APPLICATION OF MODIFIED FRACTAL SIGNATURE & REGNY SPECTRUM METHODS TO THE ANALYSIS OF BIOMEDICAL PREPARATIONS IMAGES

N. B. Ampilova

Assoc. Prof. Comp. Sci. Dept, Math. & Mech. Faculty, St.Petersburg State University
Universitetsky pr. 28, Starii Peterhof, St.Petersburg 198504
ampilova@math.spbu.ru

E. Y. Gurevich

Sofia & Jacobs Foundation
Italyanskaya str. 10/5, St.Petersburg 191186
info@organicproduct.ru

I. P. Soloviev

Assoc. Prof. Comp. Sci. Dept, Math. & Mech. Faculty, St.Petersburg State University
Universitetsky pr. 28, Starii Peterhof, St.Petersburg 198504
soloviev@math.spbu.ru

Abstract

The problem discussed in the paper is the essential part of the general research concerning the action mechanism of ultra-low doses (ULD) of medical preparations.

In this work we investigate a possibility to apply two fractal methods to analyze some classes of biomedical images. The first method — Modified Fractal Signature — was proposed in [6] for document analysis for automatic knowledge acquisition. The second method is based on the calculation of Regny spectrum for multifractal sets.

The investigation of applicability of described methods to 3 classes of preparations has been performed. Experimental results show that the method of modified fractal signature may be used for all given classes of images, whereas values of Regny spectrum for brain tumors do not allow us to classify them completely and an additional analysis has to be done. The important result of this work is that both methods are applicable to ULD-containing preparations.

1. Introduction

Traditional medicine as a rule uses simplified (linearized) models to describe the mechanism of a medical preparation effect on a living organism. At the same time the search of an adequate treatment methods requires to study the more complex, nonlinear and nonmonotonic reaction of an organism on the action of drug. To solve the problem we have to extend existing traditional methods by applying ULD of medical preparations. Investigations of ULD effects were performed by many scientists and now extensive experimental data are accumulated. But the dynamics of processes concerning to ULD effect is rather complex and at the present time there is no any adequate mathematical model describing it. Nonetheless the research of such processes is performed by many methods, being an analysis of process images obtained in different points of time is a principal one. The methods widely used are texture, fractal, morphological analysis and neural network modeling as well.

A large body of research shows that self-organization property that is (according to [5]) "spontaneous beginnings of a structure, i.e. the appearance of an ordered state in an initial random distribution of the system components without apparent external action" is intrinsic to biological systems. We may consider images of such processes states as phase portraits of a complex dynamical system. Stable invariant sets (attractors) of self-organized systems often have complex geometrical structure — they are fractals or multifractals (unions of several fractal sets, being everyone has own fractal dimension). For such sets methods of calculatiopn of fractal dimensions are used. Modified Fractal Signature method [6] is based on Minkovsky dimension that coincides with well-known box-counting dimension for non-empty bounded sets in R^n. The basic idea of this approach is that a document is mapped onto a gray-level function. Furthermore, this function can be mapped onto a gray-level surface, and from the area of such a surface (called fractal signature) the fractal dimension of the document image can be approximated.

Widely used approach is to calculate the special characteristic – multifractal (Regny) spectrum. Let $\{\mu_i\}$ be a probability measure defined on the set under investigation. The main idea of this approach is to use Regny entropy $S(q) = \ln \sum_i \mu_i^q / 1 \quad q$ which depends on a parameter q and coincides with Gibbs-Shannon entropy $S = \sum_{i=1}^{W} \mu_i \log \mu_i$ for $q = 1$ [1]. Unlike Shannon entropy, maximal value of Regny entropy corresponds to an ordered state of the system. Maximum principle, as applied to multifractal systems allows us to obtain a set of numbers (for different q) that are fractal dimensions of subsets in which Regny entropy is maximal. The spectrum is invariant with respect to brightness changing, rotation and scaling. Hence it may be used as some stable feature for a class of images. So, in [8] authors apply multifractal spectrum for texture analysis. It should be marked that there are several ways to calculate Regny spectrum ([3], [7], [8]). In this paper we follow the method developed in [7] that is based on the generation of coarsened partitions.

2. Main notions

Definition 1. Let F be a nonempty bounded set in R^n, $\Omega = \{\varpi_i : i = 1,2,3,\dots\}$ — a covering F, $N_\delta(F)$ — the number of sets from Ω whose diameters are nongreater than δ. Let $dim_H F$ and $dim_T F$ denote Hausdorf and topological dimensions F respectively. F is said to be fractal if $dim_H F > dim_T F$. Box-counting (capacity) dimension is defined by $dim_B F = \lim_{\delta\to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}$.

Definition 2 [6]. δ-parallel body F_δ can be defined by $F_\delta = \{x \in R^n : |x - y| \le \delta, y \in F\}$.

Definition 3. Let F be a nonempty and bounded set in R^n, F_δ — δ-parallel body F, $Vol^n(F_\delta)$ — n-dimensional volume of F_δ. For a constant s, if δ→0, the limit of $Vol^n(F_\delta)/\delta^{n-s}$ is positive and bounded, we say that F has Minkovsky dimension s, which is symbolized by $dim_M F$.

Theorem 1 [3] Let F be a nonempty bounded set in R^n. Then $dim_B F = dim_M F$.

3. Methods of analysis

3.1. Modified Fractal Signature Method

3.1.1. Method description

In [6] the authors call the method the Modified Fractal Signature appropach since the direct computation of fractal dimension of a document image is not used, alternately the volume of a special thing — δ-parallel body — is estimated to approximate the fractal dimension.

Let $F = \{X_{ij}, i = 0,1,\dots,K, j = 0,1,\dots,L\}$ be an image with multigray level and X_{ij} be the gray level of the (i, j)th pixel. In a certain measure range, the gray-level surface of F can be viewed as a fractal. The surface area A_δ can be used to approximate its fractal dimension. In image processing the gray level function F is a nonempty bounded set in R^3. The surface area may be calculated using the volume of a special δ-parallel body — blanket with the thickness 2δ. Denoting this volume $Vol^3(F_\delta)$ we have $A_\delta = Vol^3(F_\delta) / 2\delta$.

But according to the Definition 3 and Theorem 1 we can conclude that if

$$\lim_{\delta\to 0} Vol^3(F_\delta) / \delta^{3-D} = \beta > 0,$$

then $D = dim_M F = dim_B F$. Therefore, when δ is sufficiently small, we have $Vol^3(F_\delta) = \beta\delta^{3-D}$. So, we have $A_\delta = \frac{Vol^3(F_\delta)}{2\delta} = \beta\frac{\delta^{2-D}}{2}$, from which the fractal dimension D can be obtained. The value A_δ is said to be Fractal Signature. It should be noted that the essential distinction of images is their values of A_δ, hence in practice it is sufficient to calculate only fractal signatures.

According to Blanket Technique ([6]) the covering blanket is defined by its upper surface $u_\delta(i,j)$ and its lower surface $b_\delta(i,j)$. Initially, δ=0 and $u_0(i,j) = b_0(i,j) = X_{ij}$.

For δ=1, 2,… the blanket surfaces are defined iteratively as follows:

$$u_\delta(i,j) = \max\left\{ u_{\delta-1}(i,j)+1, \max_{|(m,n)-(i,j)|\le 1} u_{\delta-1}(m,n) \right\},$$

$$b_\delta(i,j) = \min\left\{ b_{\delta-1}(i,j)-1, \min_{|(m,n)-(i,j)|\le 1} b_{\delta-1}(m,n) \right\}.$$

The volume of the blanket Vol_δ is computed from u_δ and b_δ:

$Vol_\delta = \sum (u_\delta(i,j) - b_\delta(i,j))$.

In practice, whole image F is divided into several non-overlapping subimages and A_δ is computed for every subimage. Then all A_δ are combined into the whole signature. Sometimes it is convinient to study a "map" of the image, where in every cell the corresponding fractal signature is written.

3.1.2. Numerical experiments

Fractal signatures were calculated for some kinds of connective tissues of animals. The picture shows one of images and the graphic illustrates the dependence of normalized fractal signature on the size of the partition box. Method demonstrates good separability of values for different classes of tissues.

Figure 1. Connective tissue: pharynx

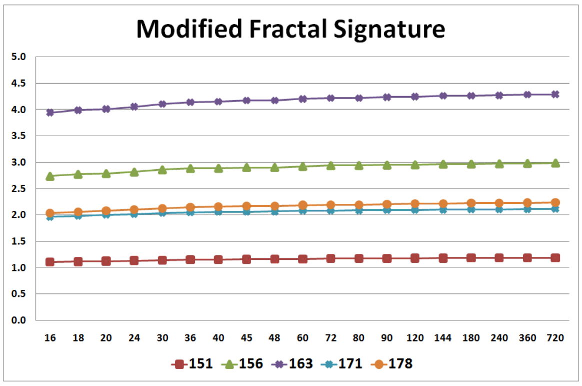

Figure 2. Fractal signatures for several types of connective tissue

3.2. Regny spectrum

3.2.1. Method description

Consider an irregular object embedded into Euclidean space and divide it into N boxes of sizes $l_i \le l, i = 1, \dots, N,$ where

$l < 1$ and the size of the whole object equals 1. Let $\{\mu_i\}$ be a probability measure defined on the partition. Suppose that $\mu_i \sim l_i^{\alpha}$, where α is a scaling value that can take the values from some range with a probability density $\rho(\alpha) l^{-f(\alpha)}$, and so the probability to get α from an interval $(\alpha', \alpha' + d\alpha')$ is $d\alpha' \rho(\alpha') l^{-f(\alpha')}$, and a continuous function $f(\alpha')$ shows the fractal dimensions of the sets on which the singularities of strength α' may lie [4].

Now we consider the function $\chi(q) = \sum_{i=1}^{N} \mu_i^{\,q}$. It follows that $\chi(q) = \int d\alpha' \rho(\alpha') l^{-f(\alpha')} l^{q\alpha'}$.

When l is small enough $\chi(q)$ is maximal for $\alpha = \alpha(q)$ such that $q\alpha' - f(\alpha')$ is minimal (i.e. maximum of Regny entropy is achieved). It is known [3] that there is a unique finite nonzero function $\tau(q)$, such that

$$\chi(q) \sim l^{\tau(q)},\ \tau(q) = \lim_{l\to 0} {\ln \chi(q)}/{\ln l}$$

and $\tau(q) = q\alpha - f(\alpha)$, $d\tau(q)/dq = \alpha$, i.e. $\tau(q)$ and $f(\alpha)$ are connected by Legendre transformation [3].

Regny spectrum $\{D_q\}$ is defined by the equation $\tau(q) = (q-1)D_q$, where D_0, D_1, D_2 are Hausdorf, information and correlation dimensions respectively [2].

There are several methods to approximate D_q. We use the method of coarsening partitions developed in [7]. Consider K partitions of an image into boxes with sizes $r_k, k = 1, \dots, K$. Let μ_{ik} be a measure of the box i for k*th* partition. Let S_k be the number of the boxes in k*th* partition for which $\mu_{ik} > 0$. Then $\alpha(q)$ and $f(\alpha(q))$ may be calculated by the formulas:

$\alpha(q) \approx \frac{A_k(q)}{\ln r_k}$, where

$$A_k(q) = \frac{\sum_i^{S_k}((\mu_{ik})^q \cdot \ln(\mu_{ik}))}{\sum_i^{S_k}(\mu_{ik})^q},$$

$f(\alpha(q)) \approx \frac{F_k(q)}{\ln r_k}$, where

$$F_k(q) = \frac{\sum_i^{S_k}\left((\mu_{ik})^q \cdot \ln\left(\frac{(\mu_{ik})^q}{\sum_i^{S_k}(\mu_{ik})^q}\right)\right)}{\sum_i^{S_k}(\mu_{ik})^q}.$$

For a fixed q values $A_k(q)$ and $F_k(q)$ are calculated for all K partitions. Hence in coordinate systems $(A_k(q), \ln r_k)$ and $(F_k(q), \ln r_k)$ there are K points that have to be approximated by a straight line. Using above formulas and least-squares method we obtain $\alpha(q)$, $f(\alpha(q))$, then $\tau(q)$ and D_q.

3.2.2. Numerical experiments

Partitions into boxes are chosen depending on the size of the image. The box measure is defined as light level of the box concerning to the number of light pixels of the whole image.

For every image Regny spectrum was calculated for q =0, 1, 2, 3, 4, 5, 10, 20, 30, 40, 50.

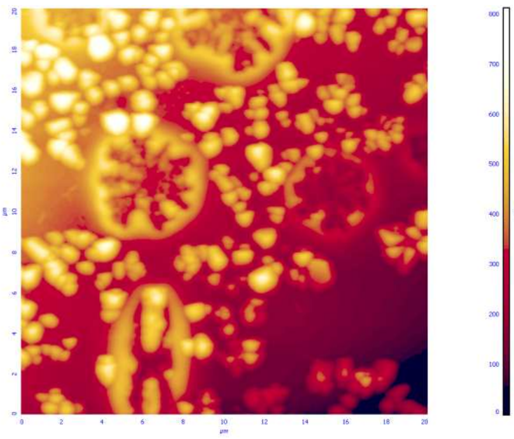

Figure 3. A compound with large dose of Ag

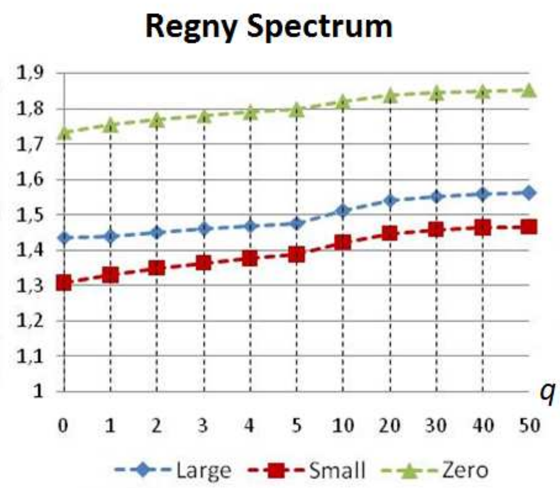

Figure 4. Regny spectrum for compounds with large, small and zero concentrations of Ag.

Conclusion

The results of numerical experiments performed for images of biomedical preparations of different classes shows that both methods are applicable to calculate special characteristic of images. They allow us to obtain numerical values that are invariant for some images transformations. We can classify images of ULD-containing preparations on accordance with the dose.

Both methods supplemented by texture and morphological analysis may be considered as a basis for image analysis.

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