi_0 \boldsymbol{p}_0), \\ \boldsymbol{H}_1 = i\omega \cdot \mathrm{rot}(\Psi_0 \boldsymbol{p}_0). \end{cases}$$

Also when the direction of the dipole moment is perpendicular to the axis *z*, we obtain ($\boldsymbol{p} = \boldsymbol{p}_\perp$) (Fig. 2):

$$\begin{cases} \varepsilon_0 \varepsilon \boldsymbol{E}_2 = k_0^2 \mathrm{rot}(\mathbf{e}_z \mathrm{rot}_z \boldsymbol{p}_\perp \Psi_2^m) - \\ \qquad (k_0^2 + \mathrm{graddiv})(\boldsymbol{p}_\perp \Psi_0), \\ \boldsymbol{H}_2 = (k_0^2 \mathbf{e}_z + \frac{\partial}{\partial z} \mathrm{grad}) \mathrm{rot}_z (\boldsymbol{p}_\perp \Psi_2^m) - \\ \qquad \mathrm{rot}(\boldsymbol{p}_\perp \Psi_0). \end{cases}$$

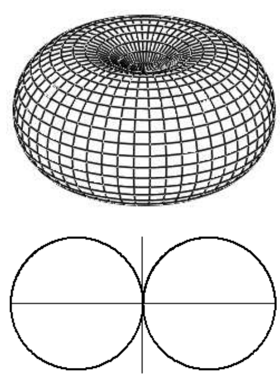

Fig. 1. Directional diagrams (DD). The dipole moment is parallel to the axis *z* ($\boldsymbol{p} = \boldsymbol{p}_0$).

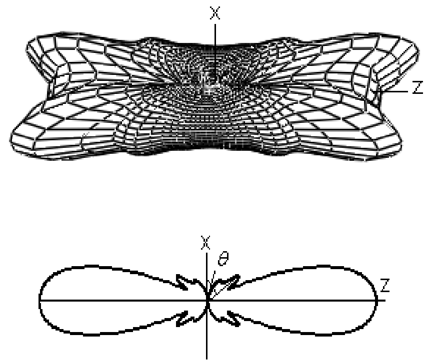

Fig. 2. DD. The axis of electric dipole is perpendicular to axis *z* ($\boldsymbol{p} = \boldsymbol{p}_\perp$), $\mu_1 / \mu = 9$.

5. RADIATION PATTERN OF POINT MAGNETIC DIPOLE MOMENT

On the basis of the obtained results, we consider now radiation of point magnetic dipole moment. For a point radiator with the oscillating magnetic dipole moment, similarly to the electric dipole case, we have:

$$\boldsymbol{m} = \boldsymbol{n}\, m \exp(-i\omega \cdot t)$$
$$(\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{m}_\perp \, , \ m = \mathrm{const} \,)$$

the electric current density is defined by using Dirac's delta-function:

$$\boldsymbol{j} = -[\boldsymbol{m}, \nabla] \cdot \delta(\boldsymbol{r}) \, . \qquad (17)$$

(i) Case ***m* = *m*₀**:

Relation between density of the electric current $\boldsymbol{j}_\perp$ and the magnetic dipole moment is defined as following, in the case that the magnetic dipole moment ***m*** is directed lengthwise *z*-axis:

$$\boldsymbol{j}_\perp = m_0\left(\mathbf{e}_x \frac{\partial}{\partial y} - \mathbf{e}_y \frac{\partial}{\partial x}\right)\delta(\boldsymbol{r})\,. \qquad (18)$$

It should be noted that the following useful formulae hold:

$$\operatorname{div} \boldsymbol{j}_\perp = 0\,,\ \boldsymbol{j}_0 = 0\,. \qquad (19)$$

Taking into account Eq. (19), intensities of the electromagnetic field by the magnetic dipole moment are defined from the solutions (12) in this case, as following:

$$\begin{cases} \boldsymbol{E} = \dfrac{k_n^2 m_0}{i\varepsilon\varepsilon_0\omega}\operatorname{rot}(\mathbf{e}_z \Psi_1^m), \\ \boldsymbol{H} = \boldsymbol{m}_0 \Delta\Psi_1^m + \boldsymbol{m}_0(\dfrac{\mu_1}{\mu} - 1)\dfrac{\partial^2}{\partial z^2}\Psi_1^m - \\ \quad m_0 \dfrac{\mu_1}{\mu}\dfrac{\partial}{\partial z}\operatorname{grad}\Psi_1^m. \end{cases} \qquad (20)$$

(ii) Case $\boldsymbol{m} = \boldsymbol{m}_\perp$:

For the point magnetic dipole moment $\boldsymbol{m}_\perp$ which is perpendicular to axis z we define intensities of electromagnetic field as following (Fig. 4):

$$\begin{cases} \boldsymbol{E} = i\dfrac{k_0^2}{\varepsilon\varepsilon_0\omega}\{\dfrac{\partial}{\partial z}([\boldsymbol{m}_\perp, \mathbf{e}_z]\dfrac{\mu_1}{\mu}\Psi_1^m - \operatorname{grad}_\perp \\ \quad \operatorname{rot}_z(\boldsymbol{m}_\perp\Psi_2^m)) - \mathbf{e}_z \operatorname{rot}_z(\boldsymbol{m}_\perp\Psi_0)\}, \\ \boldsymbol{H} = (k_0^2\mathbf{e}_z + \dfrac{\partial}{\partial z}\operatorname{grad})\dfrac{\partial}{\partial z}\operatorname{div}\boldsymbol{p}_\perp\Psi_2^m - \\ \quad \operatorname{rot}\operatorname{rot}(\boldsymbol{p}_\perp\Psi_0). \end{cases} \qquad (21)$$

Directional diagrams are represented in Figs. 3, below :

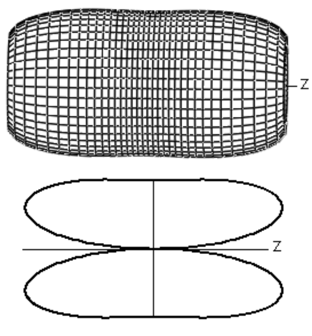

Fig. 3. DD. The axis of magnetic dipole is parallel to axis z ($\boldsymbol{m} = \boldsymbol{m}_0$).

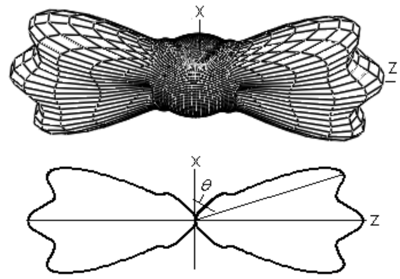

Fig. 4. DD. The axis of magnetic dipole is perpendicular to axis z, $\mu_1 / \mu = 7$.

6. CONCLUSION

The numerical calculation of the solution of Maxwell's equations satisfies the energy conservation law. Numerical computation shows that time average value energy flux on a surface of sphere from a point dipole remains independent from its radius.

As shown in the electric dipole directional diagrams, medium becomes isotropic for such radiator if its moment is directed along anisotropy axis. The dipole pattern in isotropic media is shown in Fig. 1 and directional diagram itself possesses the rotation symmetry. However, the point magnetic moment does not possess such property.

References

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