ON THE MATHEMATICAL MODEL OF INTERPOLATION OF LOW FREQUENCY MAGNETIC FIELD USING EXPERIMENTAL DATA

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Abstract

In the process of magneto-therapy often it's necessary to use space distribution of the value of magnetic induction in points of patient's area in which there is not preliminary measurement of the value of magnetic induction. It's important first of all for the points in the human body. In this case a preliminary measurement of the value of magnetic induction in some points out of the human body can be provided. In result a set of scalars can be obtained. Usually the sources of low frequency magnetic field are several coils which are situated around the human body. The measurement points are the centers of these coils and the point on the middle of the line between them. At any moment only one pair of coils is active. The order of the choice of the pairs is defined by a data table. The problem is to find the values of magnetic induction in some points between the active coils and to visualize results. The results of interpolation for every pair can be shown and a physician can observe a space distribution of the value of magnetic induction of running magnetic field.

1. INTRODUCTION

In the mathematical field of numerical analysis interpolation is a method of constructing new data points within the range of a discrete set of known data points.In engineering and science, one often has a number of data points obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to interpolate (i.e. estimate) the value of that function for an intermediate value of the independent variable. This may be achieved by curve fitting or regression analysis.

A different problem which is closely related to interpolation is the approximation of a complicated function by a simple function. Suppose the formula for some given function is known, but too complex to evaluate efficiently. A few known data points from the original function can be used to create an interpolation based on a simpler function. Of course, when a simple function is used to estimate data points from the original, there are interpolation errors. However, depending on the problem domain and the used interpolation method used, the gain in simplicity may be of greater value than the resultant loss in accuracy.

When modelling magnetic field distribution one can use mathematical models and compute magnetic induction value in the points of a given lattice. In [1-3] the model of calculation of electromagnetic induction for two coils which are arbitrary disposed in 3D was described. The algorithms of calculation and visualization were also implemented.

For a large number of the lattice nodes the calculation may be time-consuming, so an application of interpolation methods is rather actual. We may calculate magnetic induction in a small number of points and obtain the induction values in other ones by interpolation. The results obtained in [4] show that such an approach leads to considerable reducing run time.

Results of measurements of electromagnetic induction are usually obtained as scalars and we have a problem of one-dimensional interpolation.

In this work we discuss a mathematical model for interpolation electromagnetic induction in a special magnetotherapy device – magneto bed. Interpolation nodes are the centers of two given coils and the middle of connecting them line.

2. TYPES OF INTERPOLATION

We discuss some types of interpolation methods that may be used in the case of one-dimensional interpolation.

2.1. Linear interpolation

One of the simplest methods is linear interpolation (sometimes known as lerp). Generally, linear interpolation takes two data points, say (x_a,y_a) and (x_b,y_b), and the interpolant is given by:

$$y = y_a + (y_b - y_a)\frac{x - x_a}{x_b - x_a} \text{ at the point } (x, y) \qquad (1)$$

Linear interpolation is quick and easy, but it is not very precise. Another disadvantage is that the interpolant is not differentiable at the point x_k.The following error estimate shows that linear interpolation is not very precise. Denote the function which we want to interpolate by g, and suppose that x lies between x_a and x_b and that g is twice continuously differentiable. Then the linear interpolation error is

$$|f(x) - g(x)| \leq C(x_b - x_a)^2 \quad \text{where} \quad C = \frac{1}{8}\max_{y\in[x_a,x_b]}|g''(y)|. \qquad (2)$$

In words, the error is proportional to the square of the distance between the data points. The error in some other methods, including polynomial interpolation and spline interpolation (described below), is proportional to higher powers of the distance between the data points. These methods also produce smoother interpolants (Fig.1).

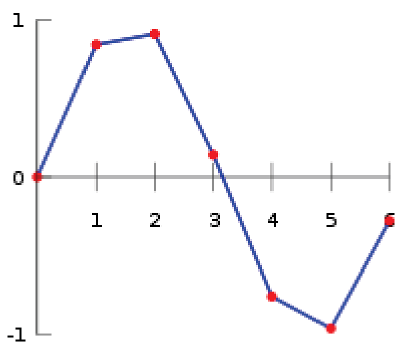

Fig. 1

2.2. Polynomial interpolation

Polynomial interpolation is a generalization of linear interpolation. Note that the linear interpolant is a linear function. We now replace this interpolant with a polynomial of higher degree.

Generally, if we have n data points, there is exactly one polynomial of degree at most $n-1$ going through all the data points. The interpolation error is proportional to the distance between the data points to the power n. Furthermore, the interpolant is a polynomial and thus infinitely differentiable. So, we see that polynomial interpolation overcomes most of the problems of linear interpolation (Fig.2).

However, polynomial interpolation also has some disadvantages. Calculating the interpolating polynomial is computationally expensive compared to linear interpolation. Furthermore, polynomial interpolation may exhibit oscillatory artifacts, especially at the end points. More generally, the shape of the resulting curve, especially for very high or low values of the independent variable, may be contrary to commonsense, i.e. to what is known about the experimental system which has generated the data points. These disadvantages can be reduced by using spline interpolation or restricting attention to Chebyshev polynomials.

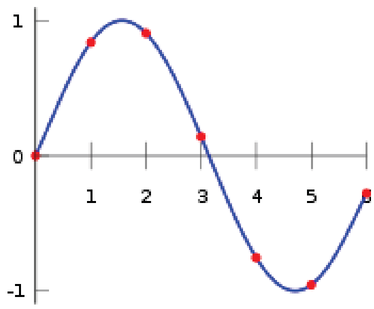

Fig. 2

2.3. Spline interpolation

Remember that linear interpolation uses a linear function for each of intervals $[x_k,x_{k+1}]$. Spline interpolation uses low-degree polynomials in each of the intervals, and chooses the polynomial pieces such that they fit smoothly together. The resulting function is called a spline. Like polynomial interpolation, spline interpolation incurs a smaller error than linear interpolation and the interpolant is smoother. However, the interpolant is easier to evaluate than the high-degree polynomials used in polynomial interpolation. It also does not suffer from Runge's phenomenon. It should be noted that minimal power for the spline is 3.

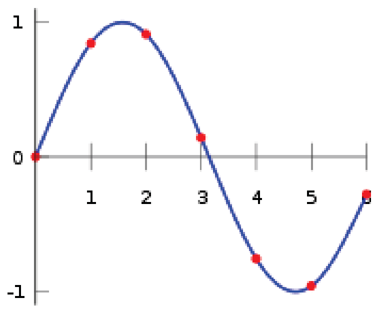

Fig. 3

3. INTERPOLATION OF THE VALUE OF MAGNETIC INDUCTION OF LOW FREQUENCY MAGNETIC FIELD IN THE CASE OF MAGNETIC BED

One new system for magneto-therapy is "magnetic bed" (Fig. 4).

The number of every coil and the coordinate system XYZ are shown on the Fig. 4. The first coil of pair is denoted by "A" in the table and the second coil in the pair by "B". In the table there are coordinates of every point of measurement.

For every pair of coils the measurements have been done at 3 points. The first and the second points are in the centers of the two coils and the third point of measurement is on the middle of the line between the centers of two coils of the pair. In every moment only one pair of coils is active. In the process of the system functioning every pair of coils is active by turns and a movement of magnetic field occurs. The order of the choice of pairs of coils is defined by a data table. It is necessary to provide interpolation on the line of every pair of coils on the base of result of measurement in three points and visualize results.

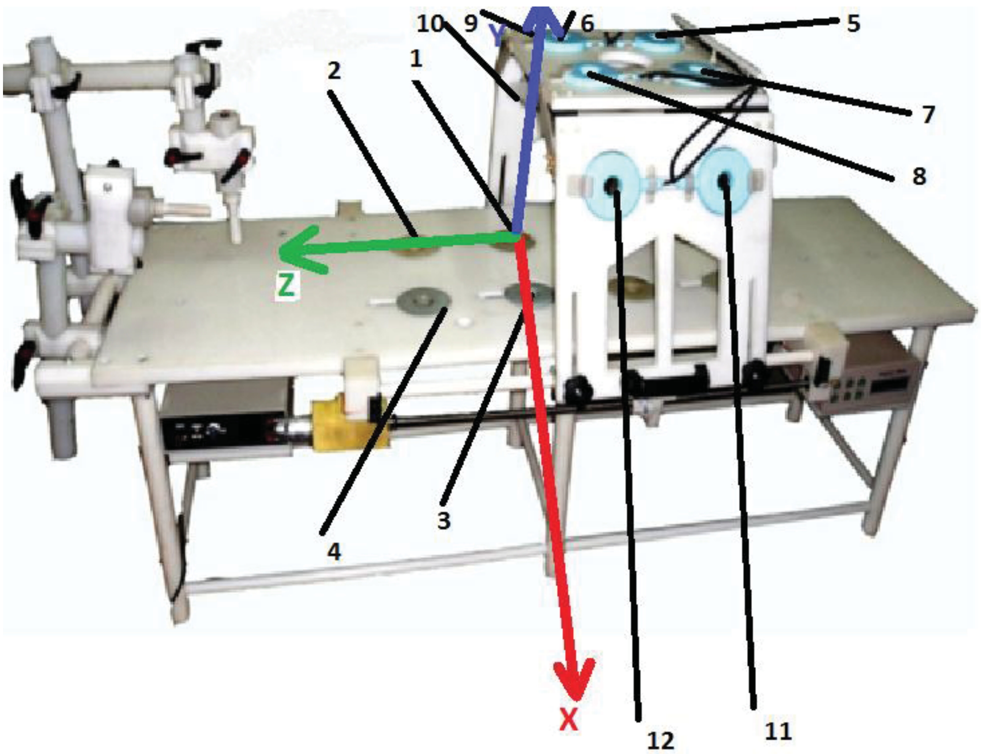

Fig. 4

The results of experimental measurement of the value of magnetic induction of low frequency magnetic field in different points according to Fig. 4 are shown in Table 1. This table can be saved as text file structured in accordance with the table (matrix 13 x 32). Interpolation can be done for points which are on the line containing interpolation nodes.

Taking into account the advantages and disadvantages of the interpolation methods we came to the decision to use one- dimensional square L'Agrange interpolation. The interpolation nodes are three points:

1. Centre of the first active coil. The coordinates of this node (x_0, y_0, z_0) can be obtained from the columns 3, 4, 5 of the input file.
   The value of magnetic induction after experimental measurement in the node (x_0, y_0, z_0) is f_0 in the column 10 of the input file.
2. Centre of the second active coil. The coordinates of this node (x_2, y_2, z_2) can be obtained from the columns 7,8,9 of the input file . The value of magnetic induction after experimental measurement in the node (x_2, y_2, z_2) is f_2 in the column 11 of the input file.

Table 1

№	Coil A				Coil B				Value A mT	Value B mT	Average value mT	Distance mm
	№	axis			№	axis						
		X	Y	Z		X	Y	Z				
1	1	0	0	0	5	0	500	0	50	50	0.5	500
2	1	0	0	0	6	0	500	234	50	50	0.38	550
3	1	0	0	0	7	238	500	0	50	50	0.25	600
4	1	0	0	0	8	238	500	234	50	50	0.12	640
5	2	0	0	234	5	0	500	0	50	50	0.38	550
6	2	0	0	234	6	0	500	234	50	50	0.5	500
7	2	0	0	234	7	238	500	0	50	50	0.12	640
8	2	0	0	234	8	238	500	234	50	50	0.25	600
9	3	238	0	0	5	0	500	0	50	50	0.12	640
10	3	238	0	0	6	0	500	234	50	50	0.25	600
11	3	238	0	0	7	238	500	0	50	50	0.5	500
12	3	238	0	0	8	238	500	234	50	50	0.38	550
13	4	238	0	234	5	0	500	0	50	50	0.12	640
14	4	238	0	234	6	0	500	234	50	50	0.25	600
15	4	238	0	234	7	238	500	0	50	50	0.38	550
16	4	238	0	234	8	238	500	234	50	50	0.5	500
17	1	0	0	0	9	-130	333	0	50	50	1	360
18	1	0	0	0	10	-130	333	234	50	50	0.63	430
19	1	0	0	0	11	368	333	0	50	50	0.5	500
20	1	0	0	0	12	368	333	234	50	50	0.38	550
21	2	0	0	234	9	-130	333	0	50	50	0.63	430
22	2	0	0	234	10	-130	333	234	50	50	1	360
23	2	0	0	234	11	368	333	0	50	50	0.38	550
24	2	0	0	234	12	368	333	234	50	50	0.5	500
25	3	238	0	0	9	-130	333	0	50	50	0.5	500
26	3	238	0	0	10	-130	333	234	50	50	0.38	550
27	3	238	0	0	11	368	333	0	50	50	1	360
28	3	238	0	0	12	368	333	234	50	50	0.63	430
29	4	238	0	234	9	-130	333	0	50	50	0.38	550
30	4	238	0	234	10	-130	333	234	50	50	0.5	500
31	4	238	0	234	11	368	333	0	50	50	0.63	430
32	4	238	0	234	12	368	333	234	50	50	1	360

3. Middle of the line between points 1 and 2.The coordinates (x_1, y_1, z_1) of this node can be calculated by using coordinates of the points 1 and 2.
   Then:

$$(x_1, y_1, z_1) = \left(\frac{x_0 + x_2}{2}, \frac{y_0 + y_2}{2}, \frac{z_0 + z_2}{2}\right) \qquad (3)$$

4. The value of magnetic induction after experimental measurement in the node (x_1, y_1, z_1) is in the column 12 of the input file.

   For the point of interpolation (x,y,z) one can use the equation

$$l_i(x,y,z) = \prod_{j=0, j\neq i}^{2} \frac{\sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2}}{\sqrt{(x_i-x_j)^2 + (y_i-y_j)^2 + (z_i-z_j)^2}}, \qquad (4)$$

$$l_i(x_j, y_j, z_j) = 0, \; j \neq i$$
$$l_i(x_i, y_i, z_i) = 1$$

where: l_i – basic polynomial for the node (x_i,y_i,z_i).

The interpolation polynomial is:

$$L(x,y,z) = \sum_{n=0}^{2} f_n \cdot l_n(x,y,z), \qquad (5)$$

where f_n – the value of magnetic induction measured in the point (x_n,y_n,z_n).

4. CONCLUSION

We propose a mathematical model for interpolation of electromagnetic field in a special device of magnetotherapy — magnetic bed. Interpolation should be performed for the points of the line connecting centres of two given coils. Measurements are made in the coils centers and in the middle of the connecting them line. These points are interpolation nodes. To solve the task we use square L'Agrange interpolation.

Acknowledgements

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References

[1] N. Ampilova, D. Dimitrov, B. Kudrin. Mathematical modeling of low frequency magnetic field in systems for magnetotherapy. Proc. 8 Int. Conf. CEMA13, 17-19 Oct. 2013, Sofia, Bulgaria. p. 48-51.

[2] B. Kudrin, A. Dimitrov. An algorithm for visualization of low-frequency magnetic signals in systems for magnetotherapy. Proc. 8 Int. Conf. CEMA13. 17-19 Oct. 2013, Sofia, Bulgaria, p. 31-35.

[3] B. Kudrin, A. Dimitrov. Computer visualization of low frequency magnetic signals in systems for magnetotherapy with variable parameters. Proc. 8 Int. Conf. CEMA13. 17-19 Oct. 2013, Sofia, Bulgaria, p. 36-39.

[4] B. Kudrin, I. Soloviev. On interpolation methods of low frequency magnetic field in systems for magneto therapy. Proc. 9 Int. Conf. CEMA14. 16-18 Oct. 2014, Sofia, Bulgaria, p..