

# COMPUTER SIMULATION OF ECG SIGNAL RESTORATION

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## Abstract

A computer simulation of loss of part of real ECG signal has been done in the paper. A restoration of ECG signal by using of mathematical method of Aizenberg for finite functions is described in the paper. The errors of signal's restoration in the cases of different number of values of loss have been calculated.

The software, which provides application of Aisenberg's method has been done in MATLAB. A comparison between obtained results and results of interpolation of ECG signals by function for interpolation in MATLAB has been done.

The application of Aizenberg's method has been confirmed by obtained results.

## 1. INTRODUCTION

When transmitting ECG signals, distortions occur, which can affect the exact diagnosis of patients. The morphology of ECG signal has been used for recognising much variability's of heart activity, so it is very important to get the parameters of ECG signal clear without noise [2, 4, 5]. This step gives a full picture and detailed information about the electrophysiology of the heart diseases and the ischemic changes that may occur like the myocardial infarction, conduction defects and arrhythmia. In order to support clinical decision-making, reasoning tool to the ECG signal must be clearly represented and filtered, to remove out all noises and artifacts from the signal. ECG signal is one of the biosignals that is considered as a non-stationary signal and needs a hard work to denoising. Interpolation of signal, which is lost at a certain time interval must be used. In regard to this, Aizenberg method for analytical continuation of finite functions may be used. The paper describes modeling of loss of real values of an ECG signal and their subsequent restoration. Signal restoration is done using Aizenberg formula [1] and the built-in function interp1 in Matlab [3]. Aizenberg method is applied for the first time in the restoration of one-dimensional real signal, recorded by the apparatus on the hard disk of the computer.

## 2. RESTORATION OF FINITE SIGNALS BY AIZENBERG METHOD

These ECG signals are one-dimensional finite signals. They are presented as functions of the time

in the positive direction of the axis  $t$ . This possibility of restoring the ECG signals is realized with the Aizenberg's formulae [1] for analytical continuation of finite functions from the Hardy's and Wiener's spaces. In short the method of Aizenberg consists in [1]:

If  $N_j$  is a limited sequence of different points in the halfplane  $\{z_j \in D: \operatorname{Im} z_j > -\sigma\}$  which has no points of density over its contour  $1 \leq j \leq n$ , let us consider the following problem of restoration of function  $f(z)$  in  $D_\sigma$  where  $D_\sigma = \prod_{j=1}^n \{z_j \in D: \operatorname{Im} z_j > -\sigma\}$ , in the set  $M = N_1 \times N_2 \times \dots \times N_n$ . For  $N_l = \{x_{lj}\}$  is valid (1):

$$\omega(m, u, p, l) = \frac{x_{lp} - \bar{x}_{lp} + 2i\sigma \prod_{j=1, j \neq p}^m (u - x_{lj})(x_{lp} - \bar{x}_{lj} + 2i\sigma)}{u - \bar{x}_{lp} + 2i\sigma \prod_{j=1, j \neq p}^m (u - \bar{x}_{lj} + 2i\sigma)(x_{lp} - x_{lj})} \quad (1)$$

For  $f \in H^2(D_\sigma)$ ,  $z \in H^2(D_\sigma)$  is valid:

$$f(z) = \lim_{m \rightarrow \infty} \sum_{k_1=1}^m \sum_{k_2=1}^m \dots \sum_{k_n=1}^m f(x_k) \prod_{l=1}^m \omega(m, z_l, k_l, l) \quad (2)$$

$$x_k = (x_{1k_1}, \dots, x_{nk_n}), k = (k_1, \dots, k_n)$$

where  $H$  is a Hardy's class of functions,  $\sigma > 0$  is a parameter,  $\bar{x}$  is a conjugate complex value of  $x$ .

The following formula can be used for interpolation of function  $f(x)$  in Wiener class  $W_\alpha^+$ :

$$f(x) = \lim_{m \rightarrow \infty} \sum_{k=1}^m f(x_k) \frac{2i\sigma \prod_{j=1, j \neq k}^m (x - x_j)(x_k - x_j + 2i\sigma)}{x - x_k + 2i\sigma \prod_{j=1, j \neq k}^m (x - x_j + 2i\sigma)(x_k - x_j)} \quad (3)$$

Formula (3) contains two parameters  $m$  and  $\sigma$ ; The accuracy of signal restoration depends on their values. Results from previous experiments show that the number  $m$  of the known values of the signal must be not more than 30.

The parameter  $\sigma$  is defined by searching of the minimal root mean-square error  $\varepsilon$  by formula (4) between the real and interpolated values:

$$\varepsilon = \sqrt{\frac{1}{n} \sum_{i=0}^n (x_i - \tilde{x}_i)^2} \quad (4)$$

where  $x_i$  and  $\tilde{x}_i$  are the values of the real and restored signal.

### 3. COMPUTER SIMULATION OF ECG SIGNAL RESTORATION

Experiments were done in the Matlab software environment for interpolation of a real ECG signal, recorded by a cardiograph using the Aizenberg method and the built-in function `interp1` from Matlab. Data from echograph in the form of Microsoft Excel Workbook (xls).

The Excel table contains 3 columns of 7680 values for the cardiac signal parameters. Each of these columns is imported into the editor Matlab's Array Editor and stored as a separate variable in Matlab's Workspace. Then it is stored in Workspace as a .mat-file.

Program in the form of m-script, calling m-function is written, which implements formula (3). The algorithm of the program is as follows:

1. .mat-file with cardiogram data opens
2. The user enters the values of the start and end points of the readings of both intervals in which the signal of the first channel is known.
3. Function is called, in which formula (3) is programmed. The missing readings can be found by it using interpolation.
4. Graphs of the theoretical ECG signal and the signal obtained by formula (3) are plotted.
5. The results for the root mean-square error of the signal from the first channel are found.
6. Steps 2 - 5 are repeated for the signal from the second channel.

Computational experiments for restoration of the signal are performed by the use of the function `interp1` as well. Again, the root mean-square error of interpolation is calculated. `interp1` function uses the following methods:

'nearest' (Nearest neighbor interpolation); 'linear' (Linear interpolation) - default; 'spline' (Cubic spline interpolation); 'pchip' (Piecewise cubic Hermite interpolation); 'cubic' (Same as 'pchip'); 'v5cubic' (Cubic interpolation used in MATLAB 5).

Best results are obtained while using the method 'spline'.

### 4. RESULTS FROM COMPUTING EXPERIMENTS

Computing experiments for restoration of the missing 1 to 5 readings (values) of cardiogram for both channels were made. If there is loss of more values, it is considered that the ECG should be repeated.

Figure 1 shows the results from interpolation of a reading by Aizenberg method. The blue line displays the graph of interpolated signal, and the red line - the real signal from the ultrasonograph. Figure 1 and the subsequent Figures do not show the entire graph of the cardiogram, but only the part of it, which is used for the computation experiment on the purpose of more clear visualization of signals. The program enables the user to set different intervals for signal restoration for each channel. For the purpose of discussion, the signal for the first channel is named  $s_0$ , and for the second -  $s_1$ .

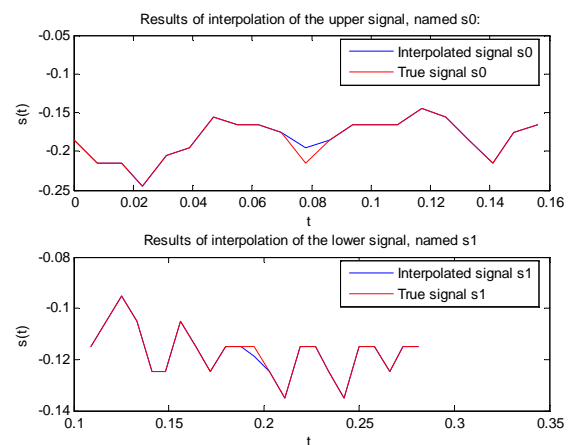


Fig. 1. Interpolation of one value according to Aizenberg's method

As it can be seen from Figure 1, interval of the known readings for the time  $t \in [1 \ 10]$  and  $t \in [12 \ 21]$  is set for the first channel, which implies that  $t \in [0 \ 0.07]$  and  $t \in [0.086 \ 0.1560]$ . The value of the signal  $s_0$  for  $t = 0.078$  is interpolated. The interval of the known readings of  $s_1$  is for  $t \in [0.1090 \ 0.1880]$  and  $t \in [0.2030 \ 0.2810]$ . The value of the signal  $s_1$  in  $t = 0.1950$  is calculated by interpolation. The parameters  $m$  and  $\sigma$  in formula (3) are different for

the two signals - for  $s_0$ :  $m = 20$ ,  $\sigma = 1$ , for  $s_1$ :  $m = 22$ ,  $\sigma = 0.1$ . The values of the estimated root mean square errors are: for  $s_0$ :  $\varepsilon = 0.0043$ , for  $s_1$ :  $\varepsilon = 0.00082757$ . As it can be seen, the values of the root mean square errors are below 1%.

Figure 2 shows interpolation with the function `interp1`, when the spline method is used. The same value as in Figure 1 for the signal  $s_0$ , i.e.  $s_0(0.078)$  is interpolated. The blue dots in Fig. 2 indicate the known data (i.e. the basis points for interpolation), the graph of the interpolated function is depicted in green, and the real graph of  $s_0$  is given in red.

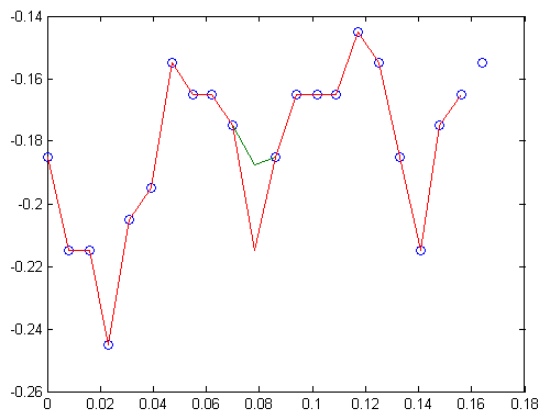


Fig. 2. Interpolation of one value from  $s_0$  with the function `interp1` from Matlab

The root mean square error for signal restoration using `interp1` is 0.0051. The comparison between this value and the estimated value of the respective error in signal restoration of  $s_0$  (and  $s_1$ ) with interpolation by Aizenberg method proves the advantage of the latter to `interp1` with respect to the accuracy of interpolation. Similar results are obtained when interpolating  $s_1$  by `interp1`.

Figure 3 shows the interpolation of 3 readings using Aizenberg method.

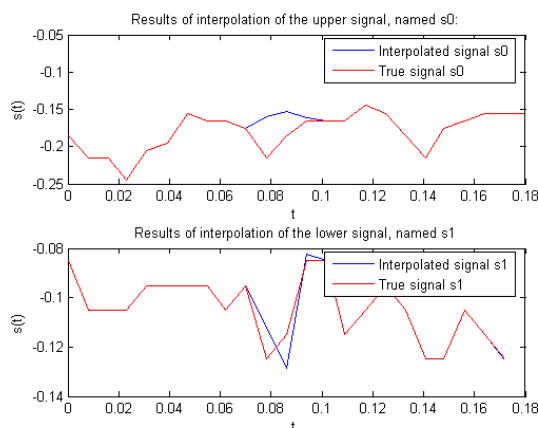


Fig. 3. Interpolation of three values according to Aizenberg's method

The readings for the two channels 11, 12, 13 are also restored, which means values of the signals  $s_0$  and  $s_1$  for  $t = 0.0780, 0.0860, 0.0940$ . Restoration errors are as follows: root mean square error for  $s_0$  is 0.0137 and for  $s_1$  is 0.000456. These values are smaller than the root mean square error calculated using `interp1` for interpolation of one value.

Figure 4 and Figure 5 show the interpolation of 5 values by Aizenberg method and by application of `interp1`, respectively.

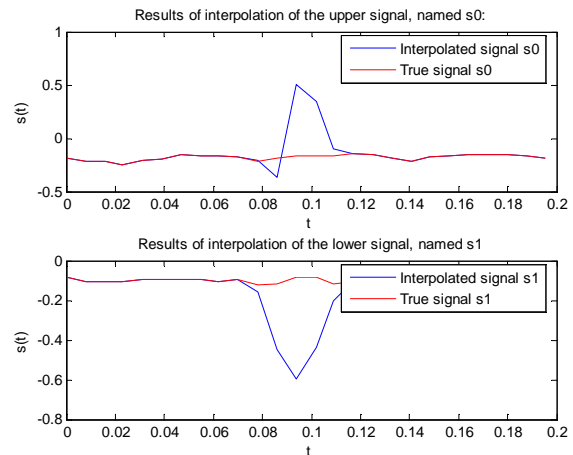


Fig. 4. Interpolation of five reads according to Aizenberg's method

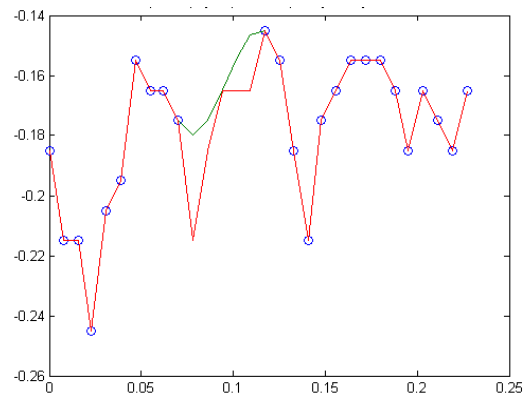


Fig. 5. Interpolation of five values from  $s_0$  with the function `interp1` from Matlab

At restoration of 5 values better results are obtained using `interp1`. The two graphs show interpolation of readings from 11 to 15, i.e. for signals with values in  $t = 0.0780, 0.0860, 0.0940, 0.1020, 0.1090$ . The root mean square error in restoration using Aizenberg method is: for  $s_0$ :  $\varepsilon = 0.2095$ , and for  $s_1$ :  $\varepsilon = 0.2577$ . The root mean square error using `interp1` is 0.0139.

## 5. CONCLUSION

More accurate restoration of ECG signal in the case of loss of 1 to 4 values is done using the Aizenberg method. To obtain a smaller error in the interpolation, the optimal ratio between the parameters  $m$  and  $\sigma$  of the formula (3) must be calculated.

The need to obtain more precise restoration of the cardiac signal requires the use of both formula (3) of the Aizenberg method in case of fewer lost values - 1 to 4 readings, and the built-in function `interp1` (with spline method) in Matlab in case of loss of 5 readings.

Research activities in this field are under development, as experiments for extrapolation of cardiac signals are also planned.

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