

COMPUTER SIMULATION OF INTERPOLATION OF THE SPECTRUM OF ECG SIGNAL

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Abstract

Computing experiments for restoration of the ECG signal's spectrum by means of interpolation were carried out. A loss of spectrum for different frequency values was simulated. The computations were made in Matlab. The built-in functions for interpolation in Matlab were used, as well as the mathematical method of L. Aizenberg for analytical continuation of finite spectrum. The software, which provides application of Aizenberg's method, has been programmed also in MATLAB. The errors of signal's restoration in the cases of different number of values of loss have been calculated.

The results received from the built-in Matlab functions and algorithm written on the Aizenberg's method was compared.

1. INTRODUCTION

It is quite often necessary to remove the noise, focused in determined band while accepting a signal with finite Fourier spectrum. Or, which is mathematically the same - to eliminate noise in the spectrum focused on the Fourier finite signal in a band. This is a task for the interpolation of the spectrum of Fourier of a finite signal or a task for signal interpolation of finite spectrum. The report presents the results of interpolation of the spectrum of cardio signal.

In order to support clinical decision-making, reasoning tool to the ECG signal must be clearly represented and filtered, to remove out all noises and artifacts from the signal. ECG signal is one of the biosignals that is considered as a non-stationary signal and needs a hard work to denoising. Interpolation of signal, which is lost at a certain time interval must be used.

In regard to this, in the computer experiments Aizenberg's formulas have been used for interpolation of the analytical functions and built-in Matlab functions `interp 1` and `interpft`.

2. INTERPOLATION OF THE SPECTRUM OF ECG SIGNAL WITH AIZENBERG METHOD

Viner's class W_{α}^{+} , consists of functions, which have Fourier spectrum, concentrated in the interval $[0, \alpha]$. The task for interpolation of the function $f(x)$

in W_{α}^{+} is solved from L. A. Aizenberg [1]. If $f \in W_{\alpha}^{+}$, the formula for interpolation of $f(x)$ has the following form (1):

$$f(x) = \lim_{m \rightarrow \infty} \sum_{k=1}^m f(x_k) \frac{2i\sigma}{x - x_k + 2i\sigma} \prod_{\substack{j=1 \\ j \neq k}}^m \frac{(x - x_j)(x_k - x_j + 2i\sigma)}{(x - x_j + 2i\sigma)(x_k - x_j)} \quad (1)$$

The known values of the spectrum are marked with $f(x_k)$, they are numbered m .

The above formula can be applied for interpolation of the spectrum of one-dimensional signals and in item 4 the results of the recovery of spectrums of ECG signals will be presented as a simulated loss of spectral values.

Formula (1) contains two parameters m and σ . The accuracy of signal restoration depends on their values. Results from previous experiments show that the number m of the known values of the spectrum must be not more than 30.

The parameter σ is defined by searching of the minimal root mean-square error ε_{sqr} by formula (2) between the real and interpolated spectral values:

$$\varepsilon_{sqr} = \sqrt{\frac{1}{n} \sum_{i=0}^n (x_i - \tilde{x}_i)^2} \quad (2)$$

where x_i and \tilde{x}_i are the values of the real and restored signal.

3. COMPUTER EXPERIMENTS FOR SPECTRUM INTERPOLATION OF ECG SIGNAL

Data from echograph are entered in the form of Microsoft Excel WorkBook (xls). The Excel table contains 3 columns of 7680 values for the cardiac signal parameters. Each of these columns is imported into the Matlab's Array Editor and stored as a separate variable in Matlab's Workspace. Then it is stored in Workspace as a .mat-file. Formula (1) is programmed on Matlab as a function, called in program in the form of m-script file. The algorithm of the program is as follows:

1. .mat-file with cardiogram data opens
2. With the function fft from Matlab the spectrum of the signal (separately for the both channels) is found
3. The user enters the values of the start and end points of the frequencies of both intervals in which the signal (named s0) of the first channel is known.
4. Function is called, in which formula (1) is programmed. The missing spectral values can be found by it using interpolation.
5. Graphs of the theoretical ECG signal' spectrum and the signal obtained by formula (1) are plotted.
6. The results for the absolute and root mean-square errors of the signal from the first channel are found.
7. Steps 3 - 6 are repeated for the signal (named s1)'s spectrum from the second channel.

Computational experiments for restoration of the signal are performed by the use of the functions interp1 and interpft as well. The values of the spectrum are introduced as complex numbers in the function using the algorithm Aizenberg. The functions interp1 and interpft required values of the function that will interpolate to be real numbers. Therefore, as input data for these functions amplitude signals from each of the two channels of the ultrasound are used.

In the spectrum of the interpolation with interp1 a method 'pchip' is used. (Piecewise cubic Hermite interpolation). It gave better results (less error recovery) compared with the methods 'linear' (Linear interpolation) and 'nearest' (Nearest neighbour interpolation). Interpolation according to the method 'spline' was not done, because during its performance interp1 function failed and issued an error. Since when interpft function was used a very large absolute error $\varepsilon_{abs} = 69.8726$ was ob-

tained, in section 4 only the results of interpolation with interp1 are presented.

A computing experiment on interpolation of a spectrum with noise of ECG signal using the method of Aizenberg, was carried out. A block diagram of the algorithm of Matlab-function that realizes the formula (1) is presented in Fig. 1. Variables in it have the following meanings: Input parameters: IN1B - the first frequency from x_k in the first interval (before the area with noise) where the values of the spectrum are known; IN1E - the last frequency of x_k in the first range; IN2B - the first incidence from x_k in the second interval (after the area with noise) where the values of the spectrum are known; IN2E - the last frequency from x_k in the second interval; M - the number m of formula (1) of all the known values of the spectrum; sf - the input spectrum (with noise) signal; sigma - parameter σ of (1); MAXABSER - first value of absolute error, MAXABSER = 0.001; MAXSQRER - first value of mean-square error, MAXSQRER = 0.001.

Output parameters: MAXSQRER1 - calculated mean square error; MAXABSER1 - calculated absolute error; S - interpolated amplitude spectrum of f(x) of (1); F2 - interpolated spectrum in complex form; Stru - the real amplitude (spectrum with noise)

Auxiliary variables: L, J and K are used for management of the cycles, P - production Π in (1), SUM - Σ of (1) X - vector of frequencies x from (1); LL - secondary variable for calculating mean square error.

4. RESULTS FROM THE COMPUTING EXPERIMENTS

On Figure 2 it is shown the interpolation, using (1) of a spectrum of a recorded ultrasound signal in time domain. The spectrums of the signals s0 and s1 from the two channels are calculated and it is modelled loss of one spectral component under number 16. On Fig. 2 in both signals the number of known spectral values $m = 28$. Errors of the recovered signals in the frequency domain are: for s0: absolute error: $\varepsilon_{abs} = 0.9505$, root mean square error $\varepsilon_{sqr} = 0.1796$; for s1: $\varepsilon_{abs} = 0.7642$, root mean square error $\varepsilon_{sqr} = 0.1419$. On all figures the graph of the interpolated values is in blue, and the real (unprocessed) signal values are in red.

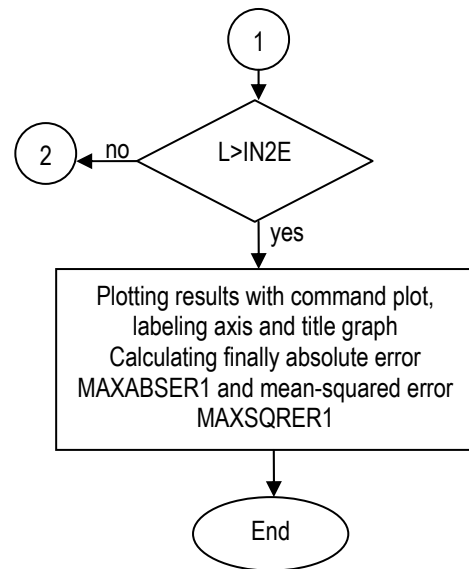
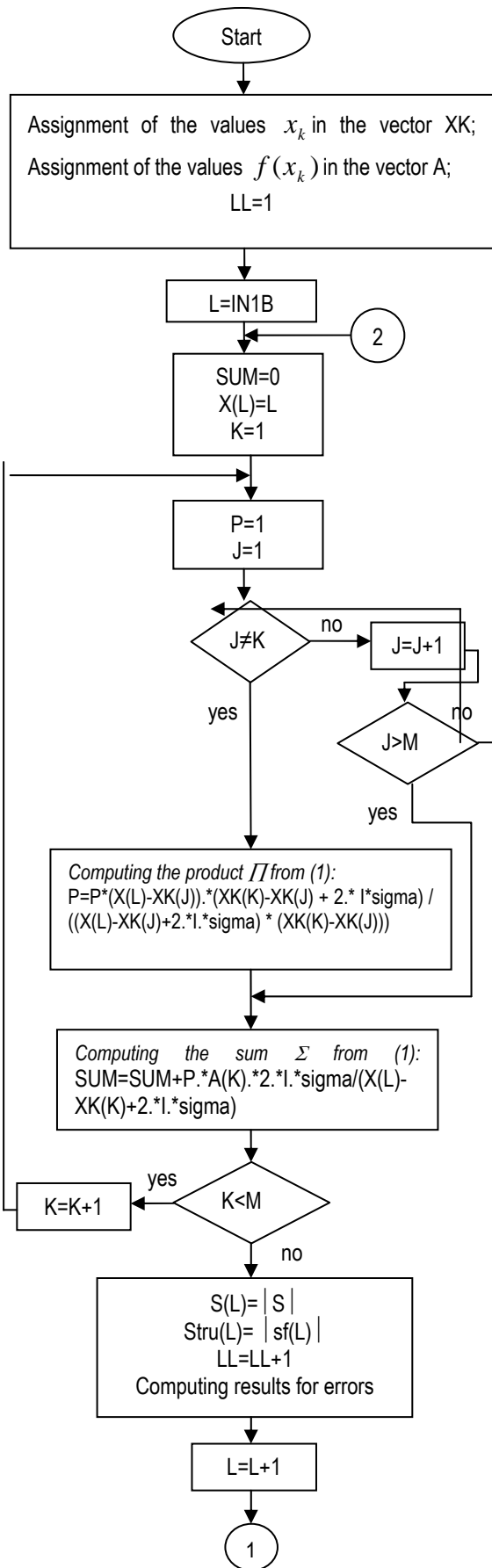


Fig. 1. Scheme of algorithm, computing formula (1)

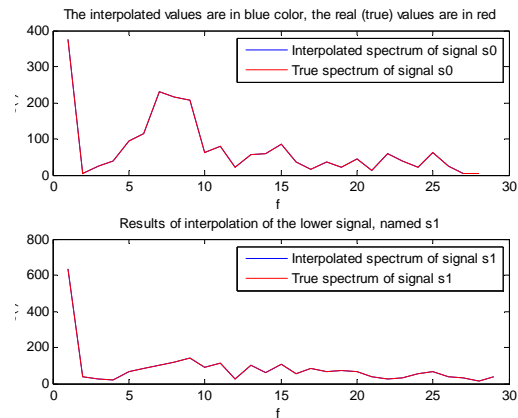


Fig. 2. Interpolation of the spectrum by the method of Aizenberg when 1 spectral value is lost

Fig. 3 shows the interpolation of 16 component of the spectrum signal s0, using built-in function interp1, method pchip. Values obtained for mean-square and absolute error were as follows: $\mathcal{E}_{sqr} = 2.6605$ и $\mathcal{E}_{abs} = 14.3271$.

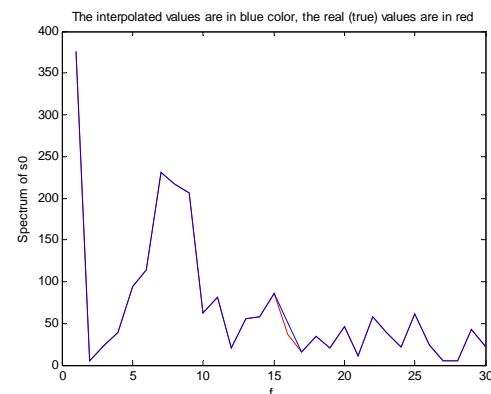


Fig. 3. Interpolation of the spectrum by interp1 when one spectral value is lost

From Fig. 2, Fig. 3 and the values of the errors show that better results in the interpolation of a value obtained through the application of the method of Aizenberg. In Fig. 4 a) is presented the spectrum of ECG with noise, and in Fig. 4 b) - interpolated spectrum for frequencies in the range [41, 42] where there is noise.

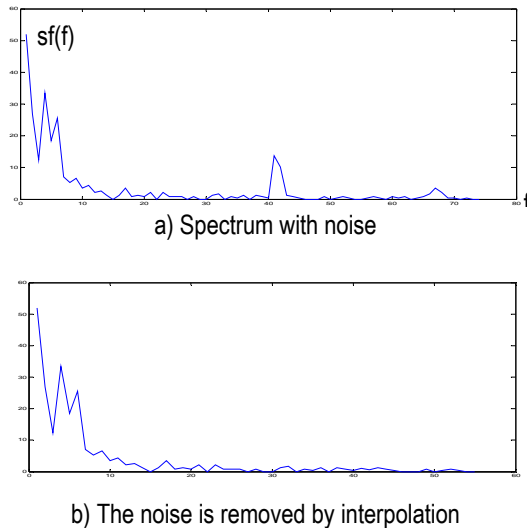


Fig. 4. Interpolation of the noised spectrum by the method of Aizenberg when two spectral values are lost

5. CONCLUSION

The Aizenberg method gives more accurate results (less error) for the recovery of the spectrum of ECG signal in comparison with built-in Matlab function `interp1` and `interpft`. For recovering the values in a larger interval the formula (1) can be repeatedly used, as x_k are taken also from the frequency of recovered intervals in previous figures.

Note: The scientific results described in this paper have been obtained on the base of financial support of contract No.102nđ41-7

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