WHITE NOISE IN HARMONIC REJECTING CURRENT COMMUTATING PASSIVE FET MIXERS

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Abstract

This paper examines the white noise contribution of different noise sources in the harmonic rejection mixers (HRM) to the total mixer noise, using simple physical models. Equations for the level of noise due to each of the separate noise transfer mechanisms have been derived. A comparison in terms of white noise level between HRM and ordinary double balanced mixers has been made.

1. INTRODUCTION

In the recent years, harmonic rejection mixers (HRM) have gained in popularity due to their suitability for highly integrated RF receivers. The use of HRM alleviates the harmonic mixing problem and thus relaxes the pre-select filtering requirements.

A current commutating passive HRM with multiple outputs is presented in Fig. 1. The RF signal is applied to the inputs of N transconductance amplifiers (TCAs) with transconductance values $g_{mi} = g_{m\max} \sin(2\pi\, i / N + \varphi_0)$. The output currents of the TCAs flow into the switching core, consisting of FET switches, driven by multiphase local oscillator (LO). Note that each column, as each row of switches can be viewed as an N-position rotary switch doing one revolution per LO cycle. The switching core is followed by current buffers (CBs), ideally with zero input impedance.

In principle, it is possible to implement an HRM with one I and one Q output only. However, in this case the TCA outputs will not be used over the whole LO cycle and should be connected to the ground by dummy switches when they are not used. Otherwise, undesirable large voltage swings at the TCA outputs will occur. On the other hand, the presence of additional outputs makes possible the implementation of techniques for improved harmonic rejection as those proposed in [1].

An HRM is equivalent to an ordinary mixer, driven by an effective LO waveform from which some harmonics are excluded [1]. It is easy to realize that the effective LO contains only harmonics of orders $kN \pm 1$.

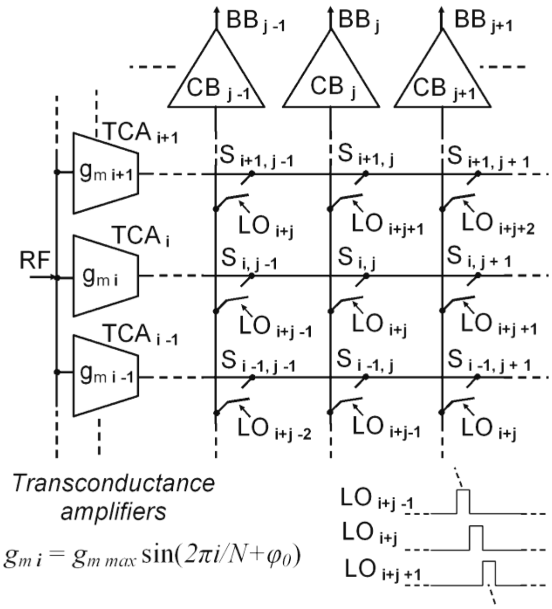

Figure 1. HRM with multiple outputs (single ended implementation).

In contrast to HRM, we will use the term “simple mixers” (SM) for ordinary double-balanced mixers without harmonic rejection capability.

In this paper we extend the analysis, made for SMs in [2] to the case of an HRM like the one shown in Fig. 1, but in a balanced implementation. Furthermore, the number of TCAs is $N/2$ as each differential TCA replaces two TCAs with transconductances of equal absolute values and opposite signs. The equations are derived for even values of N only, as the odd values are not practical.

The paper assumes that CB input resistance is much lower than switch resistance.

The discussions will be primarily oriented to direct conversion and low-IF receivers, as they are very suitable for highly integrated implementation.

The rest of the paper is organized as follows: Sections 2, 3, and 4 present noise contributions of transconductors, switches, and current buffers, respectively. Section 5 compares HRM and simple mixers in terms of white noise level in baseband (BB).

2. TCA WHITE NOISE

From the viewpoint of the succeeding blocks, the TCA noise is indistinguishable from the input RF signal. Therefore, the noise components, centered around the LO harmonics, will be down-converted to baseband. The total noise power is sum of the power of the down-converted noise components, since they are uncorrelated. So, the total single-sided PSD of the noise, converted to baseband is:

$$S_0 = N_0 \, F \, g_{m\max}^2 \sum_{k=1}^{\infty} C_k^2 / 2 \,, \tag{1}$$

where N_0 is the thermal noise PSD, F is the TCA noise factor, and C_k are the Fourier series coefficients of the effective LO waveform. Then we can express the ratio between the PSD of the TCA noise converted to baseband by a real mixer and by a perfect multiplier driven by pure sinusoidal LO.

$$S_0 / S_{0\,perfect} = \sum_{k=1}^{\infty} C_k^2 \Big/ C_1^2 \tag{2}$$

For an HRM we have found that

$$S_0 / S_{0\,perfect} = \frac{\pi^2}{N^{\,2} \sin^2(\pi / N)}, \tag{3}$$

whereas for an SM driven by a symmetrical square-wave LO this ratio is $\pi^2/8$ or 0.9 dB. In fact (2) gives an equivalent increase of the TCA noise factor due to LO harmonics. Fig. 2 presents eq. (3) for N between 5 and 16.

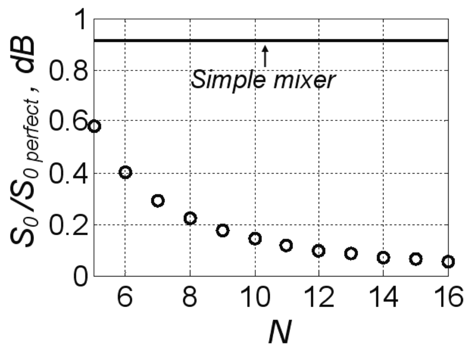

Figure 2. Plot of eq. (3) in dB

3. SWITCH WHITE NOISE

The switching FETs in a passive current commutating mixer operate in the deep triode region when they are in "on" state. The transistor can be replaced by a Norton equivalent circuit, consisting of the drain-source conductance in parallel connection with a noise current source. The single-sided PSD of its noise current is

$$S_n = 4kT\gamma \, g_{ds} \,, \tag{4}$$

where γ is the excess noise factor of the transistor and g_{ds} is its "on" conductance [3].

At first glance, it seems that the switches cannot generate noise, as the mixer output current is fixed by the TCAs. However, because the switches do not commutate instantly and there is a nonzero parasitic capacitance at the TCA outputs, the white noise of the switches appears at the output [2]. As in [2] we will call the noise transfer mechanism which is due to imperfect commutation "direct" and that which is due to parasitic capacitance "indirect".

3.1. Switch white noise, Direct

It is recommendable to ensure some overlap between the "on" times of the switches which are turning on and the switches which are turning off [2]. As a result, 4N switches are in a conductive state during the ON overlap. They form low-ohmic paths at the CB inputs. So the noise current of the switches can flow into CB inputs.

Let us examine the noise current flowing in the CB_j input during the commutation from TCA_i to TCA_{i+1} (Fig. 3a). As the CB_{j-1} and CB_{j+1} inputs are virtual grounds, only 8 of these 4N switches should be taken into account for CB_j. The corresponding equivalent circuits are presented in Fig. 3b.

The conductance of the switch is

$$g = \beta(V_G - V_B - V_{th} + V_{LO}) = \beta(V_{eff} + V_{LO}) \,, \tag{5}$$

where V_G and V_B are the gate and source bias voltages, respectively, V_{th} is the threshold voltage of the FETs, V_{eff} is the dc effective voltage of the switch. $\beta = \mu C_{ox} W / L$, in which μ is the channel mobility, C_{ox} is the gate oxide capacitance, and W and L are the width and length of the switch.

We assume as in [2] that LO waveform transitions are linear functions of the time, i. e.

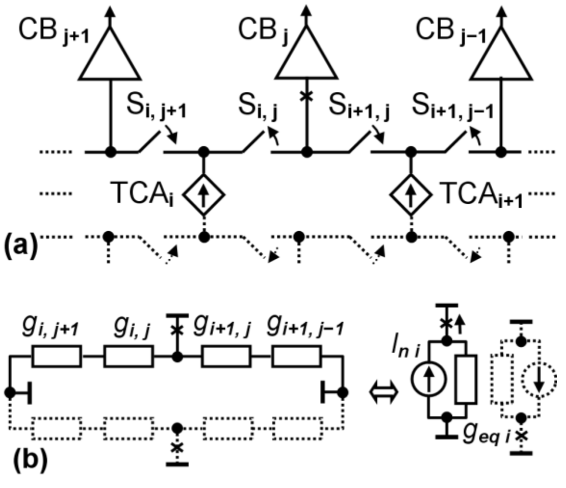

Figure 3. Commutation of CB_j from TCA_i to TCA_{i+1} (a) and corresponding equivalent circuit (b).

$$V_{LO} = V_{LO\min} + S\,t \text{ or } V_{LO} = V_{LO\max} - S\,t\,, \quad (6)$$

where S is the slope of LO transitions.

Then the equivalent conductance connected to CB_j input during the overlap time of the i-th commutation is

$$g_{eq\,i} = \left(\beta_i + \beta_{i+1}\right)\frac{V_{eff}^2 - V_{LO}^2}{2V_{eff}}\,, \quad (7)$$

where $\beta_i = \beta_{\max}\sin\left(2\pi\, i / N + \varphi_0\right)$ and $\beta_{\max}$ is the β value of the "unity" switch.

As $g_{eq\,i}$ varies periodically with the time, the noise current produced by the periodical commutation of each pair of TCAs, can be considered as white noise, multiplied by a periodical signal. Its PSD can be found as product of their respective PSD and power:

$$S_{ni} = 4kT\gamma\left(\beta_i + \beta_{i+1}\right)\frac{1}{T_{LO}}\int_{t_1}^{t_2}\frac{V_{eff}^2 - V_{LO}^2}{2V_{eff}}\,dt\,. \quad (8)$$

In the last equation t_1 and t_2 are the borders of the overlap time interval.

Since the noise currents produced by the N separate commutations in each LO cycle are uncorrelated, the total PSD at CB_j input is the sum of their PSDs.

After performing the necessary mathematical operations and simplifications, we obtain for the total differential PSD at HRM output:

$$S_{o\,diff} \approx 4kT\gamma\frac{16}{3}\frac{N}{\pi}\frac{\beta_{\max}V_{eff}^2}{ST_{LO}}\,. \quad (9)$$

If the switch widths are not scaled according to the transconductance values of the corresponding TCAs, the PSD of the direct switch noise will be about 2 dB higher.

3.2. Switch white noise, Indirect

Due to the parasitic capacitances at the TCA outputs, the switch white noise will appear at the HRM output even in the case of instantaneous commutation. Let us consider the role of the parasitic capacitance at the non-inverting TCA_i output for the white noise at the CB_j output. The corresponding part of the HRM is presented in Fig. 4a. The equivalent circuit of the highlighted part of Fig. 4a is shown in Fig. 4b. As the resistor is a memoryless component and all N switches connected to TCA output have the same "on" conductance, the resistors in Fig. 4b can be replaced by one resistor. In addition, since the values of the white noise are uncorrelated in time, the noise current sources can be replaced by a single noise source. We have assumed that the CB input resistance is low, so the CB inputs can be considered as virtual grounds. All these considerations lead to the equivalent circuit in Fig. 4c.

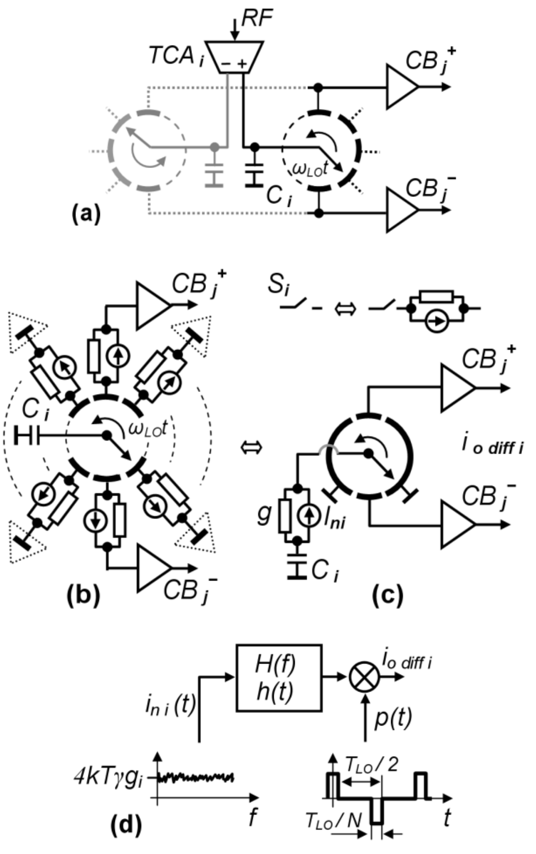

Figure 4. Indirect switches noise due to parasitic capacitance C_i

The switch conductance g_i and C_i form a high-pass filter with a transfer function

$$H(f)=\frac{j\,f/f_0}{1+j\,f/f_0}\,, \qquad (10)$$

where $f_0 = 1/(2\pi C_i/g_i)$ is the corner frequency. The corresponding impulse response is $h(t)$. The values of the corner frequency f_0 should be the same for all TCAs in a well designed HRM, so the transfer function and the impulse response of the individual TCAs are the same.

This filter causes a frequency-dependent attenuation for the components of the noise current. When the TCA$_i$ output is connected to CB$_j$, the filtered noise current flows into its input. Over the rest of the LO cycle, the filtered noise current continues to flow to the ground, so from the viewpoint of the noise source and the filter, the circuit is unchanged. Therefore the circuit in Fig. 4c can be modeled by the system shown in Fig. 4d.

The differential noise current, produced by this system is

$$i_{o\,diff\,i}(t)=[i_{ni}(t)*h(t)]p(t)\,. \qquad (11)$$

Following some mathematical simplifications, we obtain the single-sided baseband PSD of (11) presented in a form comparable to that in [2]

$$S_{o\,diff\,i}=\frac{8}{\pi^2}S_{swi}\sum_{m=0}^{+\infty}\left[\frac{\sin^2\left(\frac{(2m+1)\pi}{N}\right)}{(2m+1)^2}\left|H((2m+1)\,f_{LO})\right|^2\right], \qquad (12)$$

where $S_{swi}=4kT\gamma\,g_i$ is the single-sided PSD of the noise current produced by any of the switches connected to TCA$_i$ and
$g_i = g_{\max}\sin(2\pi i/N+\varphi_0)$.

Since the noise currents, originating from the switches at the two TCA$_i$ outputs are uncorrelated, the total noise power related to TCA$_i$ is two times as large as in (12). The noise currents produced by the switches belonging to the separate TCAs are uncorrelated, so the total PSD for one HRM output is

$$S_{o\,diff}=2\sum_{i=1}^{N/2}S_{o\,diff\,i}=4kT\gamma\,g_{\max}\frac{\cos\left(\frac{\pi}{N}-\varphi_0\right)}{\sin\left(\frac{\pi}{N}\right)}\times$$

$$\times\frac{16}{\pi^2}\sum_{k=0}^{+\infty}\left[\frac{\sin^2\left(\frac{(2m+1)\pi}{N}\right)}{(2m+1)^2}\left|H((2m+1)\,f_{LO})\right|^2\right]. \qquad (13)$$

We found an expression for PSD in closed form, but it is somewhat awkward. Fortunately, it turned out that many of its terms are negligible, so we obtained a very good approximation, which works well for all f_{LO}/f_0 values:

$$S_{o\,diff}\approx(4kT\gamma\,g_{\max})\frac{2N}{\pi^2}\times$$

$$\frac{f_{LO}}{f_0}\left[1-\exp\left(-\frac{2\pi}{N}\frac{f_0}{f_{LO}}\right)\right]. \qquad (14)$$

For $f_{LO}/f_0 > 2\pi/N$ this expression approaches $(4kT\gamma\,g_{\max})4/\pi$. On the other hand, for $f_{LO}/f_0 << 2\pi/N$ eq. (14) tends to $(4kT\gamma\,g_{\max})\frac{2N}{\pi^2}\frac{f_{LO}}{f_0}$.

If the switches are not scaled according to the transconductance values of the corresponding TCAs, the noise PSD will be about 2 dB higher.

As $\sin(\pi-x)=\sin(x)$ the number of TCAs can be further reduced to $N/4$. In this case, the first summation in (13) will be incorrect, as the pairs of symmetrical addends correspond to correlated noise currents. We examined this correlation. Without going into details, we found that it is significant only for the pair of addends with numbers $i=N/4$ and $i=N/4+1$ when f_{LO}/f_0 decreases. For $f_{LO}/f_0 < 3/N$ the sum of the powers of these two noise currents tends to the power of only one of them. In this case, the value calculated by (13) should be multiplied by approximately $(1-\pi/N)$.

The equations derived in section 3 were verified by Matlab simulations.

4. BUFFER WHITE NOISE

The white noise of bias and load circuits of the CBs (Fig. 5) appears directly at the output [2]. The white noise contribution of the common gate (CG) FETs depends on the operation of the switches and the parasitic capacitances at the TCA outputs. As in [2] and [4] there are two mechanisms, which transfer the white noise of CG FETs to the mixer output: direct and indirect.

These components of the white noise, which are close to DC, appear at the output in the same way as the flicker noise in [2] and [4]. The components, which are centred around $k\,f_{LO}$, seem to be down-converted to baseband. However, they will be shorted to ground by the large CB input capacitance and will appear at the HRM output unaffected by the operation of the switches. As a result, they will be outside the spectrum of the desired signal [2] and should not be taken into account. Therefore, the equations for the direct and indirect white noise of the buffer are the same as the equations for the buffer flicker noise derived in [4]. The only difference is that the squared voltage of the flicker noise has to be replaced by $4kT\gamma / g_m$.

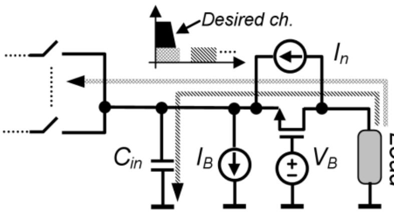

Figure 5. CB white noise

If the CBs are implemented as transimpedance operational amplifiers, the equations will be roughly the same. The RMS flicker noise voltage should be substituted by the input referred noise voltage of the amplifier.

5. COMPARISON BETWEEN HRM AND SM AND DISCUSSION

In order to make a comparison between HRM and SM, we should select $g_m \approx (\pi/4)\, g_{\max}$ for the TCA in the SM. This ensures equal conversion gains of both compared mixers. The other SM parameters should be scaled accordingly.

HRMs perform better than SMs with respect to transconductor white noise. The advantage is fairly small and approaches 0.9 dB when N is large. In practice, increasing N above 10-12 does not result in a further noticeable reduction in the noise level (Fig. 2).

The direct white noise of the switches has a higher level than in a comparable SM when N is above 4 (i.e. for all useful values of N).

The indirect white noise of the switches in an HRM has a higher PSD than in an SM when $f_{LO} < f_0$. On the other hand, for N up to 20 and practically any f_{LO} value, the HRM noise is no more than 6 dB higher than that of the SMs.

What was discussed in [2] and [4] about the CB flicker noise also applies for the white noise originated by them.

The analysis of the derived equations shows that the level of the *indirect* white noise of the switches and CG FETs can exceed the unavoidable white noise at the TCA outputs. This can result in a considerable degradation of the noise figure of the HRM. In order to prevent this, the parasitic capacitance at the TCA outputs should be minimized and too large values of N should be avoided. It is not appropriate to reduce the conductance of the switches, as this will result in a decrease of f_0, which will counteract the noise level reduction. Moreover, the large-signal performance of the mixer will be impaired.

In practice, the *direct* white noise of the switches and CG FETs is lower than the white noise at the TCA outputs in any case. The direct white noise of the switches can be several times lower than the TCA white noise, while the direct white noise of the CG FETS can be made negligible. This can be achieved primarily by using LO with sharp transitions, as long as the power budget of the HRM allows it.

4. CONCLUSION

We derived equations for white noise in HRM and made comparison between HRMs and SMs with respect to noise level.

In general, HRMs have a higher white noise level than SMs. The increase of N alleviates the harmonic mixing problem, but it also increases the level of the white noise, originating from the switches and the current buffers.

The white noise contributed by the switches and the CBs can be made lower than the TCA noise. Therefore, the noise figure of the entire HRM can be near the noise figure of the TCAs.

References

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