MATHEMATICAL METHODS AND ALGORITHMS BASED ON ANALYSIS OF ECG AND EEG SIGNALS. NOISE REMOVAL FROM ECG AND EEG SIGNALS

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Abstract

This paper attempts to provide a comprehensive survey of different types of pathologies detection algorithms and their potential role in diagnostic and therapeutic applications. Major recent algorithms use electrocardiogram (ECG) and electroencephalogram (EEG) signals to detect the event. In these algorithms, various features are extracted from the EEG signal alone or in concert with the EEG signals. Noise sources increase the difficulty in analyzing the ECG and EEG to obtaining clinical information. For this reason, it is necessary to design specific filters to decrease such artifacts in ECG and EEG records. A new technique based on simple statistical parameters is proposed to solve this problem in this application. It is concluded that the proposed filter reduces the common artifacts present in EEG signals without removing significant information embedded in these records.

Keywords*: ECG, EEG, noise removal, seizure detection algorithm.*

1. INTRODUCTION

The mathematical modelling of the ECG is known as the forward problem of electrocardiography. It relies on three main ingredients: a model for the electrical activity of the heart, a model for the torso (extra-cardiac regions) and some specific heart-torso coupling conditions. Within each of these components, several options are possible, with different levels of complexity and realism. Although many works have been devoted to the numerical simulation of cardiac electrophysiology, only a small number addresses the numerical simulation of ECGs using a whole-heart reaction-diffusion model. Among them, only a very few provide meaningful simulations of the complete 12-led ECG. The main ingredients of mathematical ECG model are standard: bidomain equations and phenomenological cell model for the heart, and a generalized Laplace equation for the torso.

The numerical methods proposed to solve the problem offer a good balance between efficiency, stability and accuracy. The PDE system made of the heart and torso models is solved using a finite element method and a second order semi-implicit time marching scheme. The coupling conditions at the heart-torso interface are enforced by a Dirichlet-Neumann domain decomposition algorithm.

Technically an electroencephalography (EEG) consists of multiple channels that monitor neurons' activities in a region, each channel represent an electrode on a patient's scalp. A nerve axon may be stimulated and the activated sodium (Na^+) and potassium (K^+) channels produced in the vicinity of the cell membrane may lead to the electrical excitation of the nerve axon. The excitation arises from the effect of the membrane potential on the movement of ions, and from interactions of the membrane potential with opening and closing of voltage activated membrane channels.

A regular EEG can have from up to 20 electrodes and last more than an hour. Quite large areas of cortex –in the order of a few square centimetres - have to be activated synchronously to generate enough potential for changes to be registered at electrodes placed on the scalp [1] EEG plays a central role in diagnosis and management of patients with seizure disorders. Routine EEG is used in the following clinical circumstances: epilepsy, to distinguish epileptic seizures from other types of

spells, differentiate encephalopathy, neurodegenerative disorders, to evaluate comatose patients, to serve as an adjunct test of brain death.

2. MAIN TEXT

The reference model for the electrical activity of the heart is the so-called bidomain model. This macroscopic model is based on the assumption that, at the cell scale, the cardiac tissue can be viewed as partitioned into two ohmic conducting media, separated by the cell membrane: intracellular, made of the cardiac cells, and extracellular which represents the space between them. After a homogenization process, the intra and extracellular domains can be supposed to occupy the whole heart volume Ω_H (this also applies to the cell membrane). Hence, the averaged intra- and extracellular densities of current, j_i and j_e, conductivity tensors, σ_i and σ_e, and electric potentials, u_i and u_e, are defined in Ω_H. The electrical charge conservation becomes

$$div(j_i + j_e) = 0, \text{ in } \Omega_H,$$

and the homogenized equation of the electrical activity of the cell membrane is given by

$$A_m\left(C_m \frac{\partial V_m}{\partial t} + I_{ion}(V_m, w)\right) + div(j_i) = A_m I_{app}$$

Complemented with the Ohm's laws

$$j_i = -\sigma \nabla u_i, \quad j_e = -\sigma \nabla u_e.$$

Here, V_m stands for the transmembrane potential, defined as

$$V_m \overset{def}{=} u_i - u_e.$$

A_m is a constant representing the rate of membrane area per volume unit and C_m the membrane capacitance per area unit [8]. The term $I_{ion}(V_m, w)$ represents the ionic current across the membrane and I_{app} a given applied current stimulus. Both currents are measured per membrane area unit.

In general, the ionic variable w (possibly vector valued) satisfies a system of ODE of the type:

$$\frac{\partial w}{\partial t} + g(V_m, w) = 0, \text{ in } \Omega_H.$$

The definition of the functions g and I_{ion} depends on the considered cell ionic model [6].

To calculate brain rhythms a discrete Fourier (DFT) transformation is used:

$$\sum_{n=0}^{N-1} x_n \left(\sin\left(-\frac{2\pi}{N} kn\right) + i \cos\left(-\frac{2\pi}{N} kn\right)\right)$$

$$k = 0,..., N-1.$$

as well as the inverse discrete Fourier transformation (IDFT):

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{(2\pi i / N) kn} \qquad n = 0,..., N-1.$$

To calculate the required brain rhythm DFT is applied over the signal. All the frequencies not corresponding to the required rhythm are set to zero. IDFT is applied over the resulting data [5].

After filtering out all the waves except the required rhythm we can calculate its power. To do this we use a statistical value – root mean square. Which is defined as:

$$RMS = \sqrt{\frac{x_0^2 + x_1^2 + ... = y_{N-1}^2}{N}}$$

Here *x* is value of the signal at a discrete time. For every interval of EEG all of the brain rhythms and powers are calculated. Out of these a rhythm is said to be dominant in an interval if it is power is the highest [7].

3. ILLUSTRATIONS

Action potential duration (APD) heterogeneity may be found at different myocardium locations, for instance: between base and apex, between septal and posterior sides, and transmurally[2]. Although not yet fully explained [3], experimental evidence suggests that trans mural APD heterogeneity is likely to be the most important factor in the genesis of the normal ECG T-wave shape and polarity.

In the present work, cell heterogeneity is only considered as transmural variation of APD in the left ventricle [9]. Hence, we assume that epicardial cells have the shortest APD and that endocardial cells have the shortest APD and that endocardial cells have an intermediate APD between mid-myocardial cells (M-cells) and epicardial cells[4]. The APD heterogeneity is modelled with a parameter τ_{close} varying across the left ventricle transmural direction: τ_{close}^{endo} near the endocardium, τ_{close}^{mcell} in the mid-myocardium (M-cells) and τ_{close}^{epi} near the epicardi-

um (figure 1 and figure 2). For simplicity, we take a constant value of τ_{close}^{RV} in the whole right ventricle. The values of the parameters are given in table 1.

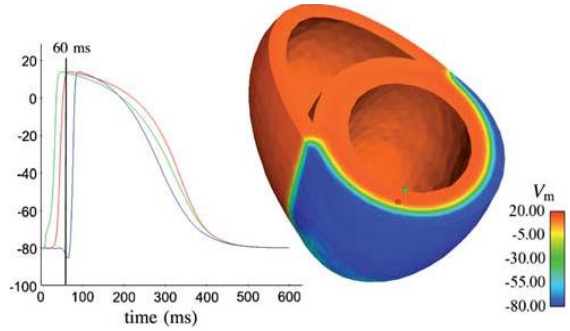

Figure 1. Transmural APD heterogeneity: comparison of the simulated transmembrane potential for endocardial cells (green), M-cells (red) and epicardial cells (blue). Snapschots of the transmembrane potential at times t = 60 ms.

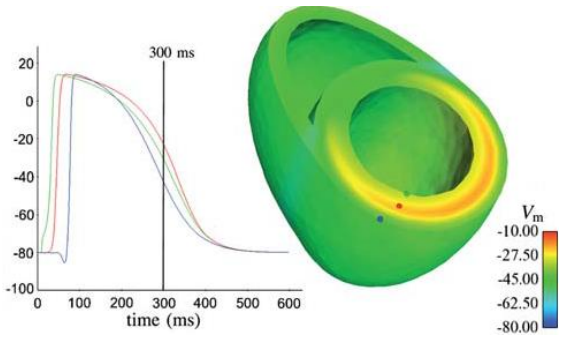

Figure 2. Transmural APD heterogeneity: comparison of the simulated transmembrane potential for endocardial cells (green), M-cells (red) and epicardial cells (blue). Snapschots of the transmembrane potential at times t = 300 ms.

Table 1. Cell membrane parameters

$A_m(cm^{-1})$	200
$C_m(mF)$	10^{-3}
τ_{in}	4.5
τ_{out}	90
τ_{open}	100
τ_{close}^{RV}	120
τ_{close}^{endo}	130
τ_{close}^{mcell}	140
τ_{close}^{epi}	90
V_{gate}	-67
$V_{\min}$	-80
$V_{\max}$	20

The membrane potential of EEG signal increases when the membrane is polarized with a net negative charge lining the inner surface and an equal but opposite net positive charge on the outer surface [10]. This potential may be simply related to the amount of electrical charge Q, using

$$E = \frac{Q}{C_m}$$

where Q is in terms of coulombs/cm^2 , and E is in units of volts. In practice, in order to model the action potentials (APs) the amount of charge Q^+ on the inner surface of the cell membrane has to be mathematically related to the stimulating current I_{stim} flowing into the cell through the stimulating electrodes. The Hodgkin and Huxley model is illustrated in Figure 3.

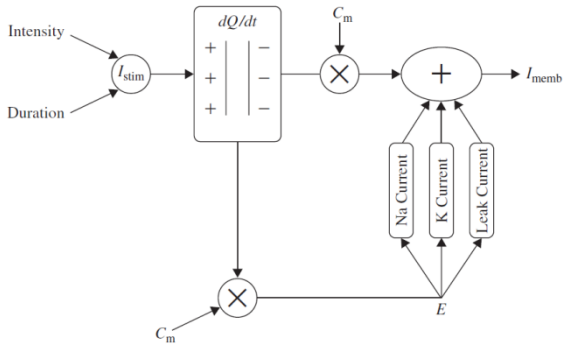

Figure 3. The Hodgkin-Huxley excitation model

In this figure I_{memb} is the result of positive charges flowing out of the cell. This current consists of three currents, namely Na, K, and leak currents. The leak current is due to the fact that the inner and outer Na and K ions are not exactly equal.

4. CONCLUSION

The electrical activity of the heart is based on the coupling of the bidomain equations with phenomenological ionic model, including anisotropic conductivities and transmural APD heterogeneity.

To further advance automatic EEG analysis these work items are planned for the future: develop methods for noise detection, high level analysis methods (i.e. epilepsy classification, a new drowsiness scale, etc.)

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