A SIMPLE AND ACCURATE METHOD FOR ACHIEVEMENT OF A UNIFORM ON-AXIS MAGNETIC FIELD OF LONG SOLENOID

Sava Vasilev Savov

Department of Electrical Engineering, Technical University of Varna,
Varna 9010, Bulgaria
e-mail: savovsv@yahoo.com
phone: +359 52 383 408
fax: +359 52 302 442

Abstract

A uniform and strong magnetic field is required for application in the magnetic resonance imaging (MRI). There are two main approaches to achieve a uniform enough DC magnetic field: a) using a permanent magnet; b) using a resistive magnet (coil). Here the second approach is considered.

Coils of different shapes can be applied to achieve such a uniform DC magnetic field over the vicinity of its center. Here a long solenoid is chosen and analyzed. Another two systems of coils inside one MRI device are: a) gradient coil; b) RF coil, but they are out of our scope here.

In this paper a simple analytical algorithm for improving the uniformity of magnetic field's distribution of solenoid, is presented. The on-axis magnetic flux density is improved by using a variable winding's density.

Suitable numerical results are included in the paper to demonstrate the efficiency of proposed method for improvement of the uniform on-axis magnetic field distribution.

1. INTRODUCTION

Magnetic resonance imaging (MRI) is an efficient nonionizing medical imaging technique used in radiology to visualize the structure of the body by providing detailed images of it in any plane [1 -7]. A strong DC magnetic field *aligns* the nuclear spins of the hydrogen protons of the water contained in the body, while a perpendicular RF magnetic field *alters* the alignment of these protons following prescribed pulse sequences. This is very similar to the phenomenon of gyrotropy in a ferrite where an RF field perpendicular to the *DC magnetic field* modulates the magnetic dipole moments (to provide an anisotropic macroscopic response).

After each pulse, the protons drift back into alignment with the DC field, thereby emitting a detectable RF signal, which is picked up by coils and recorded. The MRI images with spatially varying contrast can then be constructed by various *digital signal processing (DSP) techniques* based on the fact that protons in different tissues of the body (e.g. fat vs. muscle) realign at different speeds. A critical issue for high-resolution imaging in MRI is the capability to generate an extremely *uniform DC magnetic field* across the field of view [8 –11]. In principle, it can be also applied for the off-axis magnetic flux density, but there much more complicated expressions in terms of special functions (elliptic integrals) appear [12]. Because of that, such an opportunity is not further explored in the paper.

Another possible method to solve this problem is, for example, to use a genetic algorithm in order to find the position of every turn of the coil, but this stochastic optimization method is rather time-consuming (because of the big number of parameters involved).

The design of resistive and superconducting magnets is usually based on two well-known coils in electromagnetics. The first one is the so-called *Helmholtz pair* [9], which consists of *two adjacent loops of the same radius carrying same current I in same direction*, as illustrated in Fig. 1. It can be shown easily that when the distance between the two loops *equals* the radius of the loops ($d = R$), the magnetic field B_z is *uniform around* z = 0 through the third power of z:

$$B_z(z) = B_z(0) + O((z/d)^4) \tag{1}$$

where

$$B_z(0) = \frac{4\mu_0 I}{5\sqrt{5}R} \tag{2}$$

One can achieve even a *higher degree of homogeneity* for B_z by using two or more symmetrical pairs of loops but then it is difficult to find the optimum distance analytically. This multiple case yields to the second important coil considered below.

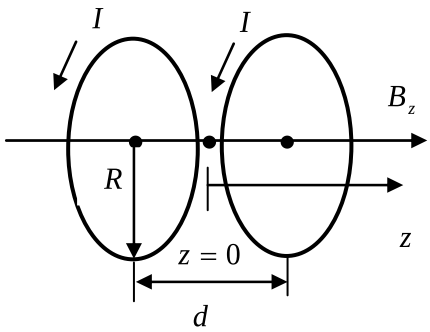

Fig. 1. Geometry of the Helmholtz pair

Another symmetrical source of a relatively *uniform* magnetic field is the cylindrical coil (*solenoid*) [10] with a length *L* and a radius *R* (Fig.2), but with a *length grater than the radius* (*L* >> *R*). The classical result for the uniform magnetic flux density of such a coil is obtained approximately by Ampere's law application

$$B_z(z) = \mu_0 N_0 I \qquad (3)$$

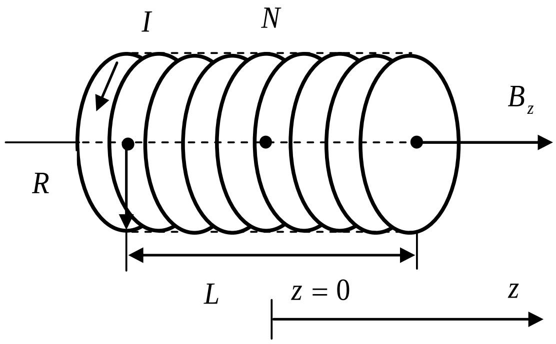

Fig. 2. Geometry of the long solenoid

where $N_0 = N/L = const$ is the *number of windings per unit length* and *N* is the total number of windings. In this paper we propose *a method to improve the uniformity of the axial magnetic field* $B_z(z)$ by exploring a more general case with a *variable winding's density*. Because of the geometrical symmetry in magnetic field distribution it is natural to assume that the last function is an *even function*. Let us assume that N_0 is replaced by a simple polynomial of second order

$$M_0(z) = N_0\left(1 + b_2^{'} z^2\right) \qquad (4)$$

where $b_2^{'}$ is a unknown coefficient that *has to be determined* in such a way so the corresponding magnetic field to be *uniform as much as possible*. Here $M_0(z) = N_0$ is the particular case when $b_2^{'} = 0$ (uniform density). In the next section a more general expression for the on-axis magnetic field of a solenoid with non-uniform winding's density $M_0(z)$ is derived.

2. MAGNETIC FIELD OF NON-UNIFORM SOLENOID

The on-axis magnetic field of a solenoid, represented in Fig.2, can be obtained by the solution of the following superposition integral [11]

$$B_z(z) = \frac{\mu_0 I R^2 N_0}{2} \int_{-L/2}^{L/2} \frac{\left(1 + b_2^{'} z'^2\right) dz'}{\left[R^2 + \left(z - z'\right)^2\right]^{3/2}} \qquad (5)$$

where z' is the integral variable. It is convenient to introduce new *relative variables*: $\varsigma = 2z/L$, $\eta = 2z'/L$ and new *relative parameter*: $\rho = 2R/L$. It is clear that for a long solenoid the interest region of the last parameter is $\rho < 1$ or $L > 2R$. A substitution with the new variables into (5) yields

$$B_z(\varsigma) = \mu_0 N_0 I \frac{\rho^2}{2} \int_{-1}^{1} \frac{\left(1 + b_2 \eta^2\right) d\eta}{\left[\rho^2 + \left(\eta - \varsigma\right)^2\right]^{3/2}} \qquad (6)$$

where new coefficient $b_2 = b_2^{'} L^2/4$ is involved. The particular case of $b_2 = 0$ can be solved by setting: $\eta = \varsigma + \rho \tan\varphi$ and by direct integration - this gives the well-known magnetic field of a uniform coil. Taking into account that $d\eta = \rho \dfrac{d\varphi}{\cos^2\varphi}$ for the non-uniform coil is obtained

$$\boxed{B_z(\varsigma) = \frac{\mu_0 N_0 I}{2} \left\{ \begin{matrix} D_0(\varsigma) + \\ b_2\left[\varsigma^2 D_0(\varsigma) + 2\varsigma\rho D_1(\varsigma) + \rho^2 D_2(\varsigma)\right] \end{matrix} \right\}} \qquad (7)$$

where new auxiliary integrals are introduced

$$\boxed{D_k(\varsigma) = \int_{\varphi_1(\varsigma)}^{\varphi_2(\varsigma)} \tan^k \varphi \cos\varphi \, d\varphi} \text{ (k=0,1,2)} \qquad (8)$$

with the following integration limits

$$\varphi_1(\varsigma) = a\tan\left(\frac{-1-\varsigma}{\rho}\right); \ \varphi_2(\varsigma) = a\tan\left(\frac{1-\varsigma}{\rho}\right) \qquad (9)$$

For the uniform case ($b_2 = 0$) only the first integral D_0 (ς) survives and for an *infinite long* coil $L \to \infty$, $D_0(\varsigma) \to 2$,that yields the *uniform* field (3).

After lengthy but straightforward calculations (omitted here) the following final expressions *in terms of elementary functions* are obtained for the three integrals above

$$D_0(\varsigma) = \frac{\alpha_2(\varsigma)}{\sqrt{1+\alpha_2^2(\varsigma)}} - \frac{\alpha_1(\varsigma)}{\sqrt{1+\alpha_1^2(\varsigma)}} \qquad (10)$$

with parameters

$$\alpha_1(\varsigma) = -\frac{1+\varsigma}{\rho};\ \alpha_2(\varsigma) = \frac{1-\varsigma}{\rho} \qquad (11)$$

Similarly:

$$D_1(\varsigma) = \frac{1}{\sqrt{1+\alpha_1^2(\varsigma)}} - \frac{1}{\sqrt{1+\alpha_2^2(\varsigma)}}, \qquad (12)$$

and

$$D_2(\varsigma) = D_3(\varsigma) - D_0(\varsigma) \qquad (13)$$

Here new auxiliary integral is introduced

$$D_3(\varsigma) = \int_{\varphi_1(\varsigma)}^{\varphi_2(\varsigma)} \frac{d\varphi}{\cos\varphi} \qquad (14)$$

which leads to the following final expression

$$D_3(\varsigma) = \ln\left|\frac{(1+t_2(\varsigma))(1-t_1(\varsigma))}{(1-t_2(\varsigma))(1+t_1(\varsigma))}\right| \qquad (15)$$

where

$$t_1(\varsigma) = \tan\left(\frac{\varphi_1(\varsigma)}{2}\right);\ t_2(\varsigma) = \tan\left(\frac{\varphi_2(\varsigma)}{2}\right) \qquad (16)$$

is set.

The magnetic field of the solenoid with non-uniform winding's density can be found by *closed-term expression* (7), where the three auxiliary integrals are defined by expressions (10), (12), (13) and (15).

3. NUMERICAL RESULTS

Good enough results are obtained for the case of a long solenoid (when $\rho < 1$) – here the parameter b_2 is in the range $0 < b_2 < 1$. Several results of simulations in the special case $\rho = 0.2$ and with different values of the parameter b_2 are shown in Figs. 3 – 5. For this particular case very uniform on-axis magnetic field is obtained for the value $b_2 = 0.12$ which may be considered as an *optimum* here (Fig. 4).The real range of the coordinate z is $0 \le |z| < 0.5L$ $(0 \le |\varsigma| < 1)$. However, we will explore the magnetic flux density $B_z(z)$ in the interval $0 \le |z| < 0.3L$. For this particular choice *the magnetic field is very uniform*. For every value of the shape parameter ρ a suitable optimum value of the other polynomial parameter b_2 could be found. The corresponding winding's density for these three cases is shown in Fig. 6. It is obvious that to *compensate* the decreasing of the on-axis magnetic flux density near the ends a *suitable increasing* of the winding's density of the coil there has to be applied (see the curve for the optimal case $b_2 = 0.12$ with a dashed line).

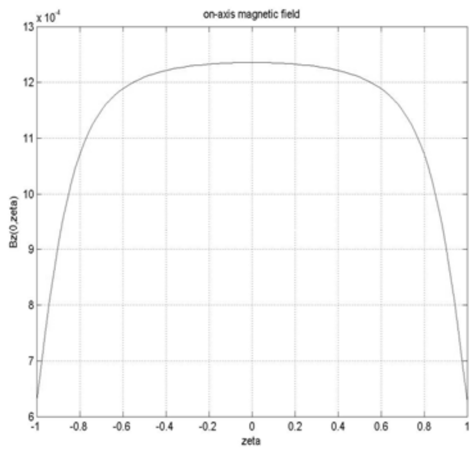

Fig. 3. Magnetic field for the case $b_2 = 0$ (uniform winding's density).

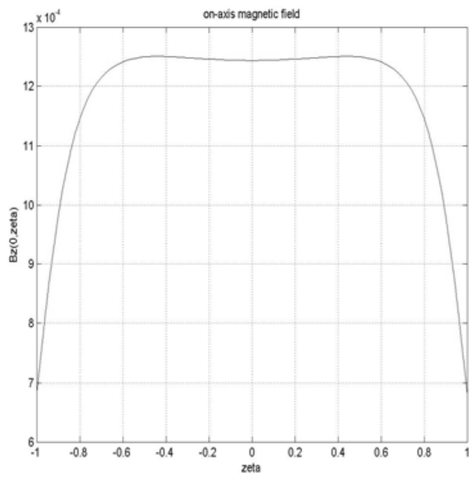

Fig. 4. Magnetic field for the case $b_2 = 0.12$ (optimal case)

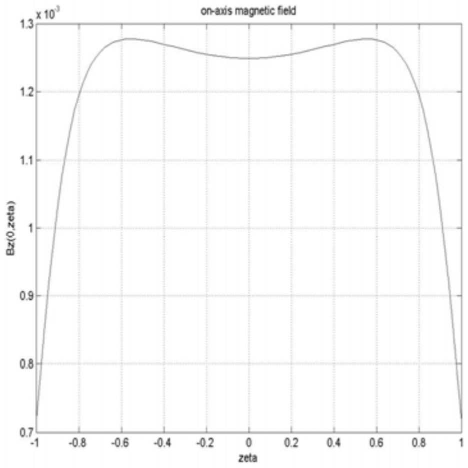

Fig. 5. Magnetic field for the case
$b_2 = 0.20$

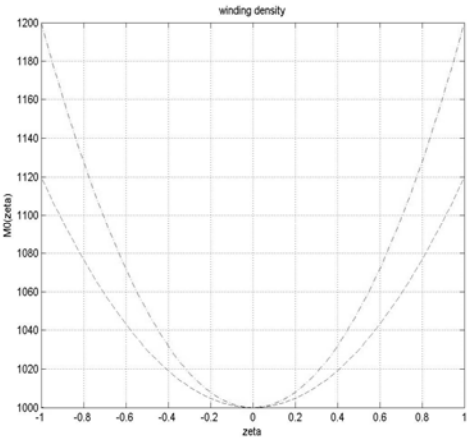

Fig. 6. Winding's density for the case $N_0 = 1000$
(b_2 = 0 – solid line; b_2 = 0.12 - dashed line; b_2 = 0.20 - dash-doted line)

4. CONCLUSION

In this paper one simple method of *improving the homogeneity* of the axial magnetic field of solenoid is considered. We have analyzed the case of variable winding's density – the numerical calculations are based on the derived equation (7) which involves *elementary functions only* for the integrals (8). They show that for every shape parameter $\rho = 2R/L$ a specific *optimum value* for the coefficient $b_2^{'} = b_2 4/L^2$ of the polynomial (4) exists. The problem with achieving a *uniform magnetic flux density* B_z on the z-axis of a permanent magnet is important in the case of MRI and also in other medical applications [13 – 16].

5. ACKNOWLEDGMENT

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