

# INDOOR MULTIPATH RADIO CHANNEL CHARACTERIZATION IN LARGE BUILDINGS

Dimitar G. Valchev

University of Food Technologies  
26 Maritsa Blvd, 4000 Plovdiv, Bulgaria  
Phone: +359 32 603 860; E-mail: dvalchev@ece.neu.edu

## Abstract

This paper presents new mathematical developments and results for modeling three-dimensional multipath radio channels inside large buildings, such as hospitals, malls and industrial sites. Particular examples for propagation scenarios are given and the corresponding characteristics of the received signal are derived from the spatial configuration of the surrounding reflecting objects in vicinity of the receiver. The obtained results are useful for designing spatially directive reception diversity systems that effectively overcome the harmful effects of multipath propagation.

## 1. INTRODUCTION

An important feature of the radio communication channel is its multipath propagation due to various scattering and reflecting objects around the mobile receiver antenna. The multipath propagation is particularly characteristic to large indoor spaces where the transmitted radio signal undergoes many reflections and scatterings before reaching the mobile unit. As a result, multiple signal replicas arrive at the mobile receiver from different directions in three-dimensional space, that is, each radio wave arrives at some azimuth and some generally non-zero elevation relative to the receiver antenna which also travels at some azimuth and elevation.

In the traditional literature [1, 2, 3] the developed multipath channel model is two-dimensional (horizontal). In reality, the multipath environment is three-dimensional since the reflecting and scattering objects have some non-zero height and therefore the many radio waves arrive at the mobile unit at different non-zero elevations. Those non-zero elevations change the received signal correlation and Doppler spectrum [4]. Therefore, a three-dimensional model is the adequate analysis approach when describing realistic multipath wireless channels. In this paper the autocorrelation function of the fading signal at the mobile receiver antenna is derived. First, a mathematical model is developed for describing the multipath propagation channel and the motion of the receiver. Next, the autocorrelation function and the Doppler spectrum of the received signal are derived based on the developed mathematical model. The approach here is alternative to that developed in [5] where the authors first find the

Doppler spectrum, and then through a Fourier transform obtain the signal autocorrelation. The approach here is better when the Doppler spectrum is difficult for direct derivation.

## 2. MATHEMATICAL MODEL

The transmitted signal is assumed to be unmodulated with amplitude  $E_0$  and carrier angular frequency  $\omega_c$ . It is represented as

$$s(t) = E_0 e^{j\omega_c t} . \quad (1)$$

The amplitude  $E_0$  is also called *envelope*. The multipath channel model consists of  $N$  radio waves coming to the receiver. Assuming no attenuation through the different propagation paths in a local volume, the received complex voltage signal at the mobile unit is

$$\tilde{V}(t) = E_0 \sum_{n=1}^N e^{j\psi_n}, \quad \psi_n = \omega_c t + \varphi_n \quad (2)$$

where  $\varphi_n$  is a random initial phase, uniformly distributed from 0 to  $2\pi$ .

The  $N$  radio waves incoming to the receiver have arbitrary directions in three-dimensional space. The mobile unit travels also in an arbitrary direction in three-dimensional space, given by the azimuth-elevation pair  $(\theta, \phi)$ , where  $\theta$  is the polar angle in the  $x$ - $y$  plane and  $\phi$  is the angle between the travel direction and the  $x$ - $y$  plane. The different radio waves come to the receiver antenna at angles  $\alpha_n$  relative to the travel direction of the mobile unit. The angles  $\alpha_n$  are statistically independent of  $\psi_n$ .

For analysis simplification, only single polarization is assumed and only the electric field component is sensed by a suitable antenna with a three-dimensional radiation pattern. The antenna is assumed to have spherically uniform unit gain. The geometrical configuration of the described channel is shown in Fig. 1.

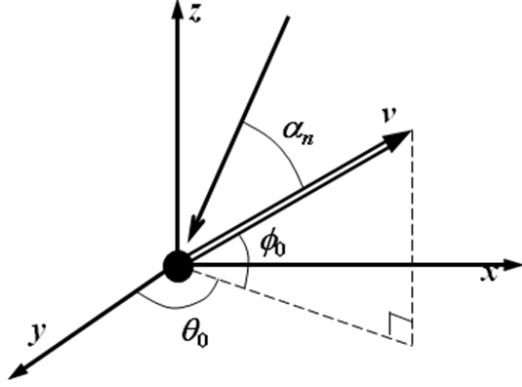


Fig. 1. Geometrical configuration of the radio propagation.

### 3. FADING SIGNAL CORRELATION AND SPECTRUM

The second-order statistics of the received fading signal relate to its fluctuation rate. The fluctuation rate depends on the autocorrelation and the spectrum (the Fourier transform of the correlation) of the received signal. Therefore, finding the received signal autocorrelation is very important for obtaining the different statistical measures of the channel performance.

If the mobile unit travels a distance  $d$  with velocity  $v$ , each radio wave arrives at the antenna at angle  $\alpha_n$  relative to the travel direction, and changes its phase  $\psi_n$  by  $kdc\cos\alpha_n$ , where  $k = 2\pi/\lambda$  is the wave number with  $\lambda$  being the wavelength of the transmitted signal. As a result, the received complex voltage in (2) changes to  $\tilde{V}^d$  and the correlation of the received signal can be found by the expression

$$\begin{aligned} R(d) &= E\{\tilde{V}^* \tilde{V}^d\} = E_0^2 E\left\{\sum_{n=1}^N e^{-j\psi_n} \sum_{m=1}^N e^{j(\psi_m + kd\cos\alpha_m)}\right\} = \\ &= E_0^2 \sum_{n=1}^N \sum_{m=1}^N E\left\{e^{j(\psi_m - \psi_n)} e^{jkd\cos\alpha_m}\right\} = \\ &= E_0^2 \sum_{n=1}^N \sum_{m=1}^N E\left\{e^{j(\psi_m - \psi_n)}\right\} E\left\{e^{jkd\cos\alpha_m}\right\} \end{aligned} \quad (3)$$

where  $E\{\cdot\}$  is the expectation taken over the distribution of the direction of arrival  $\alpha$  of the radio waves at the mobile unit and  $\tilde{V}^*$  is the complex conjugate of the received complex voltage  $\tilde{V}$  in (2).

The last equality in (3) follows from the statistical independence of  $\psi_m$  and  $\alpha_m$ . The ensemble average  $\sum_{n=1}^N \sum_{m=1}^N E\left\{e^{j(\psi_m - \psi_n)}\right\}$  is zero except for  $m = n$  when it is equal to one. Therefore, (3) reduces to

$$\begin{aligned} R(d) &= E_0^2 E\left\{\sum_{m=1}^N e^{jkd\cos\alpha_m}\right\} = E_0^2 N \int_0^\pi p(\alpha) e^{jkd\cos\alpha} \sin\alpha d\alpha = \\ &E_0^2 N \int_0^{2\pi} \int_0^\pi p(\alpha, \beta) e^{jkd\cos\alpha} \sin\alpha d\alpha d\beta. \end{aligned} \quad (4)$$

There is a factor  $\sin\alpha$  in the integral in (4) because  $\alpha$  is a measure of a three-dimensional orientation and therefore the ensemble average in (4) involves integration over the unit sphere. The functions  $p(\alpha)$  and  $p(\alpha, \beta)$  in (4) are the marginal and the joint probability density functions (PDFs), respectively, of the angle (or angles) of arrival of the multipath power at the mobile unit.

#### 3.1. A case of a uniform PDF

For a uniform PDF  $p(\alpha) = 1/\pi$  for  $0 \leq \alpha \leq \pi$ , (4) reduces to [6]

$$R(d) = \frac{E_0^2 N}{\pi} \int_0^\pi e^{jkd\cos\alpha} \sin\alpha d\alpha = E_0^2 N j_0(kd) \quad (5)$$

with  $j_0(x)$  being the zero-order spherical Bessel function of the first kind, also equal to  $\frac{\sin x}{x}$  which is the unnormalized version of the sinc function.

When the travel direction of the mobile unit and the incoming radio waves all belong to a single plane in the three-dimensional space,  $\alpha$  is a measure of a two-dimensional orientation and therefore there is no  $\sin\alpha$  factor in the integral in (5). Then, the following expression for the received fading signal autocorrelation is obtained:

$$R(d) = \frac{E_0^2 N}{2\pi} \int_{-\pi}^\pi e^{jkd\cos\alpha} d\alpha = E_0^2 N J_0(kd) \quad (6)$$

and this is the classical result, obtained in [1, 2, 3].

The spatial correlation may be transformed into a temporal correlation through the distance-time proportional dependence  $d = v\tau$  and the expression in (5) becomes

$$R(kv\tau) = E_0^2 N \frac{\sin(kv\tau)}{kv\tau}. \quad (7)$$

The received signal spectrum is obtained from the temporal correlation through a Fourier trans-

form. The sinc type correlation function implies a flat spectrum of the received signal within the limits  $\pm f_{\max} = \pm v/\lambda$  around the carrier frequency when the angular multipath power density at the mobile unit is uniform over the unit sphere, as also shown by an alternative approach in [5], where the authors first derive the Doppler spectrum and then the autocorrelation by a Fourier transform. Usually this is the case in very densely built-up areas, as well as in large and tall corporate office buildings or industrial sites. The shape of the autocorrelation and the Doppler spectrum determine the second-order fading statistics [1, 2]. Usually of particular interest are the autocorrelation function and the power spectrum of the received signal *envelope*. The envelope correlation is approximated by the power correlation [1], equal to the square of the unnormalized sinc function for a spherically uniform angular distribution of the multipath power. The corresponding envelope spectrum is obtained through a Fourier transform of the envelope correlation, or through a self-convolution of the received signal flat spectrum, resulting in a triangular spectrum. The normalized envelope correlation  $R_{|V|}(kvt)/R_{|V|}(0)$  and the envelope spectrum  $S_{|V|}(f)$  in decibels for a spherically uniform angular distribution of the angle of arrival of the multipath power are shown in Fig. 2.

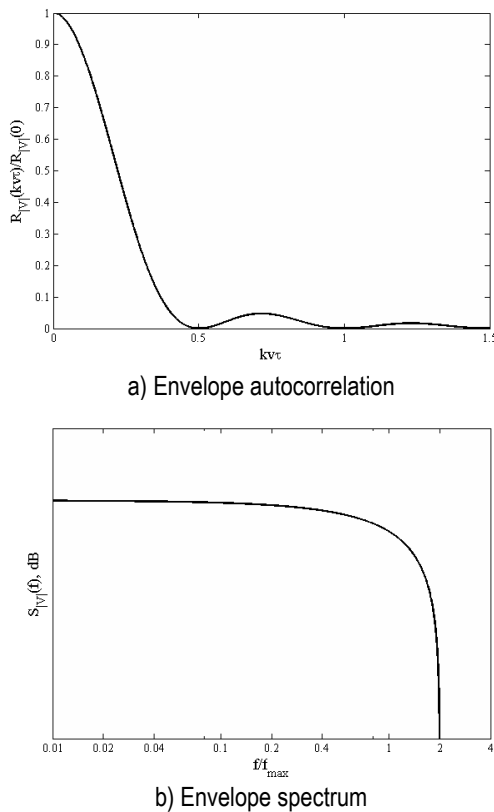


Fig. 2. Envelope autocorrelation and spectrum at the mobile unit for a uniform PDF

### 3.2. A case of a uniform PDF plus a line-of sight (LOS) path

This case is characteristic to situations in indoor spaces where there is a direct line (LOS) between the mobile unit and the base station. The only difference in the autocorrelation and the spectrum of the received signal envelope is that now there will be a constant term  $P_{LOS}$  added to the expression in (5) to obtain

$$R(d) = E_0^2 N_{j_0}(kd) + P_{LOS}. \quad (8)$$

Because of the constant term in (8), the autocorrelation here will never vanish.

The spectrum of the received signal envelope in the case of an additional LOS component is obtained by adding a single impulse function at the offset frequency corresponding to the angle of arrival of the LOS path to the Doppler spectrum from the previous subsection.

## 4. CONCLUSION

This paper develops a method for finding the fading signal correlation and the Doppler spectrum at the receiver antenna of the mobile unit in a three-dimensional multipath environment. The method is based on first deriving the autocorrelation function by averaging over the angular distribution of the incoming radio waves. It is shown that in case of uniform three-dimensional angular distribution of the multipath power at the mobile unit the autocorrelation of the received signal is a sinc function implying a flat Doppler spectrum between the minimum and the maximum Doppler offset at the receiver. For the case of a LOS component added to a uniform three-dimensional angular distribution of the multipath the autocorrelation of the received signal is a sinc function raised by the power level of the LOS component. This implies an additional impulse to the flat Doppler spectrum. From the so obtained autocorrelation function and spectrum, the corresponding approximate envelope autocorrelation and envelope spectrum at the receiver are found. The so obtained autocorrelation and spectrum form a basis for determining important second-order statistics of the fading signal at the receiver antenna of the mobile unit.

**References**

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