ANALYSIS OF COMPLEX IMPEDANCE MEASUREMENT I-V METHOD

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Abstract

Complex impedance measurement methods are widely described, but the information on what the uncertainty of the measurement in case of complex result is scarce.

In this article we will investigate two techniques of I-V method: the I-V differential and I-V single ended. Both of them are based on "voltage divider". Complex value measurement uncertainty analysis is presented based on theoretical measurement setup analysis and complex sensitivity functions. Real and imaginary uncertainty components are presented in complex impedance plane.

1. INTRODUCTION

The complex circuit impedance is widely used in electromagnetic and ultrasound applications. Impedance variation over frequency range is receiving greater attention nowadays. Because of wideband nature of measurements, impedance is varying in wide range. In addition, impedance variance is in complex plane. Usually it is undesirable to have jumps in impedance measurements (in case of hysteretic analysis or nonlinear circuits). Therefore it is important to have a technique, capable to obtain the impedance without changing the reference impedance (reference impedance switching will cause current and voltage variation in investigated circuit). When complex values are measured, the uncertainty estimation becomes complex too.

Here we aim to analyze the measurement uncertainties of I-V method when it is used for complex impedance measurements.

2. I-V IMPEDANCE MEASUREMENT

The unknown impedance Z_x can be obtained from the measured values of voltage and current. When voltage and current are obtained directly then technique is named I-V [1]. The measurement can be arranged using two techniques: the I-V differential and I-V single ended. The simplified connection diagram of the I-V *differential* technique is presented in Figure 1.

Current is calculated using the voltage measurement across an accurately known resistor, R_{ref}.

$$Z_x = \frac{U_{Zx}}{I} = \frac{U_{Zx}}{U_{Rref}} R_{ref} . \quad (1)$$

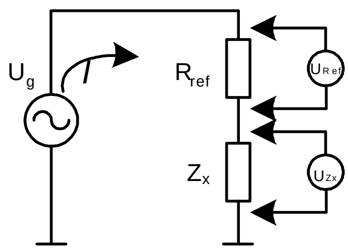

Fig. 1. Differential I-V technique

The *single-ended* technique is using only single-ended measurement channels. The implementation diagram for single-ended technique is presented in Figure 2.

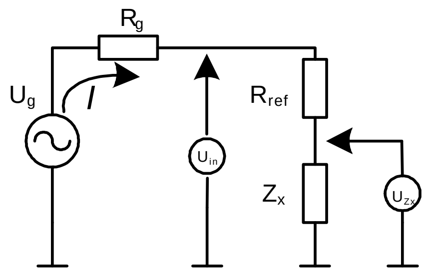

Fig. 2. Single ended I-V technique

The R_{ref} voltage dropout is obtained by voltages U_{Zx} and U_{in} subtraction:

$$Z_x = \frac{U_{Zx}}{U_{in} - U_{Zx}} R_{ref} . \quad (2)$$

The analysis below is based on circuits presented.

3. THE SENSITIVITY COEFFICIENTS

Uncertainty analysis was done to estimate the presented techniques' performance. Assuming that impedance is measured using indirect method, for uncorrelated input quantities the combined standard uncertainty $u_c(Z_x)$ is a squares sum of corresponding uncertainties $u_i(Z_x)$ [2] :

$$u_c(Z_x) = \sqrt{\sum_{i=1}^{n} u_i(Z_x)^2} = \sqrt{\sum_{i=1}^{n} c_i^2 u(x_i)^2} \ . \quad (3)$$

The quantity $u_i(Z_x)$ is the contribution to the standard uncertainty associated with the output estimate Z_x associated with *i*-th the input estimate x_i. Then the *i*-th sensitivity coefficient c_i of the corresponding input estimate x_i is the partial derivative of the measurement function *f* with respect to x_i

$$c_i = \frac{\partial Z_x}{\partial x_i} = \frac{\partial [f(x_1, x_2, x_3, ..., x_i)]}{\partial x_i} . \quad (4)$$

So, a single-ended I-V technique Z_x according to equation (2) is a function of U_{Zx}, U_{in} and R_{ref} accordingly. The sensitivity coefficients for those variables can be obtained as [3]:

$$c_{UZx} = \frac{R_{ref}}{U_{in} - U_{Zx}} + \frac{U_{Zx} R_{ref}}{(U_{in} - U_{Zx})^2} , \quad (5)$$

$$c_{Uin} = - \frac{U_{Zx} U_{Rref}}{(U_{in} - U_{Zx})^2} , \quad (6)$$

$$c_{Rref} = \frac{U_{Zx}}{U_{in} - U_{Zx}} . \quad (7)$$

And the absolute combined standard uncertainty of Z_x [4]:

$$u_c(Z_x) = \frac{\sqrt{u^2(U_{Zx}) \left(\frac{R_{ref}}{U_{in} - U_{Zx}} + \frac{U_{zx} R_{ref}}{(U_{in} - U_{Zx})^2} \right)^2 +}}{+ \sqrt{u^2(U_{in}) \left(\frac{U_{Zx} R_{ref}}{(U_{in} - U_{Zx})^2} \right)^2 + u^2(R_{ref}) \frac{U_{Zx}^2}{(U_{in} - U_{Zx})^2}}} , \quad (8)$$

where $u(U_{Zx})$, $u(U_{in})$ and $u(R_{ref})$ are corresponding components' absolute uncertainties.

4. UNCERTAINTY ANALYSIS

For performance evaluation the relative uncertainty of impedance Z_x was used:

$$u_{\%}(Z_x) = \frac{u_c(Z_x)}{|Z_x|} \cdot 100\% , \quad (9)$$

Equation (8), together with voltage measurement standard uncertainty and resistor accuracy have been used. Applying equation (8) for equation (9) gives the percentage relative uncertainty for impedance measurement when I-V impedance measurement using single-ended implementation is used

$$u_{\%}(Z_x) = \frac{\frac{U_{in} - U_{Zx}}{U_{Zx} R_{ref}} \sqrt{u^2(U_{Zx}) \left(\frac{R_{ref}}{U_{in} - U_{Zx}} + \frac{U_{zx} R_{ref}}{(U_{in} - U_{Zx})^2} \right)^2 +}}{\sqrt{+ u^2(U_{in}) \left(\frac{U_{Zx} R_{ref}}{(U_{in} - U_{Zx})^2} \right)^2 + u^2(R_{ref}) \frac{U_{Zx}^2}{(U_{in} - U_{Zx})^2}} \cdot 100\%} , \quad (10)$$

Voltage measurement uncertainty required for equation (8) and (10) was obtained from experimental results presented in [4], since similar system is planned for the measurements. The input voltage U_{in} has been assumed of 1 V value, reference resistor R_{ref} was assigned 10Ω value. The variation for unknown impedance was given the variance range as the fraction of reference resistor R_{ref}. The resulting voltages and currents were use to obtain final value of measurement uncertainty. Real and imaginary parts (Figure 3) were treated separately, giving the variance in corresponding range.

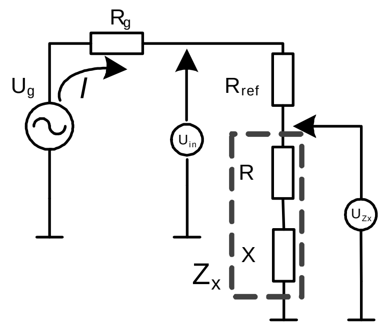

Fig. 3. Zx complex measurement using single-ended implementation

If variables in equation (10) are complex, the resulting uncertainty will be complex too. In order to analyse the uncertainties, tree ways were chosen: absolute value:

$$u_{abs\%}(Z_x) = |u_{\%}(Z_x)| , \quad (11)$$

and real and imaginary parts:

$$u_{RE\%}(Z_x) = \Re(u_{\%}(Z_x)), u_{IM\%}(Z_x) = \Im(u_{\%}(Z_x)), \quad (12)$$

The uncertainty absolute value is presented in Figure 4.

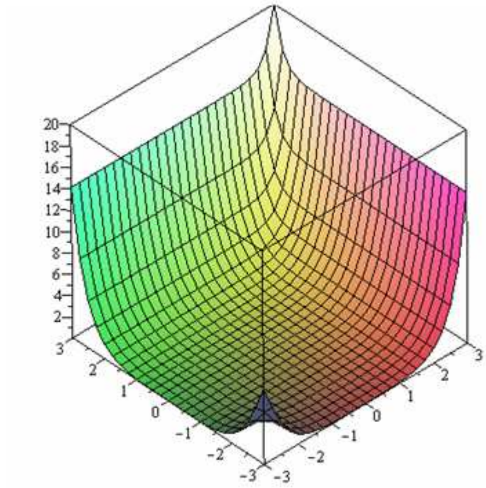

Fig. 4. Relative standard uncertainty of Zx complex measurement influence of u%(Rref)

Wide range (x1000 times above and below the R_{ref}) for variance was given. Real and imaginary parts and uncertainty value itself produce the 3-dimensional space. Contour plot might be more convenient (Figure 5).

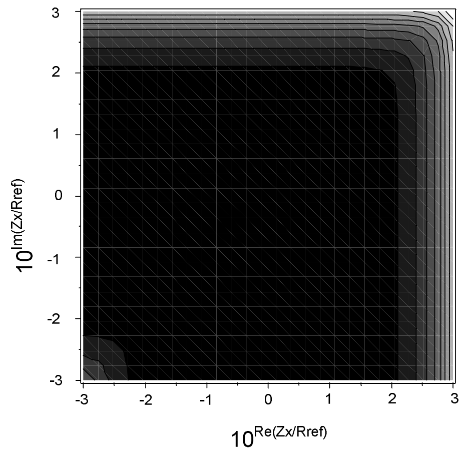

Fig. 5. Contour plot of absolute value of relative standard uncertainty

It can be seen that there is significant area of flat plateau in measurement uncertainties, and only when approaching ratios of 1000 times random errors become significant.

When analyzing in details (Figure 6, x10 times) it can be noted that there is a insignificant local minima, located at 1.6xR_{ref}. It depends on reference resistor accuracy.

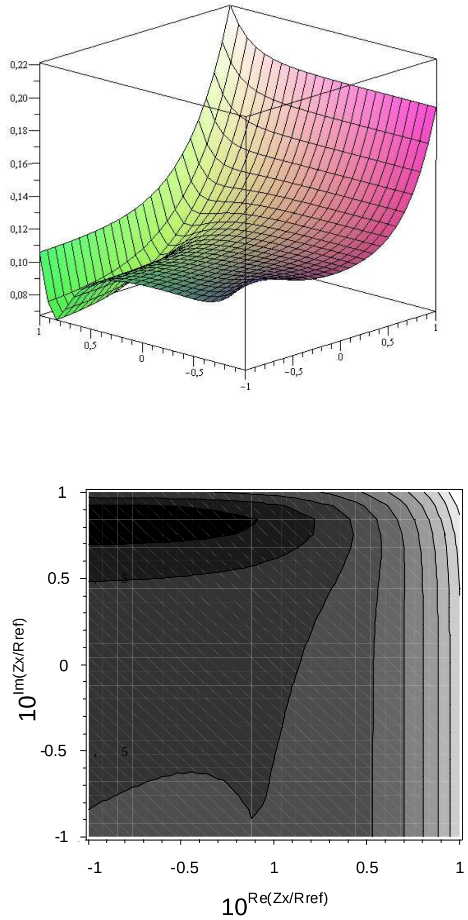

Fig. 6. 3D plot and contour plot of absolute value of relative standard uncertainty for smaller range

Theoretically uncertainty of 0,2% can be reached, if 0.1% uncertainty reference resistor is used.

Real (Figure 7) and imaginary (Figure 8) parts for the same analysis are presented.

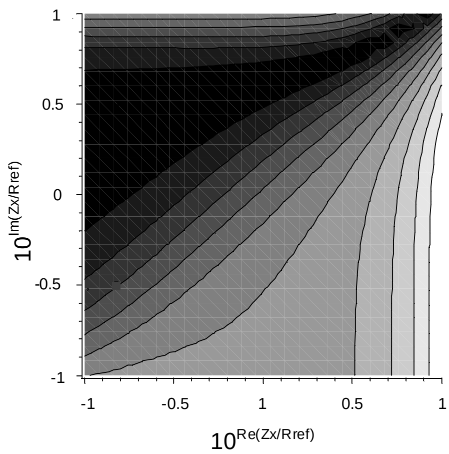

Fig. 7. Contour plot of real value of relative standard uncertainty for smaller range

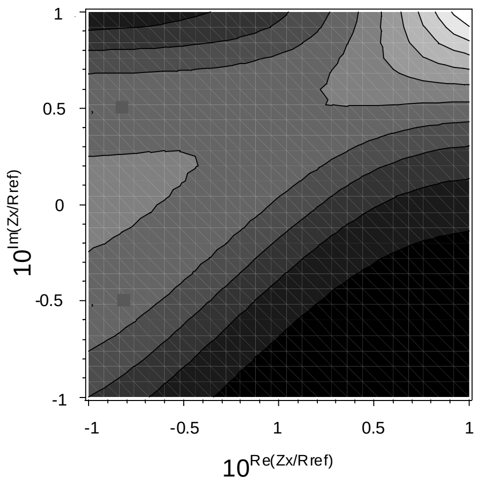

Fig. 8. Contour plot of imaginary value of relative standard uncertainty for smaller range

It can be seen that real part of uncertainty varies along real impedance axis and imaginary part varies along imaginary axis correspondingly.

5. ACKNOWLEDGMENTS

We would like to thank the Dr. Vytautas Dumbrava and Dr. Linas Svilainis for their consultations and useful literature on uncertainty analysis support.

6. CONCLUSIONS

The analysis uncertainties of complex impedance measurement estimation were presented.

It has been shown that both real and imaginary parts of uncertainty exist which define the optimal range for the technique application.

References

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