# THE RADIATION PROBLEM FROM A VERTICAL SHORT DIPOLE ANTENNA ABOVE FLAT AND LOSSY GROUND: NOVEL FORMULATION IN THE SPECTRAL DOMAIN WITH NUMERICAL SOLUTION AND CLOSED – FORM ANALYTICAL SOLUTION IN THE HIGH FREQUENCY REGIME

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## Abstract

In this paper we consider the problem of radiation from a vertical short (Hertzian) dipole above flat lossy ground, which represents the well-known in the literature 'Sommerfeld radiation problem'. The problem is formulated in a novel spectral domain approach, and by inverse three-dimensional Fourier transformation the expressions for the received electric and magnetic (EM) field in the physical space are derived as one-dimensional integrals over the radial component of wavevector, in cylindrical coordinates. This formulation appears to have inherent advantages over the classical formulation by Sommerfeld, performed in the spatial domain, since it avoids the use of the so – called Hertzian vector and its subsequent differentiation for the calculation of the received EM field. It also gives new insights regarding the propagation mechanism. Subsequent use of the Stationary Phase Method (SPM) in the high frequency regime yields closed-form analytical solutions for the received EM field vectors, which coincide with the corresponding reflected EM field originating from the image point. In this way, we conclude that the so-called in the literature 'space wave' (line of sight plus reflected EM field) represents the total solution of the Sommerfeld problem in the high frequency regime, in which case the surface wave can be ignored. Furthermore, numerical results in the high frequency regime are presented in this paper, in comparison with corresponding numerical results based on Norton's solution of the problem (space and surface waves). Finally, numerical results based on the numerical integral are also presented for comparison purposes. These results essentially provide a means of estimating the frequency limits of applicability of the SPM method for the problem in question. Subsequent suggestions on the preferred method (SPM vs Numerical) for calculating received signal level for various frequency ranges are made.

# **1. INTRODUCTION**

The so-called 'Sommerfeld radiation problem' is a well - known problem in the area of propagation of electromagnetic (EM) waves above flat lossy ground for obvious applications in the area of wireless telecommunications [1-5]. The classical Sommerfeld solution to this problem is provided in the physical space by using the so- called 'Hertz potentials' and it does not end - up with closed form analytical solutions. K. A. Norton [6] concentrated in subsequent years more in the engineering application of the above problem with obvious application to wireless telecommunications, and provided approximate solutions to the above problem, which are represented by rather long algebraic expressions for engineering use, in which the so - called 'attenuation coefficient' for the propagating surface wave plays an important role.

In this paper the authors take advantage of previous research work of them for the EM radiation problem in free space [7] by using the spectral domain approach. Furthermore, in Ref. [8] the authors provided the fundamental formulation for the problem considered here, that is the solution in spectral domain for the radiation from a dipole moment at a specific angular frequency ( $\omega$ ) in isotropic media with a flat infinite interface. At that paper, the authors end – up with integral representations for the received electric and magnetic fields above or below the interface [Line of Sight (LOS) plus reflected field - transmitted fields, respectively], where the integration takes place over the radial spectral coordinate  $k_{\rho}$ . Then, in the present paper the authors concentrate to the solution of the classical 'Sommerfeld radiation problem' described above, where the radiation of a vertical dipole moment at angular frequency w takes place above flat lossy ground [this is equivalent to the radiation of a vertical small (Hertzian) dipole above flat lossy ground]. The proposed spectral – domain approach and particularly the derived integral representations for the EM field is not just a more effective means of reaching the same results compared to the classical spatial domain Sommerfeld's method. It also helps deducing new inferences regarding the propagation mechanism, as explained in Section 3.

Next, by using the Stationary Phase Method, (SPM method, [9]-[11]) integration over the radial spectral coordinate  $k_{\rho}$  is performed and the high frequency solution to the problem ['space wave', which represents the interference of the Line – of – Sight (LOS) and the wave scattered from the ground] is derived in a novel, to our knowledge, closed - form analytic solution, as exhibited in Section 4, below. In addition, numerical results which show both the 'space wave' mentioned above, as well as the Norton's 'surface wave' [6] are presented in Section 5. Finally, numerical results based on the numerical integration of the spectral integral are also presented for comparison purposes, as well as for obtaining an indication of the frequency limits of the SPM method, which is an inherently 'high frequency approximation' technique ([9]).

# 2. PROBLEM GEOMETRY

The geometry of the problem is given in Fig. 1. Here a Hertzian (small) dipole with dipole moment p directed to positive x – axis, at altitude x<sub>0</sub> above the infinite, flat and lossy ground, radiates time – harmonic electromagnetic (EM) waves at angular frequency  $\omega=2\pi f$  [exp(-iwt) time dependence is assumed in this paper]. Here the relative complex permittivity of the ground is  $\varepsilon_r = \varepsilon'/\varepsilon_0 = -\varepsilon_r + ix$ , where x= $\sigma/\omega\varepsilon_0 = 18 \times 10^9 \sigma/f$ ,  $\sigma$  being the ground conductivity, f the frequency of radiation and  $\varepsilon_0 = 8.854 \times 10-12$  F/m is the absolute permittivity in vacuum or air. Then the wavenumbers of propagation of EM waves in air and lossy ground, respectively, are given by the following equations:

$$k_{01} = \omega / c_1 = \omega \sqrt{\varepsilon_1 \mu_1} = \omega \sqrt{\varepsilon_0 \mu_0} \tag{1}$$

$$k_{02} = \omega / c_2 = \omega \sqrt{\varepsilon_2 \mu_2} = k_{01} \sqrt{\varepsilon_r + ix}$$
 (2)

The Maxwell equations for the time – harmonic EM fields considered above are given by:

$$\begin{cases} \operatorname{rot} \underline{E} - i\omega\mu_0\mu_r \underline{H} = 0\\ \operatorname{rot} \underline{H} + i\omega\varepsilon_0\varepsilon_r \underline{H} = \underline{j} \end{cases}$$
(3)

where  $\underline{j}$  is current density (source of EM fields considered here).



#### 3. INTEGRAL FORMS FOR THE RECEIVED ELECTRIC AND MAGNETIC FIELDS IN THE SPECTRAL DOMAIN

Following [7]-[8], the EM field in physical space is derived from current density in spectral domain and Green's function, also in the spectral domain, through inverse three – dimensional (3D) Fourier transformation as following :

$$\underline{H} = -i \,\mathrm{F}^{-1} \left[ \tilde{\Psi} \cdot (\underline{k} \times \underline{\tilde{J}}) \right] \tag{4}$$

$$\underline{\underline{E}} = -\frac{i}{\omega\varepsilon_{r}\varepsilon_{0}} \mathbf{F}^{-1} \left\{ \widetilde{\Psi} \left[ \varepsilon_{r} \mu_{r} k_{0}^{2} \, \widetilde{\underline{J}} - \langle \underline{\underline{k}}, \underline{\underline{J}} \rangle \underline{\underline{k}} \right] \right\}$$
(5)

where the symbol < > denotes the inner product and  $F^{-1}$  is the inverse 3D Fourier Transform (FT) operator and

$$\tilde{\Psi} = \left(k_{01}^2 - k^2\right)^{-1} = \left(k_{01}^2 - k_{\rho}^2 - k_x^2\right)^{-1}$$
(6)

is the 3D Green's function in spectral domain and cylindrical coordinates.

Application of the problem specifics (i.e. current density vector in spectral domain has only x - component, the wavevector  $k=(k_{p},k_{\alpha}=0,k_{x})$  does not possess azimuthal  $\alpha$  component, see Fig. 1) and the corresponding boundary value problem ([8], [12]) yields the following integral representations for the space wave (LOS field plus reflected field) above the ground level (x > 0)

$$\underline{H}(\underline{r}) = \underline{H}^{LOS} - \frac{i\omega p e_{\alpha}}{8\pi} \int_{-\infty}^{\infty} \frac{\varepsilon_{r2}\kappa_1 - \varepsilon_{r1}\kappa_2}{\kappa_1(\varepsilon_{r2}\kappa_1 + \varepsilon_{r1}\kappa_2)} k_{\rho}^2 \cdot \mathbf{H}_0^{(1)}(k_{\rho}\rho) e^{i\kappa_1(x_0 + x)} dk_{\rho}$$
(7)

$$\underline{\underline{E}(\underline{r})} = \underline{\underline{E}}^{LOS}(\underline{r}) - \frac{ip}{8\pi\varepsilon_{r1}\varepsilon_{0}} \hat{e}_{\rho} \int_{-\infty}^{\infty} k_{\rho}^{2} \frac{\varepsilon_{r2}\kappa_{1} - \varepsilon_{r1}\kappa_{2}}{(\varepsilon_{r2}\kappa_{1} + \varepsilon_{r1}\kappa_{2})} e^{i\kappa_{1}(x+x_{0})} H_{0}^{(1)}(k_{\rho}\rho) dk_{\rho} + + \frac{ip}{8\pi\varepsilon_{r1}\varepsilon_{0}} \hat{e}_{x} \int_{-\infty}^{\infty} k_{\rho}^{3} \frac{\varepsilon_{r2}\kappa_{1} - \varepsilon_{r1}\kappa_{2}}{\kappa_{1}(\varepsilon_{r2}\kappa_{1} + \varepsilon_{r1}\kappa_{2})} e^{i\kappa_{1}(x+x_{0})} \cdot H_{0}^{(1)}(k_{\rho}\rho) dk_{\rho}$$
(8)

where

$$\kappa_1 = \sqrt{k_{01}^2 - k_{\rho}^2}$$
(9)

$$\kappa_2 = \sqrt{k_{02}^2 - k_{\rho}^2}$$
(10)

and  $H_0^{(1)}$  is the Hankel function of first kind and zero order.

The line-of-sight (LOS) EM field of the Hertzian dipole in the far field is given in spherical coordinates by [9,13]:

$$H_{\alpha}^{LOS}(r,\theta) = \frac{\omega^2 p}{4\pi} \sqrt{\varepsilon_0 \mu_0} \frac{\exp(ikr)}{r} \sin \theta = \frac{\omega k_{01} p}{4\pi} \frac{\exp(ikr)}{r} \sin \theta \quad (11)$$
$$\underline{E}^{LOS}(r,\theta) = \zeta H_{\alpha}^{LOS} \cos \theta \stackrel{\circ}{e_{\rho}} - \zeta H_{\alpha}^{LOS} \sin \theta \stackrel{\circ}{e_x} \quad (12)$$

Conversion to cylindrical coordinates is made by means of the following expressions (see Fig 1 above):

$$r \approx \rho + \frac{(x - x_0)^2}{2\rho}$$
 (13)

$$\theta = \pi - \tan^{-1}\left[\frac{\rho}{(x_0 - x)}\right],$$
 for  $x_0 > x$ 

$$\theta = \tan^{-1}\left[\frac{\rho}{(x-x_0)}\right], \qquad \text{for } x > x_0$$

The physical interpretation of eqns. (7) and (8) is that the scattered EM field at the observation point consists of a complex summation of the EM waves scattered from the different points of the flat and lossy ground, each one with its own local reflection coefficient (here the term 'complex summation' means that both the amplitude and phase of these individual scattered waves must be taken into account).

Similar expressions are derived for the transmitted fields below the ground interface (x < 0):

$$\underline{\mathbf{H}}^{\mathrm{T}}(\underline{r}) = -\frac{i\omega p}{4\pi} \hat{\mathbf{e}}_{\alpha} \int_{-\infty}^{\infty} k_{\rho}^{2} \frac{\varepsilon_{r2}}{\varepsilon_{r2}\kappa_{1} + \varepsilon_{r1}\kappa_{2}} e^{i(\kappa_{1}x_{0} - \kappa_{2}x)} \cdot \mathbf{H}_{0}^{(1)}(k_{\rho}\rho) dk_{\rho} \quad (15)$$

$$\underline{\mathbf{E}}^{\mathrm{T}}(\underline{\mathbf{r}}) = -\frac{ip}{4\pi\varepsilon_{0}} \int_{-\infty}^{\infty} (k_{\rho} e_{x} - \kappa_{2} e_{\rho}) \frac{k_{\rho}^{2}}{\varepsilon_{r2}\kappa_{1} + \varepsilon_{r1}\kappa_{2}} \cdot e^{i(\kappa_{1}x_{0} - \kappa_{2}x)} \mathbf{H}_{0}^{(1)}(k_{\rho}\rho) dk_{\rho} \quad (16)$$

# 4. ANALYTICAL CLOSED – FORM EXPRESSIONS FOR THE SCATTERED EM FIELDS OBTAINED THROUGH THE APPLICATION OF THE STATIONARY PHASE METHOD (SPM)

Following [8], [12], [14], [16] and [17] and by using the Stationary Phase Method (SPM) [9]), we finally end – up with the following closed - form expressions for the space wave (in the higher – half space, x>0), as given below:

$$\underline{\underline{E}}_{x>0} = \underline{\underline{E}}^{LOS} - \frac{ip}{8\pi\varepsilon_{o}\varepsilon_{r1}} \mathbf{I}_{1} \cdot \hat{e}_{\rho} - \frac{ip}{8\pi\varepsilon_{o}\varepsilon_{r1}} \mathbf{I}_{2} \cdot \hat{e}_{x}$$
(17)

$$\underline{\mathbf{H}}_{x>0} = \underline{\mathbf{H}}^{LOS} - \frac{i\omega p}{8\pi} \mathbf{I}_3 \cdot \hat{\boldsymbol{e}}_{\alpha}$$
(18)

where

$$I_{1} = \frac{i2}{k_{01}\rho^{1/2}} \frac{1}{\left(x + x_{0}\right)^{1/2}} \kappa_{1s}^{3/2} k_{\rho s}^{3/2} \cdot$$
(19)

$$\cdot \frac{\varepsilon_2 \kappa_{1s} - \varepsilon_1 \kappa_{2s}}{\varepsilon_2 \kappa_{1s} + \varepsilon_1 \kappa_{2s}} e^{i k_{\rho s} \rho} e^{i \kappa_{1s} (x + x_0)}$$

$$I_2 = \frac{i2}{k_{01}\rho^{1/2}} \frac{1}{\left(x + x_0\right)^{1/2}} \kappa_{1s}^{1/2} k_{\rho s}^{5/2} \cdot$$
(20)

$$\cdot \frac{\varepsilon_2 \kappa_{1s} - \varepsilon_1 \kappa_{2s}}{\varepsilon_2 \kappa_{1s} + \varepsilon_1 \kappa_{2s}} e^{i k_{\rho s} \rho} e^{i \kappa_{1s} (x + x_0)}$$

$$I_{3} = \frac{i2}{k_{01}\rho^{1/2}} \frac{1}{(x+x_{0})^{1/2}} \kappa_{1s}^{1/2} k_{\rho s}^{3/2} \cdot \frac{1}{(x+x_{0})^{1/2}} \kappa_{1s}^{1/2} k_{\rho s}^{3/2} \cdot \frac{1}{\varepsilon_{2}\kappa_{1s} - \varepsilon_{1}\kappa_{2s}} e^{ik_{\rho s}\rho} e^{i\kappa_{1s}(x+x_{0})}$$

$$(21)$$

and

$$k_{\rho s} = \frac{k_{01}\rho}{\left[\left(x + x_{0}\right)^{2} + \rho^{2}\right]^{\frac{1}{2}}} = k_{01}\frac{1}{\left[1 + \left(\frac{x + x_{0}}{\rho}\right)^{2}\right]^{\frac{1}{2}}} = k_{01}\cos\phi$$
(22)

is the (unique) stationary point [9, 12, 16, 17]. Moreover the following expressions hold:

$$\kappa_{1s} = \sqrt{k_{01}^2 - k_{\rho s}^2} = k_{01} \sin \phi$$
 (23)

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$$\kappa_{2s} = \sqrt{k_{02}^2 - k_{\rho s}^2}$$
(24)

Note that in the above expressions the angle  $\varphi$  is the well – known in the literature 'grazing angle' [13], as shown in Fig. 2 below.

Then our final closed-form analytical solution for the reflected fields can also be written in the compact form of the following expressions:

$$\underline{E}_{x>0}^{sc} = \frac{p}{4\pi\varepsilon_{o}\varepsilon_{r1}} \frac{1}{\rho^{1/2}} \frac{1}{(x+x_{0})^{1/2}} \frac{\kappa_{1s}^{1/2} k_{\rho s}^{1/2}}{k_{01}} \cdot \frac{\varepsilon_{2}\kappa_{1s} - \varepsilon_{1}\kappa_{2s}}{\varepsilon_{2}\kappa_{1s} + \varepsilon_{1}\kappa_{2s}} e^{i\kappa_{\rho s}\rho} e^{i\kappa_{1s}(x+x_{0})} \cdot (\kappa_{1s}\hat{e}_{\rho} + k_{\rho s}\hat{e}_{x}) = \\
= \frac{pk_{01}}{4\pi\varepsilon_{o}\varepsilon_{r1}} \frac{(\sin\phi)^{1/2}}{\rho^{1/2}(x+x_{0})^{1/2}} \cdot \frac{\varepsilon_{2}\kappa_{1s} - \varepsilon_{1}\kappa_{2s}}{\varepsilon_{2}\kappa_{1s} + \varepsilon_{1}\kappa_{2s}} e^{ik_{\rho s}\rho} e^{i\kappa_{1s}(x+x_{0})} \cdot (\kappa_{1s}\hat{e}_{\rho} + k_{\rho s}\hat{e}_{x}) = \\
= \frac{pk_{01}}{4\pi\varepsilon_{o}\varepsilon_{r1}} \frac{(\cos\phi)}{\epsilon_{1}} \cdot \frac{\varepsilon_{2}\kappa_{1s} - \varepsilon_{1}\kappa_{2s}}{\varepsilon_{2}\kappa_{1s} + \varepsilon_{1}\kappa_{2s}} e^{ik_{\rho s}\rho} e^{i\kappa_{1s}(x+x_{0})} \cdot (\kappa_{1s}\hat{e}_{\rho} + k_{\rho s}\hat{e}_{x}) = \\
= \frac{pk_{01}\cos\phi}{4\pi\varepsilon_{o}\varepsilon_{r1}(A^{\prime}A)} \cdot \frac{\varepsilon_{2}\kappa_{1s} - \varepsilon_{1}\kappa_{2s}}{\varepsilon_{2}\kappa_{1s} + \varepsilon_{1}\kappa_{2s}} e^{i\kappa_{\rho s}\rho} e^{i\kappa_{1s}(x+x_{0})} \cdot (\kappa_{1s}\hat{e}_{\rho} + k_{\rho s}\hat{e}_{x}) \tag{25}$$

$$\underline{H}_{x>0}^{sc} = \frac{\omega p}{4\pi} \frac{1}{\rho^{\frac{1}{2}}} \frac{1}{(x+x_0)^{\frac{1}{2}}} \frac{\kappa_{1s}^{\frac{7}{2}} k_{\rho s}^{\frac{7}{2}}}{k_{01}} \cdot \frac{\varepsilon_{2} \kappa_{1s} - \varepsilon_{1} \kappa_{2s}}{\varepsilon_{2} \kappa_{1s} + \varepsilon_{1} \kappa_{2s}} e^{ik_{\rho s}\rho} e^{i\kappa_{1s}(x+x_0)} \hat{e}_{\alpha} = 
= \frac{\omega k_{01} p}{4\pi} \frac{(\sin \phi)^{\frac{1}{2}} (\cos \phi)^{\frac{3}{2}}}{\rho^{\frac{1}{2}} (x+x_0)^{\frac{1}{2}}} \frac{\varepsilon_{2} \kappa_{1s} - \varepsilon_{1} \kappa_{2s}}{\varepsilon_{2} \kappa_{1s} + \varepsilon_{1} \kappa_{2s}} e^{ik_{\rho s}\rho} e^{i\kappa_{1s}(x+x_0)} \hat{e}_{\alpha} = 
= \frac{\omega k_{01} p \cdot \cos \phi}{4\pi (A'A)} \frac{\varepsilon_{2} \kappa_{1s} - \varepsilon_{1} \kappa_{2s}}{\varepsilon_{2} \kappa_{1s} + \varepsilon_{1} \kappa_{2s}} e^{i\kappa_{\rho s}\rho} e^{i\kappa_{1s}(x+x_0)} \hat{e}_{\alpha} \qquad (26)$$

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where (A'A) is the distance between the image point and the observation point (Fig. 2).



Fig. 2. Geometry of the radiation where the image A' of the radiating Hertzian dipole is also shown.

#### 5. NUMERICAL RESULTS – COMPARISON OF SPM WITH (i) NORTON'S APPROXIMATE SOLUTION (ii) NUMERICAL INTEGRATION TECHNIQUES

In this Section two types of indicative numerical results are provided for comparison purposes. Firstly, the magnitudes for the following fields:

- Scattered Electric field, eq. (25) above,

- Line of Sight (LOS) field,
- 'Space Wave', i.e. the complex summation of the previously mentioned fields,

are presented for various distance points (p) between the transmitting dipole and the receiving point and compared with Norton's results [6, 13]. Two demonstration frequency sets were selected for the radiating dipole, namely f=80 MHz (Fig. 3) and f=30 MHz (Fig. 4).



horizontal distance (p) for frequency f=80 MHz. Here the various components of received electric field are shown as following: Line – of – Sight (LOS) field (circle), field scattered from ground (asterisk), 'space wave' (square) and 'surface wave' (diamond).



Fig. 4. Same as Figure 3, except that here frequency f = 30 MHz.

The rest of the problem parameters are the following: height of transmitting dipole  $x_0=60m$ , height of observation point (receiver position) x=15m, current of the radiating Hertzian dipole I=1A, length of the Hertzian dipole 2h=0.1m (much smaller than the wavelength  $\lambda$ =c/f in both cases), relative dielectric constant of ground  $\varepsilon_r$ =20 and ground conductivity  $\sigma$ =0.01 S/m. Finally, note that the relation between current I and dipole moment p is given by: I(2h) = iwp, where w=2\pi f and i is the unit imaginary number. Comparison of numerical results derived from our formulation, and the Norton's results [6,13] shows very good agreement, as it can be seen in Figs. 3 and 4. The surface wave represented in Figs. 3 and 4 is the so-called 'Norton surface wave' [6, 13]. Note that at the higher frequency of 80 MHz (Fig. 3) the surface wave, according to Norton's formulation [6, 13] is rather negligible, as compared to the 'space wave', while it becomes rather more important at the lower frequency of 30 MHz (Fig. 4). Our proposed SPM method of Sections 4 and 5 (which is inherently a 'high frequency method') ignores this surface wave contribution in the high frequency regime.

The second type of numerical data, shown in this paper, compare the results obtained using the analytical, closed – form formulas of the SPM method, described in section 4 above, i.e. eqns. (17) through (26), with those taken by numerically evaluating the corresponding integral expressions of eqns. (7) through (14), given in section 3. Figure 5, below, provides indicative results for the scattered electric field of, eq. (8) (here the LOS field is not shown).

In order to numerically estimate the integral of eq.(8), the adaptive Simpson's algorithm was used ([18]). The error tolerance was set to 10<sup>-6</sup>. Moreover to mitigate the 'small scale' oscillating behaviour of the resulting graphs, which is an outcome of the fact that the integrated expression consists of complex numbers of fast – varying phases, a typical three (3) sample smoothing (averaging) was applied, where appropriate.

Careful examination of eq. (8) reveals the fact that there exists a singular point at  $\kappa_1=0$ , that is at  $k_p=k_{01}$ (see eq.(9) above) and hence it must be excluded by a sufficient range around  $k_{01}$ . Our preliminary tests, showed that this range needs to be no less than10<sup>-4</sup> times the limiting value of  $k_{01}$  (according to eq. (22),  $k_{ps} < k_{01}$ ). However, this indicates that a segment around the stationary point will be excluded from the calculation (see respective labels in the diagrams of Fig. 5, below). The inferences for this are described below.

Examining the curves of Fig. 5 below, it is evident that for frequencies of about 100 KHz and above the results taken under the numerical integration approach underestimate the received signal level compared with the SPM method. The reason for this behaviour is related with the properties of the SPM method, which indicate that for large arguments (in our case frequencies) the integral expression can be asymptotically approximated by taking into consideration just the contributions of the areas around the stationary points and their neighbourhoods [9]. However, as mentioned above, when numerically evaluating the integral expression of eq. (8) a sufficiently large range around  $k_{01}$  had to be excluded for the algorithm to converge. In most cases this range overlaps with the stationary point, essentially meaning that a significant contributing part is missed.

On the contrary, according to the last curve of Fig. 5 (f = 10 KHz), it is now the SPM method that seems to underestimate the EM field values (we also reach the same findings for f < 30 KHz). Indeed, for such low frequencies, the large argument approximation of the SPM method cannot be invoked, in other words, eqns. (19) through (26) do not hold. It is still necessary to exclude a range around  $k_p=k_{01}$ , for the numerical integration algorithm to converge, however this time this range is not a major contributor to the overall outcome.

In conclusion, our research group suggests the following 'safe' recommendations (see Fig. 5):

- For frequencies in the MF (300 KHz 3 MHz, [13]) frequency range and higher, the SPM method significantly provides more accurate results for the received signal level and hence should be the selection of choice for prediction purposes.
- For the VLF frequency range (<30 KHz, [13]) the SPM fails and the estimation ought to be based on numerical integration techniques.
- Finally, in the LF range (30 300 KHz [13]), the results, given by the two methods, seem comparable and our research group proposes that a closer examination and potential fine – tunning of the numerical integration algorithm is necessary, before reaching 'safe' inferences (see Section 6 below)

Further clarifications about the above presented numerical results and the related inferences that can be made based on them will be provided during the Conference. CEMA'14 conference, Sofia



Fig. 5. Value of the scattered electric field as a function of horizontal distance p between transmitter and receiver, calculated by (i) the SPM method and (ii) numerical integration techniques.

#### 6. CONCLUSIONS - FUTURE RESEARCH

In this paper we formulated the radiation problem from a vertical short (Hertzian) dipole above flat and lossy ground in the spectral domain, which resulted in an easy to manipulate integral expression for the received EM field above or below the ground. As also explained above, this formulation appears to have inherent advantages over the classical formulation by Sommerfeld [5], since it avoids the use of the so - called Hertz potential and its subsequent differentiation for the calculation of the received EM field. Subsequently, by applying the Stationary Phase Method (SPM) in the high frequency regime, the classical solution for the 'space wave' was rederived in a new fashion, thus showing that this is the dominant solution in this high frequency regime. Moreover, we explained why the SPM method even appears to provide more accurate results than many common numerical integration techniques for

most frequencies of interest in the area of wireless telecommunications and hence can be the basis for an efficient simulation tool for radio signal propagation.

Corresponding research in the near future by our research group will concentrate to the calculation of the received EM field below the ground at the high frequency regime (by using again the SPM method). Furthermore, we intend to calculate the received EM field, above or below the ground, for any frequency of the radiating dipole, in an exact and analytical manner [19]. In this context, the behavior of surface waves will become evident through the use of the residue theorem, when applied to eqns. (7) and (8) above, in a way similar to Ref. [5].

Moreover, we also intend to investigate the formulation of the same radiation problem in spectral domain, but now in the case of a horizontal radiating Hertzian dipole above flat and lossy ground. In ad-

dition, further investigations will be performed in the case of rough (and not flat) ground and in the case of curvature of the earth's surface for large distance communication applications. Finally, in the near future our research group will focus on the design of a software product for accurate prediction of pass loss in different types of environment, like urban, suburban and rural environments. The above software tool will be based on the exact electromagnetic (EM) method proposed in this paper, and therefore it is expected that it will exhibit important advantages over previously developed corresponding software tools. Some of these advantages might include accuracy, speed, efficiency and low complexity, since the various calculations will be based on closed form analytical expressions, instead of resource starving and time consuming numerical methods. We also intend to fine tune the numerical method presented in this paper (e.g. numerical experiment with convergence tolerances), as well as to test alternative algorithms and techniques (e.g. the 'adaptive Lobatto' algorithm will be examined) and to use the most appropriate as the back up method in situations where the SPM is not sufficient (e.g. at low frequency regime). In this framework, comparisons with existing commercial software tools will also be performed [21].

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