ON THE LIMITATIONS OF GAIN CALIBRATION IN HARMONIC REJECTION MIXERS

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Abstract

The paper deals with the number of harmonics that can be completely rejected simultaneously in harmonic rejection mixers using only gain calibration. The restrictions on the combinations of rejected harmonics are examined too. The results are also applicable to harmonic rejection in digital domain.

1. INTRODUCTION

Harmonic rejection mixers (HRM) have been increasingly used in wideband receivers as they alleviate the harmonic mixing problem and thus considerably relax the preselect filtering requirements. An HRM is a complex mixer, consisting of several parallel operating conventional hard switching mixers, driven by a multiphase local oscillator (LO). The HRM output signal is a weighted sum of the signals down converted by the individual mixers [1]. The HRM can be seen as a single perfect multiplier, driven by an *effective* LO waveform, in which some harmonics are eliminated.

There are numerous implementation options for HRMs, but the basic principle can be illustrated by Fig. 1. It can be easily realized that the effective LO waveform is by its nature a sampled sinusoid. Therefore it contains only harmonics of orders $kN \pm$ □, where *k*=1, 2, ... So, the nearest interferer, which should be suppressed by the preselect filters is at □N □□f_{LO} (assuming a zero-IF receiver).

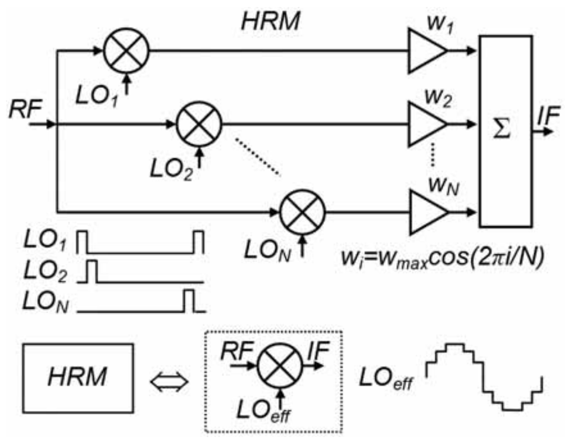

Fig. 1. HRM operation principle

Unfortunately harmonic rejection ratios (HRR) of HRMs are typically limited to 30-40 dB due to gain and phase mismatches [2]. HRR can be enhanced by gain and phase calibration [3]. Simultaneous calibration of both phases and gains however, increases complexity of HRM. Hence, many HRMs employ only gain calibration [4, 5, 6].

Another efficient way for HRR improvement is to perform the suppression of harmonic interferers partly [1, 2, 7, 8] or entirely [9] in digital domain. In [1, 2, 7, 8] adaptive cancellation is used. In [9] the harmonic rejection (HR) problem is formulated and treated as a problem from the area of multiuser detection and an MMSE equalizer is derived for the suppression of the harmonic interferers. These two approaches to digital HR appear to be completely different, but in both cases, although different tools are used, the proper weighting factors are found and applied to form a weighted sum of the individual mixer outputs. From this perspective, the proposed HRMs with digital interference suppression can be seen as HRMs with gain calibration only.

In this paper we determine the number of harmonics that can be completely rejected simultaneously in presence of phase mismatches using gain calibration only. In addition, the restrictions on the combination of these harmonics are investigated. The results are also applicable to HRMs employing digital domain HR techniques.

The rest of the paper is organized as follows:

In Section 2 the system model is presented and the conditions for HR as a system of linear equations are expressed. In Section 3 this equation system is examined from the viewpoint of finding the allowable combinations of harmonics, that can be completely rejected simultaneously. In section 4 examples for some typical scenarios are presented and discussed.

2. THE HRM MODEL

The effective LO waveform can be expressed as:

$$LO_{eff} = \sum_{i=1}^{N} LO_i(t) w_i \ , \tag{1}$$

where $LO_i(t) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_{LO} - t_i)$ and $p(t)$ denotes the single effective pulse, which multiplies the RF signal in each HRM path. The convolution with $p(t)$ is equivalent to a mild filtering of the effective LO waveform and usually does not play a significant role in the rejection of the LO harmonics. Therefore, it will be ignored in the rest of the paper, excluding an important special case. Also, without loss of generality $T_{LO} = 1$ will be assumed. Furthermore, the path count N will be assumed to be even, as odd values result in implementation disadvantages.

The Fourier series coefficients of the effective LO waveform, ignoring the scale factors, are given by:

$$c_k = \sum_{i=1}^{N} w_i e^{jk\theta_i} \ , \tag{2}$$

where $\theta_i = -(2\pi i / N + \varphi_i)$ and φ_i is the phase error of the i-th LO pulse train.

The path gains, needed to reject the DC component and the harmonics of the orders from 2 to $N-2$ are the solution of the following system of $N-1$ linear equations:

$$\mathbf{A}\,\mathbf{w} = \mathbf{b} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\theta_1} & e^{j\theta_2} & \cdots & e^{j\theta_N} \\ e^{j2\theta_1} & e^{j2\theta_2} & \cdots & e^{j2\theta_N} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(N-2)\theta_1} & e^{j(N-2)\theta_2} & \cdots & e^{j(N-2)\theta_N} \end{bmatrix},$$

$\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$, $\mathbf{b} = [0, e^{j\alpha}, 0, \ldots, 0]^T$, and α is the phase angle of the LO fundamental component.

We seek real solutions only. Complex solutions are inapplicable, because the two quadrature components of the LO harmonics are not available separately in the HRM hardware. So, the multiplication with complex weighting factors cannot be implemented.

A widely used HRM implementation option is to use double LO pulses of the form $p_D = p(t) - p(t - T_{LO}/2)$ ensuring even HR. Then the number of HRM paths is reduced by half and will be denoted by M. Such HRMs will be designated here as M-path HRMs in contrast to N-path HRMs described before. The LO phase shifts in M-path HRMs are π/M. The HR equation system consists of $M-1$ equations for odd HR only.

Both N-path and M-path HRMs can be simplified if one of the path gains is constrained to be zero (i. e. this path does not exist in the hardware). Then the number of unknown path gains reduces by one.

3. HR EQUATION SYSTEM EXAMINATION

3.1. HRM without phase errors

Within the restriction to real solutions it is convenient to split the equations into real and imaginary parts using the Euler's formula. We obtain a new system

$$\mathbf{C}\,\mathbf{w} = \mathbf{d} \tag{4}$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & \cdots & 1 \\ \cos(\theta_1) & \cdots & \cos(\theta_N) \\ \sin(\theta_1) & \cdots & \sin(\theta_N) \\ \vdots & \ddots & \vdots \\ \cos[(N-2)\theta_1] & \cdots & \cos[(N-2)\theta_N] \\ \sin[(N-2)\theta_1] & \cdots & \sin[(N-2)\theta_N] \end{bmatrix}$$

and $\mathbf{d} = [0, \cos\alpha, \sin\alpha, 0, \ldots, 0]^T$, are a $(2N-3)\times N$ real-valued matrix and a $(2N-3)\times 1$ real-valued vector, respectively. Furthermore, it is suitable to scale the first and the N-th equations by $1/\sqrt{N}$ and the remaining ones by $\sqrt{2/N}$.

The equations from $N+1$-st to the the $2N-3$-rd are superfluous, as the $N+1$-st equation has all zero coefficients, and each of the rest equations duplicates one of the equations from the fourth to the $N-1$-st.

Let us create a new reduced equation system

$$\mathbf{Gw} = \mathbf{h} \tag{5}$$

where $\mathbf{G}$ is a matrix formed from the first N rows of $\mathbf{C}$ and $\mathbf{h} = [\square\square\square\square\alpha\ \square\square\square\alpha\ \square\square\square\square\square]^T$ is an N-element vector.

It can be easily proven that in the absence of phase errors $\mathbf{G}$ is an orthogonal matrix. As a result the system (5) has a unique solution.

3.2. Examination of the perturbed system

An orthogonal matrix is the best conditioned matrix. This implies that in the presence of sufficiently small phase errors $\boldsymbol{\varphi} = [\varphi_{\square\square}\ \varphi_{\square\square}\ \square\square\square\ \varphi_{N\square}]^T$ the perturbed system $\mathbf{G}(\boldsymbol{\varphi})\mathbf{w} = \mathbf{h}$ will not only have a unique solution $\mathbf{w}(\boldsymbol{\varphi})$ but its scatter will be small. Therefore, in the presence of phase errors, the path gains in an HRM can be adjusted for perfect suppression of the DC component and the harmonics up to and including $N/\square - \square$-st. In addition, the path gains will be nearly equal to their nominal values.

It is useful to prove that additional harmonics cannot be completely rejected. If one of the two equations, related to a harmonic of order k_{add}, $N/\square < k_{add} < N - \square$, is satisfied by $\mathbf{w}(\boldsymbol{\varphi})$, then $\mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi}) = \square$, where $\mathbf{r}(\boldsymbol{\varphi})$ is the corresponding row of $\mathbf{C}$, e. g. the one with cosine coefficients. It is difficult to find an expression for $f(\boldsymbol{\varphi}) = \mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi})$. So we find its derivative

$$\left.\frac{\partial[\mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi})]}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} = \left.\frac{\partial \mathbf{r}}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} \mathbf{w}(\square) + \mathbf{r}(\square)\left.\frac{\partial \mathbf{w}}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} \tag{6}$$

Using implicit differentiation we find

$$\left.\frac{\partial \mathbf{w}}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} = -[\mathbf{G}(\mathbf{0})]^{-\square} \left.\frac{\partial \mathbf{G}}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} \mathbf{w}(\mathbf{0}). \tag{7}$$

After performing substitutions and simplifications, we obtain:

$$\left.\frac{\partial[\mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi})]}{\partial \varphi_i}\right|_{\boldsymbol{\varphi}=\square} = -\sqrt{\square N}\ \square\square\left(k_{add}\, i \frac{\square\pi}{N}\right) w_i, \tag{8}$$

hence $\dfrac{\partial[\mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi})]}{\partial \varphi_i} \neq \square$ in the general case. Therefore in the general case $\mathbf{r}(\boldsymbol{\varphi})\mathbf{w}(\boldsymbol{\varphi})$ cannot be constant and, in particular, cannot be zero. In a similar way this can be proven for additional equations with sine coefficients. Hence, additional harmonics cannot be completely rejected.

However, the perfect suppression of the k-th harmonic can be traded off for perfect suppression of its "symmetrical", $(N-k)$-th harmonic, where $k < N/\square$. This is possible, because the corresponding matrix rows for the two symmetrical harmonics are identical in the unperturbed case. The scatter of the solution will be somewhat larger, as the phase errors will be multiplied by $(N-k)$ instead of k.

The simultaneous complete suppression of two symmetrical harmonics is, however, problematic. Assume, that in the system present e. g. the cosine equations for the k-th and $(N-k)$-th harmonics. By subtracting the first one from the second one we obtain a new equation. The solution of the system will not change if one of the two original equations is replaced by the new one. The coefficients of the latter will be approximately $-\varepsilon_i N\ \square\square(\square\pi\, k\, i\ \square N)$, so they will be completely random, unlike the coefficients of the remaining equations, where the random phase errors are added to much larger fixed angles. As a result some combinations of phase errors will cause very large path gains and even inconsistency of the system. It can be proven that the power of white noise, transferred to the HRM output from the output of the preceding RF amplifier is proportional to the sum of the squares of the path gains. Hence, in this case the HRM output noise can exceed many times that in the case of a solution, nearly equal to the nominal path gains. Since the fundamental magnitude is constant, a catastrophic SNR degradation can occur.

Similar problems arise when the HRM is calibrated for a complete rejection of the "middle", $N/2$-nd harmonic, because the corresponding sine equation has coefficients, roughly proportional to the respective phase errors. So the receiver will be susceptible to strong interferers at $N f_{LO}/\square$.

3.3. Implementation implications

In the case of an N-path HRM the consistent system of HR equations has N or $N-\square$ equations in the form (4), depending on whether there is a zero-gain path. The first three equations ensure the

DC suppression and define the fundamental magnitude and angle. The remaining equations ensure the harmonic rejection. Given that two equations per a rejected harmonic are required and only even values of N are preferred for implementation reasons, it is advisable for the *N*-path HRMs to be implemented with a zero-gain path. Otherwise one path will be superfluous.

A limitation of the HRM with one zero-gain is that they have a "native" phase angle of the fundamental tone due to the fixed position of the effective LO zero crossing. In this case the modification of the fundamental angle by path weight adjustment results in an increased level of the $N/2$-th harmonic.

For M-path HRMs the consistent HR equation system consists of M or $M-1$ equations depending on whether there is a zero-gain path. Based on considerations, similar to those for the N-pat HRMs, it is advisable to choose even values of M, or to implement HRMs with one zero-gain path, if M is odd.

The M-th harmonic causes the same problems in the M-path HRM, like the $N/2$-nd harmonic in their N-path counterparts. However this can be avoided here, if an even value of M is chosen. In this case the problematic harmonic would be already suppressed owing to the use of bipolar LO pulses, as far as good symmetry is achieved.

4. EXAMPLES AND DISCUSSION

To illustrate the theory we shall examine some examples. We assume that phase errors are uncorrelated normally distributed with $\sigma_{\varphi}=1$, and the path gains are initially equal to the nominal values corresponding to zero phase errors. In each of the examples, the calibrated path gains for 20 consecutive realizations of phase errors will be presented.

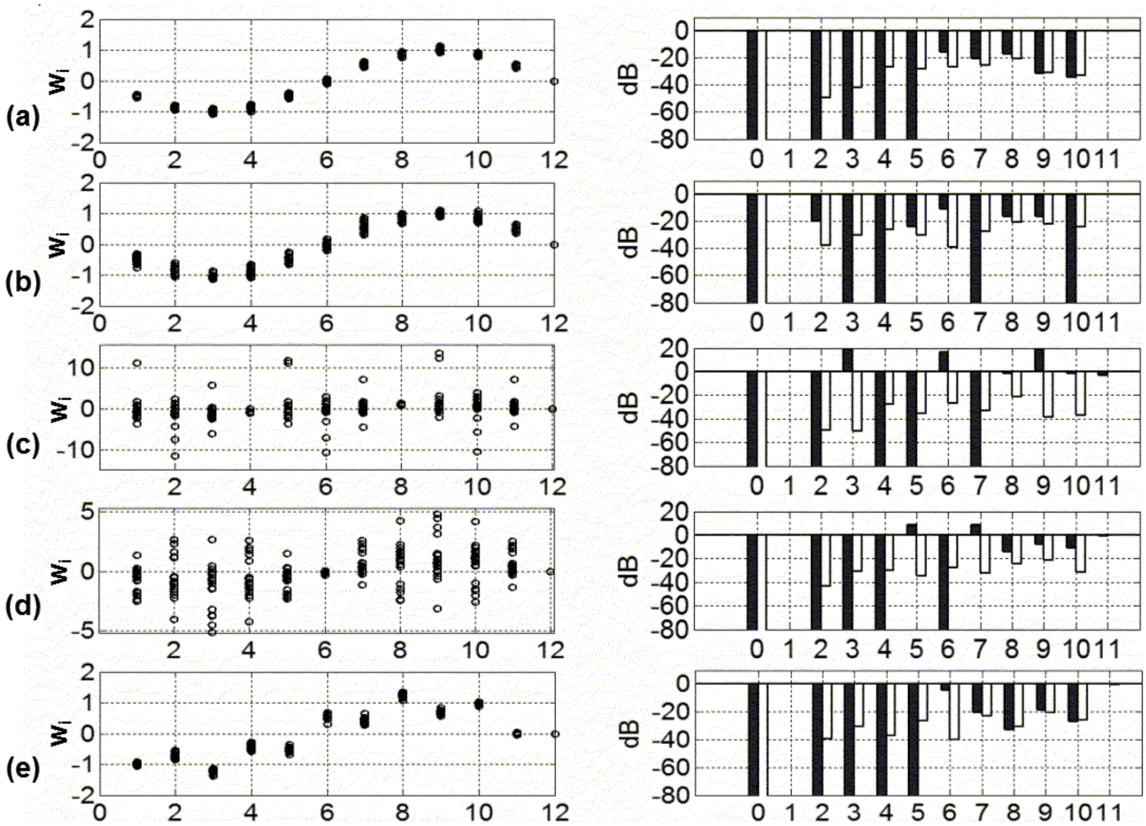

Fig. 2. Path gains after calibration for 20 realizations of a 12-path HRM and the effective LO harmonic levels for one typical its realization after and before calibration. Completely rejected harmonics of the orders 2, 3, 4 and 5 (a) and 3, 4, 7 and 10 (b). Simultaneous rejection of one pair of symmetrical harmonics (c). Complete rejection of the harmonic of the order N/2 (d). Fundamental tone shifted by 15° from its native value (e).

An N-path HRM with N=12 with one zero-gain path will be examined. The consistent system consists of 11 equations – the first one is for the DC rejection, the next two equations define the fundamental magnitude and angle, and the remaining 8 equations ensure complete suppression of 4 harmonics.

We choose the harmonics of the orders 2, 3, 4 and 5 to be completely rejected. The solution has minimal scatter (Fig. 2a). Each of the above harmonics can be exchanged for its symmetrical one without compromising the HRM performance. For example, we can completely suppress the harmonics of the orders 10, 3, 4 and 7 (Fig. 2b). However, the calibration for simultaneous rejection for example of the 5-th and 7-th harmonics results in very large gain scatter (Fig. 2c). If the $N\square 2$-nd, the 6-th harmonic in our case, is entirely rejected the solution has very large scatter again (Fig. 2d).

As can be expected, the calibration for a complete rejection of a set of harmonics worsens (but not necessarily) the HRRs for the remaining harmonics (Fig. 2 a-e). This can be especially strongly pronounced if a pair of symmetrical harmonics and/or the middle, $N\square 2$-nd harmonic is completely rejected (Fig. 2 c, d). An HRR degradation for the $N\square 2$-nd harmonic occurs in the case when a HRM with a zero-gain path is forced to operate with fundamental angle, different from its "native" one (Fig. 2 e)

For the M-path HRMs similar observations can be made as for their N-path counterparts.

5. CONCLUSIONS

The investigations of limitations of the gain calibration in HRMs showed that perfect suppression can be achieved only for $N/2-2$ and $\lfloor M/2-1 \rfloor$ harmonics in N-path and M-path HRMs, respectively. Calibration for perfect suppression of a given set of harmonics worsens the rejection of the remaining harmonics. Attempts for complete suppression of the middle harmonic or a pair of symmetrical harmonics can result in extremely large path gains and SNR degradation. In addition, convergence issues in the algorithms for calibration or digital domain HR can be expected.

As the RF spectrum gets more and more crowded, the probability for interference scenarios with a simultaneous presence of strong blockers at pairs of symmetrical LO harmonics will increase. This implies that the stopband of the preselect filters should begin at $N\square_{\square\square}/2$ if only gain calibration is employed. Additional phase calibration or increase of path count of the HRM would be needed if more relaxed filter requirements are desired.

It is advisable that N-path HRMs are implemented with even N and one zero-gain path, whereas M-path HRMs are implemented with even M.

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