A NOVEL CLOSED – FORM ANALYTICAL SOLUTION TO THE RADIATION PROBLEM FROM A VERTICAL SHORT DIPOLE ANTENNA ABOVE FLAT GROUND USING SPECTRAL DOMAIN APPROACH

S. Sautbekov, P. Frangos* Ch. Christakis* and K. Ioannidi***

*School of Electrical and Computer Engineering,
National Technical University of Athens,
9, Iroon Polytechniou Str., 157 73 Zografou, Athens, Greece
Tel. +30 210 772 3694, Fax. +30 210 772 2281, E-mail: pfrangos@central.ntua.gr

**Eurasian National University, 5, Munaitpassov Str., Astana, Kazakshtan, E-mail : sautbek@mail.ru

Abstract

In this paper we consider the problem of radiation from a vertical short (Hertzian) dipole above flat ground with losses, which represents the well – known in the literature 'Sommerfeld radiation problem'. We end - up with a closed – form analytical solution to the above problem for the received electric and magnetic field vectors above the ground in the far field area. The method of solution is formulated in the spectral domain, and by inverse three – dimensional Fourier transformation and subsequent application of the Stationary Phase Method (SPM) the final solutions in the physical space are derived. To our knowledge, the above closed – form solutions are novel in the literature for the Sommerfeld radiation problem. Finally, preliminary numerical results showing comparison of our derived new analytical solutions for the EM field scattered from the ground and corresponding EM field based on K. A. Norton's approximate solution [3] are provided in this paper.

1. INTRODUCTION

The so - called 'Sommerfeld radiation problem' is a well – known problem in the area of propagation of electromagnetic (EM) waves above flat lossy ground for obvious applications in the area of wireless telecommunications [1,2]. The classical Sommerfeld solution to this problem is provided in the physical space by using the so- called 'Hertz potentials' and it does not end – up with closed form analytical solutions. K. A. Norton [3] concentrated in subsequent years more in the engineering application of the above problem with obvious application to wireless telecommunications, and he provided approximate solutions to the above problem, which are represented by rather long algebraic expressions for engineering use, in which the so – called 'attenuation coefficient' for the propagating surface wave plays an important role.

In this paper the authors take advantage of previous research work of them for the EM radiation problem in free space [4] by using the spectral domain approach.

Furthermore, in Ref. [5] the authors provided the fundamental formulation for the problem considered here, that is the solution in spectral domain for the radiation from a dipole moment at a specific angular frequency (ω) in isotropic media with a flat infinite interface. At that paper, the authors end – up with integral representations for the received electric and magnetic fields above or below the interface [Line of Site (LOS) plus reflected field – transmitted fields, respectively], where the integration takes place over the radial spectral coordinate k_ρ. Then, in the present paper the authors concentrate to the solution of the classical 'Sommerfeld radiation problem' described above, where the radiation from a vertical dipole moment at angular frequency ω takes place above flat lossy ground [this is equivalent to the radiation of a vertical small (Hertzian) dipole antenna above the flat lossy ground]. By using the Stationary Phase Method (SPM method [6]) integration over the radial spectral coordinate k_ρ is performed and novel, to our knowledge, closed – form analytical solutions for the received electric and magnetic fields in the far field zone (where SPM method is applicable) are derived. Finally, physical interpretation of these novel closed – form analytical expressions are provided in Ref. [7], where a variant version of the present paper of ours can be found.

2. PROBLEM GEOMETRY

The geometry of the problem is given in Fig. 1. Here a Hertzian (small) dipole with dipole moment p directed parallel to positive x – axis, at altitude x_0 above the infinite, flat and lossy ground, radiates time – harmonic electromagnetic (EM) waves at angular frequency $\omega=2\pi f$ [exp(-iωt) time dependence is assumed in this paper]. Here the relative complex permittivity of the ground (medium 2) is $\varepsilon'_r=\varepsilon'/\varepsilon_0=\varepsilon_r+ix$, where $x=\sigma/\omega\varepsilon_0=18\times10^9\ \sigma/f$, σ being the ground conductivity, f the frequency of radiation and $\varepsilon_0=8.854\times10^{-12}$ F/m is the absolute permittivity in vacuum or air. Then the wavenumbers of propagation of EM waves in air and lossy ground, respectively, are given by the following equations :

$$k_{01}=\omega/c_1=\omega\sqrt{\varepsilon_1\mu_1}=\omega\sqrt{\varepsilon_0\mu_0} \qquad (1)$$

$$k_{02}=\omega/c_2=\omega\sqrt{\varepsilon_2\mu_2}=k_{01}\sqrt{\varepsilon_r+ix} \qquad (2)$$

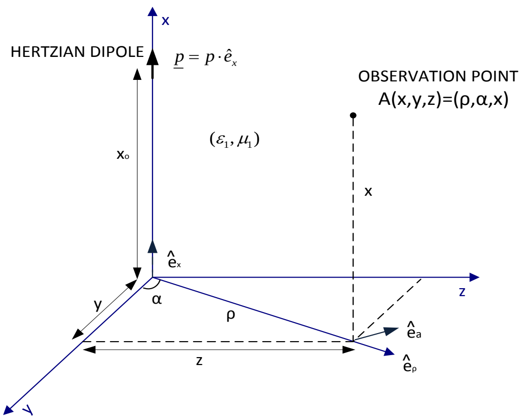

Figure 1. Geometry of the problem

3. ANALYTICAL CLOSED – FORM EXPRESSIONS FOR THE SCATTERED EM FIELDS OBTAINED THROUGH THE APPLICATION OF THE STATIONARY PHASE METHOD (SPM)

Following [5], [7], [8], and by using the Stationary Phase Method (SPM) [6,7], we finally end - up with the following expressions for the reflected fields (in the higher half - space, x>0), as given below :

$$\underline{E}_{x>0}=\underline{E}^{LOS}-\frac{ip}{8\pi\varepsilon_o\varepsilon_{r1}}\mathrm{I}_1\cdot\hat{e}_\rho-\frac{ip}{8\pi\varepsilon_o\varepsilon_{r1}}\mathrm{I}_2\cdot\hat{e}_x \qquad (3)$$

$$\underline{\mathrm{H}}_{x>0}=\underline{\mathrm{H}}^{LOS}-\frac{i\omega p}{8\pi}\mathrm{I}_3\cdot\hat{e}_\alpha \qquad (4)$$

where

$$I_1=\frac{i2}{k_{01}\rho^{1/2}}\frac{1}{(x+x_0)^{1/2}}\kappa_{1s}^{3/2}k_{\rho s}^{3/2}\cdot \frac{\varepsilon_2\kappa_{1s}-\varepsilon_1\kappa_{2s}}{\varepsilon_2\kappa_{1s}+\varepsilon_1\kappa_{2s}}e^{ik_{\rho s}\rho}e^{i\kappa_{1s}(x+x_0)} \qquad (5)$$

$$I_2=\frac{i2}{k_{01}\rho^{1/2}}\frac{1}{(x+x_0)^{1/2}}\kappa_{1s}^{1/2}k_{\rho s}^{5/2}\cdot \frac{\varepsilon_2\kappa_{1s}-\varepsilon_1\kappa_{2s}}{\varepsilon_2\kappa_{1s}+\varepsilon_1\kappa_{2s}}e^{ik_{\rho s}\rho}e^{i\kappa_{1s}(x+x_0)} \qquad (6)$$

$$I_3=\frac{i2}{k_{01}\rho^{1/2}}\frac{1}{(x+x_0)^{1/2}}\kappa_{1s}^{1/2}k_{\rho s}^{3/2}\cdot \frac{\varepsilon_2\kappa_{1s}-\varepsilon_1\kappa_{2s}}{\varepsilon_2\kappa_{1s}+\varepsilon_1\kappa_{2s}}e^{ik_{\rho s}\rho}e^{i\kappa_{1s}(x+x_0)} \qquad (7)$$

and

$$k_{\rho s}=\frac{k_{01}\rho}{\left[(x+x_0)^2+\rho^2\right]^{1/2}}= =k_{01}\frac{1}{\left[1+\left(\frac{x+x_0}{\rho}\right)^2\right]^{1/2}}=k_{01}\cos\phi \qquad (8)$$

is the (unique) stationary point [7], while φ is the angle defined by the image point of the radiating dipole, the observation point and the horizontal line drawn from the above mentioned image point [8].

Furthermore, regarding eqs. (5)-(7) above, the following quantities have been introduced :

$$\kappa_{1s}=\sqrt{k_{01}^2-k_{\rho s}^2}=k_{01}\sin\phi \qquad (9)$$

where angle φ has been defined just above, and

$$\kappa_{2s}=\sqrt{k_{02}^2-k_{\rho s}^2} \qquad (10)$$

Then our final closed-form analytical solution consists of eqs. (3) - (10).

Furthermore, note in the derivation of our analytical solution above, the following restriction holds [8] :

$$\frac{(x+x_0)^2}{\rho^4}+\frac{1}{\rho^2}<<k_{01}^2 \qquad (11)$$

For wireless telecommunication applications, which are of primary interest in this paper, condition (11) appears to be a rather weak condition, especially for frequencies f=100 MHz (radio FM) and above.

4. PRELIMINARY NUMERICAL RESULTS – COMPARISON WITH THE APPROXIMATE RESULTS PREVIOUSLY DERIVED IN THE LITERATURE

Below are provided preliminary numerical results showing comparison of the scattered electric field based (i) on our novel analytical solution, eqs. (3) – (10), and (ii) the approximate solution by K. A. Norton [3], where the 'space wave' and the approximate formula for the 'Norton surface wave' are involved (the latter rather 'dominating', in a sense, for relatively low transmitter and receiver antenna heights). Both results show excellent accuracy between the two methods. Note that the problem parameters in Figs. 2 and 3 below (except the parameters directly shown in these figures) were selected as following : current of the radiating Hertzian dipole I=1A, frequency of radiation f=80 MHz (wavelength λ=c/f=3.75 m), length of radiating Hertzian dipole 2h=0.1m. Note here that the relation between current I and dipole moment p of the radiating Hertzian dipole is : I(2h)=iωp , where ω=2πf and i is unit imaginary number.

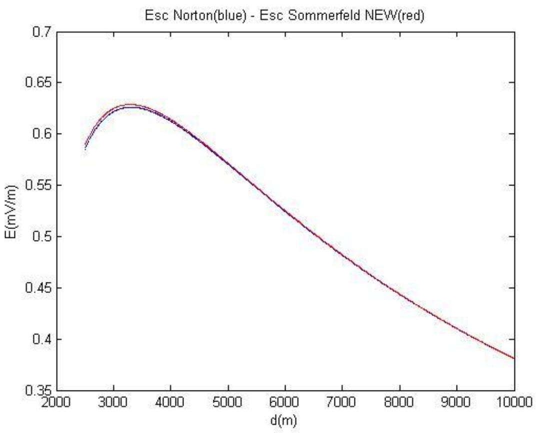

Figure 2

Comparison of electric field scattered from the ground as a function of the horizontal distance ρ=d, based on (i) our novel analytical solution (red curve), (ii) Norton's approximate solution (blue curve) [8]. Here both transmitter and receiver antenna heights (x_o and x, respectively) equal to 150 m. Note that in this case Norton's 'space wave' dominates over his corresponding 'surface wave' [8].

Similarly with Fig. 2 above, except that here both transmitter and receiver antenna heights (x_o and x, respectively) equal to 1 m. In this case Norton's 'space wave' and 'surface wave' are comparable (not shown here).

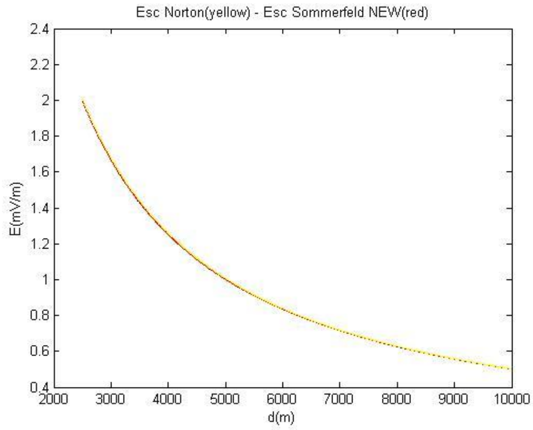

Figure 3

Further clarifications about the above presented numerical results will be provided during the Conference.

5. CONCLUSIONS – FUTURE RESEARCH

In this paper we have derived analytical closed - form solutions for the received electromagnetic (EM) field for the problem of radiation of vertical Hertzian (small) dipole antenna above infinite, flat and lossy ground. To our knowledge these expressions are novel in the literature, and they have been derived here from a formulation in the spectral domain [4,5]. Futhermore, very interesting remarks regarding the physical interpretation of the analytical expressions mentioned above can be found in [7], including wavenumbers of propagation (in horizontal and vertical directions), surface wave behavior and formula for the Fresnel reflection coefficient in the problem examined here, as well as in the limiting case of 'space waves' (where the usual expression for the Fresnel reflection coefficient is obtained). Finally, preliminary numerical results based on our novel analytical solution are provided in this paper.

Related research in the near future by our research group will include : further (detailed) comparison of values for the received EM field with K. A. Norton' results [2], derivation of corresponding EM field expression for the transmitted EM field (region x < 0), solution of the corresponding problem for horizontal radiating Hertzian dipole above flat and lossy ground, propagation in isotropic and anisotropic crystals with interface (at x=0) etc.

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