MATHEMATICAL INVESTIGATION ON CALCULATION OF MAGNETIC INDUCTION OF LOW FREQUENCY MAGNETIC FIELD IN SYSTEMS FOR MAGNETOTHERAPY

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Abstract

It is well known that the calculation and visualization of the low-frequency or permanent magnetic field is both a time- and memory consuming. The computation depends to the environment, to the parameters of sources of magnetic field and to the visualization method.

Usually different kind of coils are used for providing of low frequency magnetic field in magnetotherapy. It's known [1] that the magnetic permeability of alive tissues is approximately the same as magnetic permeability of air. The coils used in magnetotherapy are without iron core. Therefore the environment around the coils can be described as linear. Because of that the magnetic induction in every points around one or more coils can be calculated as sum of magnetic inductions of magnetic fields of all coils. A mathematical model for the computation and visualization in 3D of the magnetic field generated by several coils is described in the paper. Usually in magnetothearpy the distances between the used coils are enough long. Therefore the mutual induction between different coils is not take in account in mathematical model.

1. INTRODUCTION

In most cases, in practice the required space configuration of the magnetic field is created by means of one or more of the air coils (without core), which are arranged appropriately in the space. Because of that there is a linear relationship between the magnetic flux density of the excited magnetic field and current in the coils. It is assumed that the environment in which is seen the space-time configuration of the magnetic field is linear. The result is the superposition of the fields of the individual coils and forms a more complex time-spaced magnetic field. Let's think that the environment around the coils is homogeneous and the relative magnetic permeability constant.

$$\mu_r(x, y, z, t) = const \qquad (1)$$

Below are described the results of mathematical, computer and experimental studies of the spatial configuration of the magnetic field excited by a cylindrical coil, one of the most common structures of the low-frequency excitation magnetic field. Of Fig.1 is a staging used in the study of the spatial configuration of the magnetic field excited by a cylindrical coil.

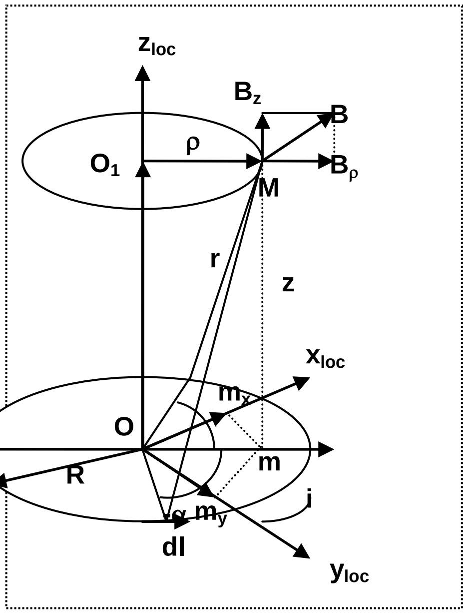

Fig.1

Introduced the following indications:

i – current value in the present single current loop

R – radius of the current loop

O – center of the current loop and the beginning of the cylindrical coordinate system;

M – arbitrary point, in which is calculated the magnetic induction

m – projection of the point M in the plane of current loop;

O_1 – center of the circle lying in a plane parallel to the plane of the current loop and passing through the point M;

ρ – radius of the circle lying in a plane

parallel to the plane of the current loop and passing through the point M; dl - elementar segment of the current loop; елементарен участък от токовия контур;

2α – angle between two symmetrical parts of the elementary current loop

r – distance from any of the two basic parts of the current loop to the point M;

Every coil has its own coordinate system (local). The system from several coils is considered in a global coordinate system. For an arbitrary point of the area the magnetic field in this point is obtained as the sum of magnetic fields concerning to every coil. To calculate the magnetic field in the point concerning to a coil it's necessary to be defined the point's coordinates in a local coordinate system of the chosen coil. The next step should be calculation of magnetic induction magnetic field in this local system. Then should be transformation of result of calculation in local coordinat system to the global system. The calculation of magnetic induction in local coordinat system is described in [1].

Visualization of results f calculation can be performed using ParaView package.

2. CALCULATION OF MAGNETIC FIELD FOR SEVERAL COILS

The calculation of total magnetic field for several coils is performed in the area bounded by a rectangular parallelepiped. The area is divided on cells with the same size, which are also rectangular parallelepipeds. The calculation is performed in the nodes of the constructed mesh. The origin of global coordinate system (GCS) is supposed to be in the left bottom angle of the area (Fig.2).

The position of every coil is defined by the coordinates of intersection point of the coil axis ($\boldsymbol{z_{loc}}$) with the bottom base of the coil and the coordinates of $\boldsymbol{z_{loc}}$ vector. Coils may be located both inside the area and outside it. Magnetic field values are calculated only inside the area. The direction of the coil axis is defined in accordance with the direction of current.

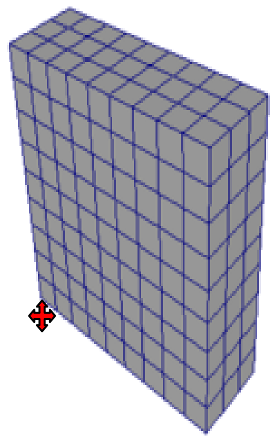

Fig.2

In the local cylindrical coordinate system (LCCS) of coil the coordinates of the point ***M*** are ***ρ*** (the distance from ***M*** to the coil axis) and ***z*** (the distance to the coil base plane). Hence, for fixed coil the coordinates of point ***M*** in LCCS are uniquely determined. For every coil a local rectangular coordinate system (LRCS) with 3 mutually orthogonal basis vectors($\boldsymbol{z_{loc}}$,$\boldsymbol{x_{loc}}$, $\boldsymbol{y_{loc}}$) has been used, where $\boldsymbol{x_{loc}}$, $\boldsymbol{y_{loc}}$ are mutually orthogonal and lie in the coil's base plane.

As the coordinates of any point ***M*** both in GCS and LCCS are uniquely determined, it's possible to be choosed an arbitrarily LRCS. The same coordinates ***ρ*** and ***z*** of ***M*** can be used for the transition chain GCS ->LRCS ->LCCS and in the case of vice versa, for the chain LCCS->LRCS->GCS the same coordinates in GCS can be obtained.

It's possible to be considered that such a LCRS to reduce calculations – the transition from one Cartesian coordinate system to another is coordinate shift and multiplication on a fixed transition matrix. In this way the transition and inverse transition matrix can be calculated only once for every coil.

The transition matrix has the form (1):

$$T = \begin{matrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ z1 & z2 & z3 \end{matrix} \qquad (1)$$

Where columns are basis vectors of LRCS in the GCS basis $\boldsymbol{O_x=(x_1,y_1,z_1)}$, $\boldsymbol{O_y=(x_2,y_2,z_2)}$, $\boldsymbol{O_z=(x_3,y_3,z_3)}$).

In going from LRCS to LCCS and inversely the coordinate on ***z*** axis does not change , coordinates $\boldsymbol{m_x}$ and $\boldsymbol{m_y}$ are transformed in ***m*** and inversely: vector B_ρ is decomposed on the basis ($\boldsymbol{m_x}$, $\boldsymbol{m_y}$)) .

It's clear that before the calculation of magnetic induction ***B*** in the point ***M*** one preliminary transition chain GCS ->LRCS should be provided. The algorithm for this calculation during the process of transition chain GCS ->LRCS for the point ***M*** of the mesh (Fig.2) is the following:

1. TRANSICTION CHAIN

GCS->LRCS

In the case when the global coordinates of an arbitrary point be ***M*** are $\boldsymbol{(x_M,y_M,z_M)}$, the basis vectors LRCS in GCS are $\boldsymbol{v_1 = (x_1,y_1,z_1)}$, $\boldsymbol{v_2 = (x_2,y_2,z_2)}$, $\boldsymbol{v_3 = (x_3,y_3,z_3)}$, the origin of LRCS is point $\boldsymbol{O(x_0,y_0,z_0)}$. The vector of coordinates of point ***M*** in LRCS can be determined, using the following equation:

$$(m_x,m_y,m_z)^T = T(M - O) \tag{2}$$

Where:
M and ***O*** are radius vectors of corresponding points;
m is the projection of the point M on the base plane of the coil;

2. TRANSICTION CHAIN

LRCS ->LCCS

The radius ρ of the circle lying in a plane can be calculated using the equation (3):

$$\rho = \sqrt{m_x^2 + m_y^2} z = m_z \tag{3}$$

3. CALCULATION OF MAGNETIC INDUCTION *B*

The vector of magnetic induction ***B*** in the point ***M*** has two components, taking in account Fig.1. These components can be calculated in LCCS according to [1]:

$$B_\rho = \frac{\mu_0 i}{2\pi} \frac{z}{\rho\sqrt{(R+\rho)^2+z^2}} \left(\frac{R^2+\rho^2+z^2}{(R-\rho)^2+z^2} L - K\right) \tag{4}$$

$$B_{zcyl} = \frac{\mu_0 i}{2\pi} \frac{1}{\sqrt{(R+\rho)^2+z^2}} \left(\frac{R^2-\rho^2-z^2}{(R-\rho)^2+z^2} L + K\right) \tag{5}$$

In equations (4) and (5) ***K*** and ***L*** are complete elliptic integrals of 1 and 2 sort as functions of ***k.***
Where:

$$k^2 = \frac{4\rho R}{(R+\rho)^2 + z^2} \tag{6}$$

It's well known [1] that the equation (1) can be taken in account for the air environment and approximately for the alive tissues, also. Therefore the total value of magnetic induction ***B*** in every point in the case of several coils can be calculated as sum of partial values of magnetic induction in the same point, created by respective coils.

The above mentioned calculations of the value of vector of magnetic induction can be used for every point, but often it would be too difficult because of long time for computer calculation [3],[4]. Because of that, some times the values of magnetic induction can be calculated for limited number of points using equations (4), (5) and (6), which are very complicated. Then the values of magnetic induction for every additional point can be obtained using more simple approximately method for calculation, using the results of preliminary exact calculations. In the process of approximately method a simple linear interpolations can be used.

The table values of magnetic induction in the additional points and values of limited differences (first order) can be obtained for instance from [2].

4. TRANSICTION CHAIN

LCCS->LRCS

$$B_{z\,loc} = B_{z\,cyl} \tag{7}$$

$$B_{x\,loc} = B_\rho \times \frac{m_x}{\sqrt{m_x^2+m_y^2}} \tag{8}$$

$$B_{y\,loc} = B_\rho \times \frac{m_y}{\sqrt{m_x^2+m_y^2}} \tag{9}$$

$$B_{loc} = (B_{xloc}, B_{y\,loc}, B_{zloc}) \tag{10}$$

5. TRANSICTION CHAIN LRCS->GCS

$$B_{glob} = T^{-1} B_{loc} \tag{11}$$

The calculations 1-5 are should be repeated for the value of magnetic induction of every coil. As it has been above mentioned the total value of magnetic induction in point ***M*** can be obtained as the sum of partial values of magnetic inductions of respective coils. This procedure should be performed for all the points of the mesh and the results can be saved in 3-dimension array and written in a file *.vtk for visualization in ParaView package.

7. CONCLUSION

A mathematical investigation on methods for calculation of the value of magnetic induction of low frequency magnetic field created by several coils in one point is described in the present paper. This is tipical situation for the modern systems for magneto-therapy. Usually there is an influence of magnetic magnetic field created by several coils on the human body. The axis of these coils can be parallel or not parallel. Usually the system for magnetotherapy contain one microprocessor unit, which can switch different coils according to the special software during the process of therapy. The number of "active" coils and their space dispositions are different in every moment. This configuration is determined by microprocessor's unit in every moment. Therefore the results of investigations in the present paper would be useful not only for physicians for optimisation of process of therapy, but for engineers in the process of design of systems for magnetotherapy, also.

Acknowledgements

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