MULTIFRACTAL SPECTRUM AS A CLASSIFICATION SIGN FOR BIOMEDICAL PREPARATIONS IMAGES

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Abstract

The paper is devoted to the application of methods of multifractal image analysis to classify biomedical preparations images.

We use two methods for calculation of multifractal spectrum: the direct determinarion and the moment-based method, being the last one does not use Regny dimensions with the following application of the Legendre transform. Such an approach results in increasing the precision of calculation.

For 4 classes of biomedical preparation images (histological preparations, blood cells, connective tissue and bone tissue) two classes (histological preparation and bone tissue) for which multifractal spectrum is a reliable classification sign have been marked out. At the same time the analysis of images of blood cells with leukemia and connective tissue shows that it is necessary to apply additional methods to obtain proved results.

1. INTRODUCTION

It is now widely accepted that physical systems that exhibit chaotic behavior are generic in Nature. Since it is possible to follow and predict their motion in any detail only for short time scales, one must use suitable statistical descriptions to describe the system asymptotic.

Considering an image as the phase portrait of a complex dynamical system we can use for its description multifractal formalism as a statistical characteristics. The multifractal formalism relies on the fact that highly nonuniform probability distributions arising from the nonuniformity of the system often possess rich scaling properties such as self-similarity. Hence we can associate a characterization of the fractal properties of a measure with the nonuniform distribution.

Spatial distribution of dissipative regions in a turbulent flow, the invariant probability distribution on a strange attractor and the distribution of growth probabilities on the external surface of a diffusion-limited aggregate are some examples.

The multifractal formalism describes the statistical properties of these singular measures in terms of their singularity (multifractal) spectrum or their generalized dimensions.

Cover the support of the measure with disjoint boxes of size *l* and define for each box a measure as l^{α_i}, where α_i are real numbers. Then the subsets of points (E_α) for which values α_i are equal or close are defined. For each subset its fractal dimension $f(\alpha_i)$ is calculated. So, the image is considered as a union of interwoven subsets, being each of them has own fractal dimension. The set of these dimensions forms multifractal spectrum (MFS).

The method of MFS direct determination is described in [6]. It should be noted that the finding of E_α sets means a categorization of an image points. In fact, it is the box measure that describes a categorization. Different methods lead to different multifractal spectra. For digital images the measure is defined through pixel intensities [4].

Generalized (Regny) dimensions D_q provide an alternative description of the singular measure. They correspond to scaling exponents for the *q*th moment of the measure. The generalized dimensions were introduced earlier than multifractal spectrum and they are easier to compute than the latter.

Since these two characteristics describe the same measure, there is a connection between them. Namely, when $f(\alpha)$ and D_q are smooth functions of α and q, they are related by a Legendre transformation. It was believed that for measures arising from real experiments it is easier to calculate generalized dimensions and then obtain multifractal spectrum using the Legendre transform.

But, as was shown in [1], the D_q corresponding to large negative *q* may have large error bars and Legendre transforming the D_q curve may decrease precision considerably. The authors elaborated a new direct method of determining multifractal spectrum based on a canonical method of computing thermodynamic averages. Both $f(\alpha)$ and α are obtained as functions of the parameter q.

It should be noted that multifractal spectrum is widely used in image analysis because it is globally invariant under bi-Lipschitz transform (general transform including brightness changing, rotation, scaling and general texture surface deformations) [3].

In this work we use both the MFS direct determination method and the moment-based method without using Regny dimensions. They may be successfully applied for a classification of 4 classes of biomedical preparation images.

2. METHODS OF ANALYSIS

2.1. MFS direct determination

2.1.1. Method description

Let μ be a measure defined through pixel intensities. For $x \in R^2$ we denote $B(x,r)$ a square of length *r* with center x. We describe $\mu(B(x,r)) = kr^{d(x)}(x)$ with $d(x)$ the density function and k some constant. The local density function of x is defined as

$$d(x) = \lim_{r\to 0} \frac{\log \mu(B(x,r))}{\log r}. \qquad (1)$$

The set of all image points x with local density α is

$$E_\alpha = \{x \in R^2 : d(x) = \alpha\}. \qquad (2)$$

Thus we obtain a point categorization $\{E_\alpha : \alpha \in R\}$ of the image with a multifractal spectrum defined as

$$\{f(\alpha) : \alpha \in R\} = \{\dim(E_\alpha) : \alpha \in R\}. \qquad (3)$$

The density function describes how locally the measurement μ satisfiers the power law behavior. It measures the non-uniformity of the intensity distribution in the square $B(x,r)$. It should be noted that in this method the measurement $\mu(B(x,r))$ is the sum of intensity pixels in the square. The density $d(x)$ is obtained as the slope of the line fitted to the data $\{\log r, \log \mu(B(x,r))\}$ by the least square method. Then we take a discrete set $\{\alpha_i\}$ from an interval and find for each α_i the point set E_{α_i} according to (2).This set contains all pixels whose densities are close to α_i. The example of such a set near the source image is shown below with $\alpha_i \in (1.5, 1.6)$. The fractal dimension $f(\alpha_i)$ is computed as the slope of the line fitted to the data $\{\log \frac{1}{\delta}, \log N(\delta, E_{\alpha_i})\}$, where $N(\delta, E_{\alpha_i})$ is the smallest number of sets of diameter less than δ that cover E_{α_i}.

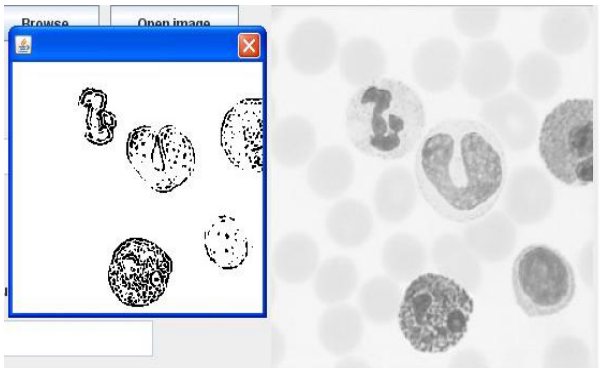

Figure 1. The example of E_α set with $\alpha \in (1.5, 1.6)$

2.1.2. Numerical experiments

Multifractal spectra were obtained for health and attected histological preparations (liver). The following pictures shows the images of the preparations and the graphic illustrates their spectra.

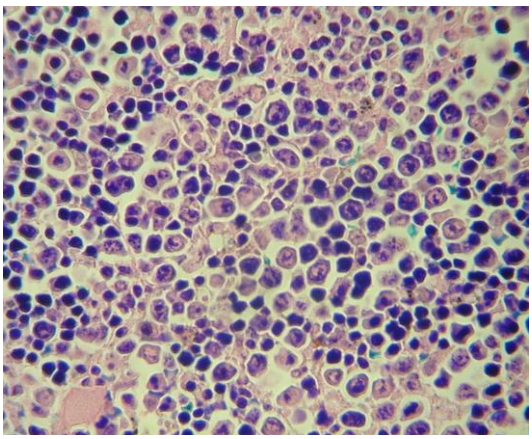

Figure 2. The health liver preparation

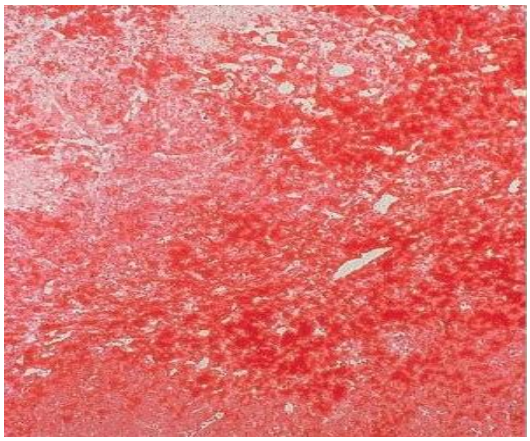

Figure 3. The affected liver preparation

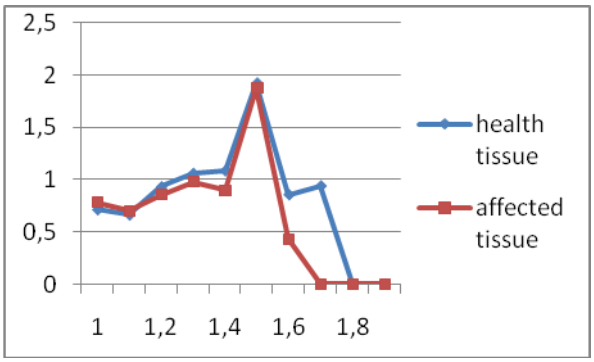

Figure 4. MFS spectra

2.2. Moment-based method

2.2.1. Method description

Let $p_i(r)$ be a normalized *i*th box measure (the sum of box pixel intensities divided by the sum of all pixel intensities), $i \in [1, N]$. Construct a one-parameter family of normalized measures $\mu(q)$, where for *i*th box we have

$$\mu_i(q,r) = \frac{(p_i(r))^q}{\sum(p_j(r))^q}. \qquad (4)$$

According to [2] the Hausdorff dimension of the support of $\mu(q)$ is given by

$$f(q) = \lim_{r\to 0} \frac{\sum_{i=1}^{N} \mu_i(q,r) \ln \mu_i(q,r)}{\ln r}. \qquad (5)$$

In addition we can compute the average value of the singularity strength $\alpha_i = \frac{\ln p_i(r)}{\ln r}$ as

$$\alpha(q) = \lim_{r\to 0} \frac{\sum_{i=1}^{N} \mu_i(q,r) \ln p_i(r)}{\ln r}. \qquad (6)$$

Thus, both α and $f(\alpha)$ may be obtained as explicit functions of the parameter q.

2.2.2. Numerical experiments

Multifractal spectra were calculated by using formulas (4)-(6) for images of 4 classes of biomedical preparations. The most reliable results were obtained for bone tissue and histological preparation images. Below the graphics for health and affected bone tissue (osteoporosis) are shown.

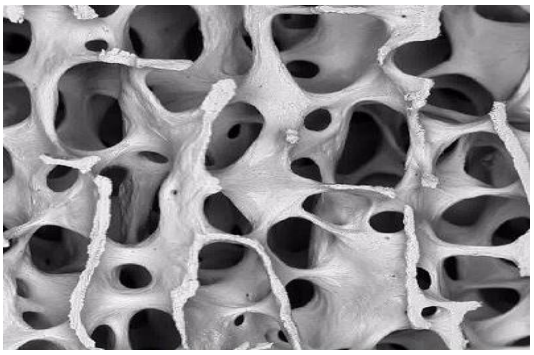

Figure 5. The health bone tissue

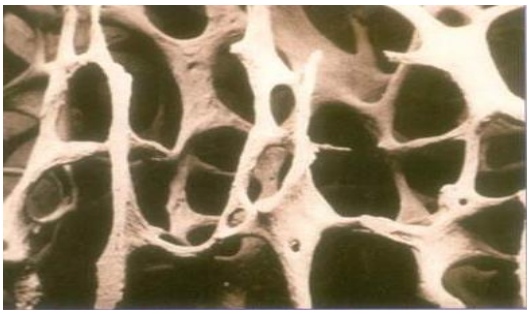

Figure 5. The affected bone tissue

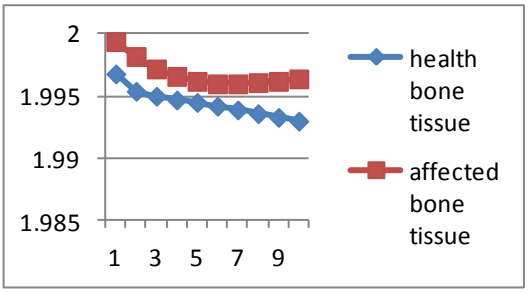

Figure 6. Graphics of (q)

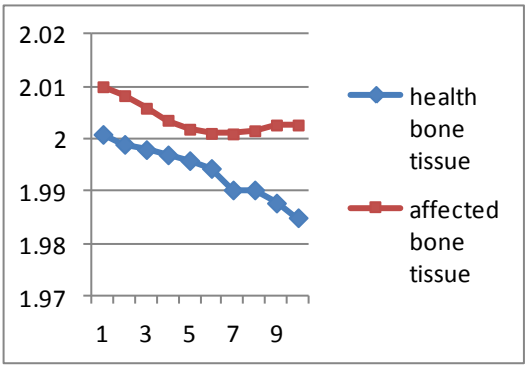

Figure 7. Graphics of f(q)

3. CONCLUSION

For 4 classes of biomedical images: histological preparations, blood cells, connective tissue and bone tissue classification signs have been obtained. The comparison of multifractal spectra for bone tissue allows us to define osteoporosis. At the same time the analysis of images of blood cells with leukemia and connective tissue shows that it is necessary to apply additional methods to obtain reliable results.

References

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