ON SOME ASPECTS OF THE FRACTAL SIGNATURE METHOD

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Abstract

This work is devoted to a substantiation of the fractal signature method and an investigation of its applicability for digital images of different classes. The dependence on the color scale (grayscale and HSV) is demonstrated. The applicability of the method to the images with similar textures but belonging to different classes is also discussed.

1. INTRODUCTION

The main dimension in fractal analysis is the Hausdorff dimension, but in practice so called box computing (or box-counting) dimensions are used. The well-known capacity dimension [3] belongs to this class. To analyze digital images the Minkovsky dimension seems to be more preferable, because it is equal to capacity dimension for nonempty bounded sets in R^3 and is simple to calculate.

To analyze and classify textures the method of calculation of Minkovsky's dimension was proposed in [6]. It is based on the Mandelbrot idea [5] about the approximate calculation of the length of coastline by measuring the area of a strip that contains the line and has width $2\delta,$ where δ is a fixed number. Then the length is approximately the area divided by 2δ. The authors applied this method to measure the area of a gray level surface constructed by a digital image. Then the sequence of special blankets is constructed over the surface. For every blanket its volume is calculated, the surface area and the so called "fractal signature" (the ratio of the log of the surface area to the log of the scale) is defined. The Minkovsky dimension of the area may be easily obtained from the fractal signature. So we have a sequence of areas and signatures in accordance with the number of blankets. For two images we can compare the obtained signature vectors: the closeness between vectors shows the textures adjacency. In [7] the authors applied the method to calculate the document fractal dimension and called it the "modified fractal signature method". They used only two consecutive blankets. In such a variant the method demonstrated high reliability and was successfully used in [1] and [4] to classify both biomedical preparations and ISAR radar images.

In this work problems of substantiation and applicability of the method are discussed. Though empirical enough, this approach demonstrated good results in experiments described in [7], [1], [4]. The dependence experimental results on the color scale is demonstrated as well.

2. METHODS OF ANALYSIS

2.1. Mandelbrot task

When studying the problem of coastline length measurement B. Mandelbrot came to conclusion that such lines have complex geometry, a direct measurement is rather difficult, and hence one can find this value only approximately [5]. He suggested the following methods.

1. Take a stick of length δ (step) and walk along the coastline, i.e change the coastline by a polygon made from segments. If $N(\delta)$ – the number of steps then the length $L(\delta) \sim N(\delta)\delta$. The problem is that when the step size decreases, the observed length $L(\delta)$ increases without limit. The empirical study of the lengh changing per-

formed by L. Richardson showed that $(\delta)\sim F\delta^{-D}$, where F and D are constants depending on the type of the line. Then we have $L(\delta)\sim F\delta^{1-D}$. It is interesting to mark that the Richardson results count in favour of the main assumption which is introduced when the class of box-computing dimensions is considered, namely: $N(\delta)\sim\delta^{-D}$, where D is a fractal dimension.

2. Consider all the points with distances to the coastline of no more than δ. They form a strip of width 2 δ. Then the strip area divided by 2 δ is an approximation to $L(\delta)$. Here, too, the length increases as δ decreases. At the same time Mandelbrot noted that there is an interval for δ in which the value $L(\delta)$ becomes stable.

2.2. The approximation of a line length

In [6] the authors considered a modified variant of the second Mandelbrot method, namely: a) the approximation of a given curve by a piecewise constant function; b) the using a modified formula for calculation of the strip area.

To explain their method we consider the following example.

Let a piecewise constant function x(i) be defined on intervals [i,i+1), i=1…10. Let $\delta = 0$ and denote upper and low approximating boundaries for the initial curve x(i) by u_δ and v_δ respectively. Construct the strip of width 2 δ by the following formulas: following formulas:

$$u_\delta(i) = \max\{u_{\delta-1}(i) + 1, \max\{u_{\delta-1}(i+1), u_{\delta-1}(i+1)\}\},$$

$$b_\delta(i) = \min\{b_{\delta-1}(i) - 1, \min\{b_{\delta-1}(i+1), b_{\delta-1}(i-1)\}\},$$

$$u_0(i) = b_0(i) = x(i).$$

The strip area is calculated as

$S_\delta = \sum_{i=1}^{n-1}\left(u_\delta(i) - b_\delta(i)\right)$. To calculate the approximate curve length we can use two formulas:

$$L(\delta) = \frac{S_\delta}{2\delta}\ (1) \text{ or } L(\delta) = \frac{S_\delta - S_{\delta-1}}{2}. \qquad (2)$$

Formula (1) was supposed by Mandelbrot, formula (2) was used in [6] as a more appropriate variant. Let x(i) be defined as (1,-1,1,1,1,0,0,3,-2,0), i.e x(1)=x([1,2))=1, x(2)=x([2,3))= -1, etc.(See Figure 1.) Then for $\delta = 1$ we have

$$u_1 = \{2,1,,2,2,2,1,3,4,3,1\},$$

$$b_1 = \{-1,-2,-1,0,0,-1,-1,-2,-3,-2\},$$

$$S_1 = 31, L(1) = 15.5.$$

On this step the length values obtained according to (1) and (2) ($L^1(\delta)$ and $L^2(\delta)$) coincide.

For $\delta = 2,3,4$ we have $S_2 = 50, L^1(2) = 12.5, L^2(2) = 9.5;\ S_3 = 68, L^1(3) = 11.3, L^2(3) = 9; S_4 = 86, L^1(4) = 10.8, L^2(4) = 9.$

It is easy to check that the length of the initial curve is equal to 9, so the using (2) leads to the result faster.

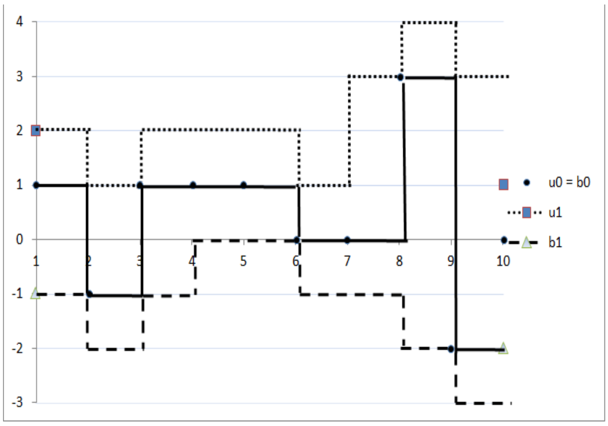


Figure 1. Covering blanket for one-dimensional case

2.3. The measuring a surface area

The described method may be applied to calculate approximately the area of gray level surface for an image. Let $F = \{X_{ij}, i = 0,1,..,K, j = 0,1,.,L\}$ be an image with multigray level and X_{ij} be the gray level of the (i, j)-th pixel. In a certain measure range, the gray-level surface of *F* can be viewed as a fractal. In image processing the gray level function F is a nonempty bounded set in R^3. The surface area A_δ may be calculated using the volume of a special δ-parallel body — blanket with the thickness 2δ.

For δ=1, 2,… the blanket surfaces are defined iteratively as follows:

$$u_\delta(i,j) = \max\left\{ \begin{array}{c} u_{\delta-1}(i,j) + 1, \\ \max\limits_{|(m,n)-(i,j)|\le 1} u_{\delta-1}(m,n) \end{array} \right\},$$

$$b_\delta(i,j) = \min\left\{ \begin{array}{c} b_{\delta-1}(i,j) - 1, \\ \min\limits_{|(m,n)-(i,j)|\le 1} b_{\delta-1}(m,n) \end{array} \right\}.$$

The volume of the blanket Vol_δ is computed as

$$Vol_\delta = \sum (u_\delta(i,j) - b_\delta(i,j)).$$

By analogy with (1) and (2) one can use the following formulas for the surface area:

$$A_\delta = \frac{Vol_\delta}{2\delta} \quad (3)$$

and

$$A_\delta = \frac{Vol_\delta - Vol_{\delta-1}}{2}. \quad (4)$$

Fractal dimension of the surface is defined by the formula

$$D \approx 2 - \frac{\log_2 A_\delta}{\log_2 \delta}. \quad (5)$$

As it was noted in [6], the formula (3) is more preferable for pure fractal objects, whereas (4) is used both fractal and non-fractal surfaces. The application of (4) is necessary if Vol_δ depends on all smaller scales features: subtracting $Vol_{\delta-1}$ isolates those features that change from scale δ-1 to δ. In what follows we use (4).

In [6] the authors calculated Vol_δ, A_δ and $S_\delta = \frac{\log_2 A_\delta}{\log_2 \delta}$ when δ changed from 1 to 49. For images I and J they computed the distance between them as the distance between obtained vectors $S_\delta(I)$ and $S_\delta(J)$:

$$\rho(I,J) = \sum_\delta (S_\delta(I) - S_\delta(J))^2 \log\frac{(\delta+0.5)}{(\delta-0.5)}. \quad (6)$$

For images having similar structures obtained vectors seemed to be close.

The authors also considered asymmetric method to calculate surface area, namely they use upper volume and low volume

$$Vol_\delta^+ = \sum (u_\delta(i,j) - \mathrm{x}(i,j)), \quad (7)$$

$$Vol_\delta^- = \sum \mathrm{x}(i,j) - b_\delta(i,j)), \quad (8)$$

and top area and bottom area respectively

$$A_\delta^+ = Vol_\delta^+ - Vol_{\delta-1}^+, \quad (9)$$

$$A_\delta^- = Vol_\delta^- - Vol_{\delta-1}^-. \quad (10)$$

Hence we have

$$S_\delta^+ = \frac{\log A_\delta^+}{\log \delta}, \quad (11)$$

$$S_\delta^- = \frac{\log A_\delta^-}{\log \delta}, \quad (12)$$

and

$$\rho(I,J) = \sum_\delta \left(\left(S_\delta^+(I) - S_\delta^+(J) \right)^2 + \right.$$
$$S\delta{-}I{-}S\delta{-}J)2\log(\delta{+}0.5)(\delta{-}0.5).$$
(13)

For $\delta = 1{,}2$ formulas (4) and (5) were used in [7] to estimate fractal dimension of different parts of text documents — text, graphic and background.

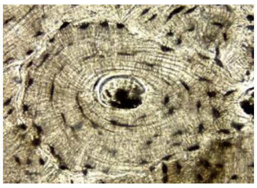
2.4. Color scale

It is known that the production of a color image is often a result of software application, whereas the image is monochrome (as in microscope, for example). The majority of mathematical methods of image analysis uses gray scale, but a transformation from one color scale to another may influence on the obtained results. It was shown in [2] that statistical characteristics calculated by the Haralic method do not allow classifying images from different classes when RGB is used. At the same time this classification was done for HSV palette.

3. EXPERIMENTAL RESULTS

It is easy to understand that the images having similar texturs may have close vectors of features. The opposite is wrong: the similarity of texturs not always follows from small distance between vectors, i.e. this is necessary but unsufficient condition Sometimes this situation may be corrected by the applying the asymmetric method or changing of palette (color). Consider some examples.

3.1. Health bone tissue (1)



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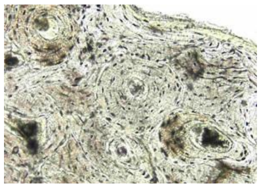

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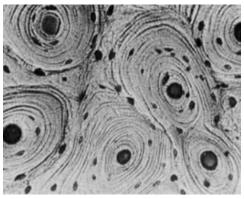

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The symmetric method gives the following results: ρ(25,26)=0.09026, ρ(25,40)=0.06453, ρ(26,40)=0.07607. Taking the minimal distance we come to conclusion that 25 and 40 are close. The classification result is right. Moreover, in this case we could assume that these images belong to the same class (with the distance 0.09026).

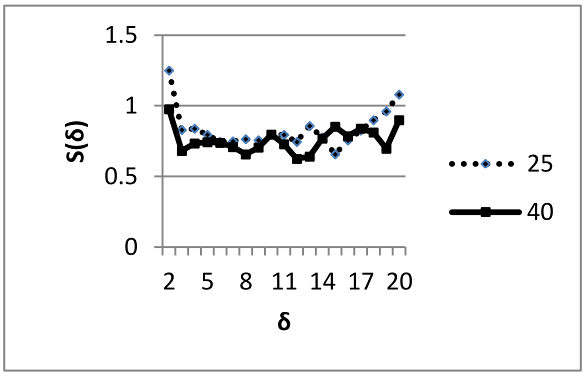
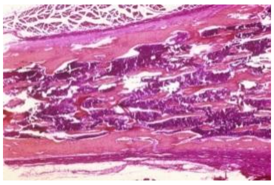


Figure 2. Close vectors for similar texturs

3.2. Health bone tissue (2)



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The result of symmetric method is 0.04113 (minimal in this class). As a misclassification occurred, asymmetric method was applied. The obtained distance 0.45156 seemed to be minimal in the set of images, hence a misclassification occurred again.

3.3. Affected bone tissue

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The symmetric method showed the closeness of vectors – the distance 0.0649 is minimal for the given class. But it is easy to see that the pictures are quite different. The asymmetric method corrected the misclassification with the distance 0.235 which is not minimal. It is interesting to note that when using HSV palette we obtain the result by applying only symmetric method (Figure 5).

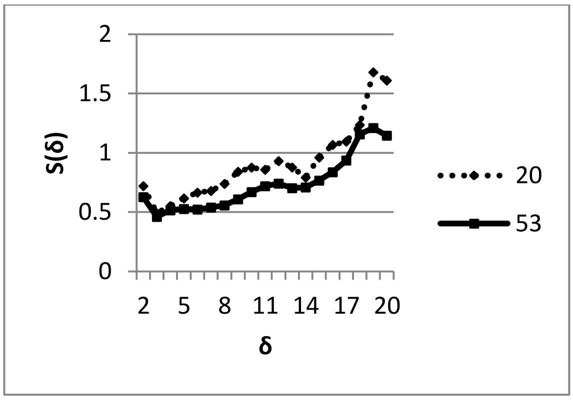

Figure 3. Close vectors for nonsimilar textures – symmetric method

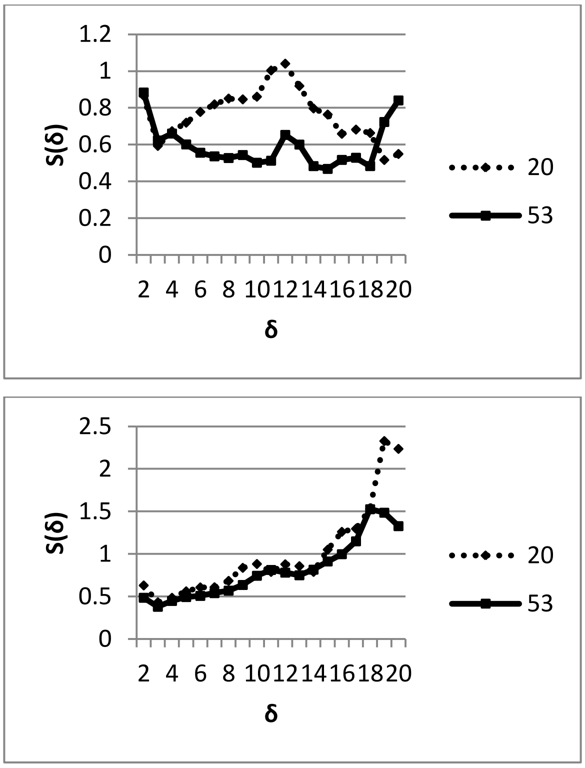

Figure 4. Nonsimilar textures —asymmetric method; S^+ (upper) and S^- (low)

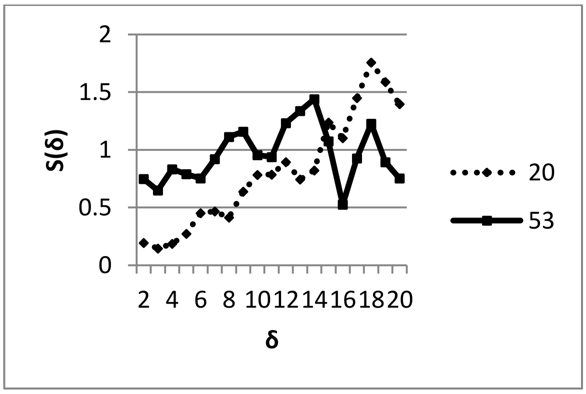

Figure 5. Symmetric method for HSV

3.4. Color scale

As we see in the case 3.2 misclassification occurred both symmetric and asymmetric method. Hence we used HSV scale (component h). The symmetric method in this scale showed the distance 1.92, not minimal. So, the images are not similar.

CONCLUSION

The fractal signature method is based on the Mandelbrot idea about the approximate measurement of the coastline length. In application to digital image analysis the construction of approximate surfaces is based on gray scale of pixels. The changing this scale by the definite rule (consecutive construction of blankets) may be considered as a changing of the image resolution. The application of the method to different classes of bone tissue images demonstrated that a primary classification have to be very accurate (size, color). Under these conditions it is sufficiently to use symmetric method. If a misclassification occurs symmetric variant or a color scale changing may help. But for more rigorous results it is necessary to apply additional methods.

Acknowledgements

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