ON INTERPOLATION METHODS OF LOW FREQUENCY MAGNETIC FIELD IN SYSTEMS FOR MAGNETOTHERAPY

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Abstract

In this work we apply methods of 3-dimensional linear interpolation, one-dimensional quadratic Lagrange interpolation and cubic spline to interpolate electromagnetic induction values in 3D. For a given 3-dimensional lattice (base points) values are obtained by calculation in accordance with a mathematical model. Then for a new set of points we obtain electromagnetic induction values both interpolation and calculation. Numerical results show that interpolation algorithms reduce run time considerably without loss of accuracy.

1. INTRODUCTION

At the moment there are many types of magnetotherapy devices that are successfully used in medicine. In most cases, in practice the required space configuration of the magnetic field is created by means of one or more air coils (without core) which are appropriately arranged in the space. Because of that there is a linear relationship between the magnetic flux density of the excited magnetic field and current in the coils. It is assumed that the environment in which the space-time configuration of the magnetic field is seen is linear. The result is the superposition of the fields of the individual coils that forms a more complex time-spaced magnetic field. We suppose that the environment around the coils is homogeneous and the relative magnetic permeability is constant.

In this work we apply numerical algorithms to interpolate low frequency magnetic field obtained in a magnetotherapy device that was designed and constructed in Sofia Technical University and used in clinical practice.

The solution of the problem of the modelling and visualization in 3D of electromagnetic field generated by this apparatus is given in articles [1,3,4]. The obtained data allows a physician to study the graphical representation of the magnetic induction field. By changing the position of coils, distance between them and the current intensity the physician can select an appropriate regime to achieve the best results for minimal treatment time.

For this model the algorithm of calculation of electromagnetic induction in nodes of a base 3-dimensional lattice was designed and implemented. The obtained results are saved in an array having size [nx3], where n – the number of nodes. Any element of the array contains coordinate values of electromagnetic induction vector. When visualizing results, the value of the module defines the color selection – the more value the more saturation of the color.

In this model the superposition principle is used and the magnetic induction in a point is the sum of the induction values generated by all the coils, and in every coil – by all its contours. Hence this algorithm is time-consuming. In this connection in [5] a method of 3-dimensional linear interpolation was implemented. Base values were obtained by calculations described and implemented in [1,3,4]. Such an approach resulted in performance increasing without loss of accuracy.

Here we use values obtained by calculations as base ones. These values are in nodes of 3-dimensional lattice and form interpolation nodes. Then we present the results of application of 3 interpolation methods using the nodes: a) 3-dimensional linear interpolation, b) one-dimensional quadratic Lagrange interpolation and c) coordinate-wise cubic spline. Numerical experiments show that any of methods reduces the run-time and does not lead to a loss of accuracy.

2. MATHEMATICAL MODELS

2.1. 3-dimensional linear interpolation (3D I)

Select a cube of the base lattice. All its vertices contain values of electromagnetic induction obtained by calculation. We select the center of the cube as interpolation point. This scheme is shown on Fig. 1. The interpolation nodes are 8 nearest nodes of the base lattice: $\{(x_i, y_j, z_k)\}$, i,j,k={0,1}.

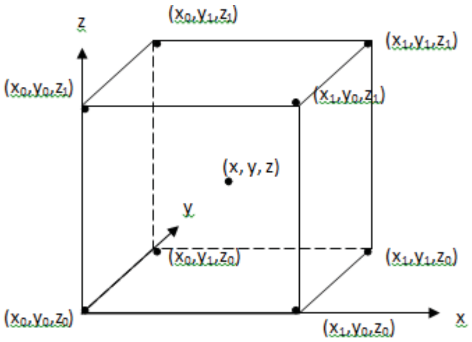


Fig. 1. Base nodes and the interpolation point for 3-dimensional linear interpolation

Consider Lagrange interpolation polynomial

$$l_{nmp}(x,y,z) = \prod_{i=0,i\neq n}^{1}\prod_{j=0,j\neq m}^{1}\prod_{k=0,k\neq p}^{1}\frac{(x-x_i)(y-y_j)(z-z_k)}{(x_n-x_i)(y_m-y_j)(z_p-z_k)}. \qquad (1)$$

It follows that

$$l_{nmp}(x_i, y_j, z_k) = 0, i \neq n \vee j \neq m \vee k \neq p$$

$$l_{nmp}(x_n, y_m, z_p) = 1.$$

The interpolation polynomial has the form

$$L(x,y,z) = \sum_{n=0}^{1}\sum_{m=0}^{1}\sum_{p=0}^{1} f_{nmp}\cdot l_{nmp}(x,y,z), \qquad (2)$$

where f_{nmp} – the vector of values obtained by the calculation in the point (x_n, y_m, z_p).

Hence we use linear interpolation that takes into account contribution of all the 8 interpolation nodes. It should be noted that in (2) we use such a form for brevity, because this formula is applied for every coordinate of f_{nmp} by turns.

2.2. One-dimensional quadratic Lagrange interpolation (1D I)

The method of one-dimensional quadratic Lagrange interpolation is applied – the interpolation point and the nodes are on the same line.

In this case the interpolation point is also the center of the cube. We use 3 nodes of the base lattice so that the nodes and the interpolation point belong to the same diagonal.

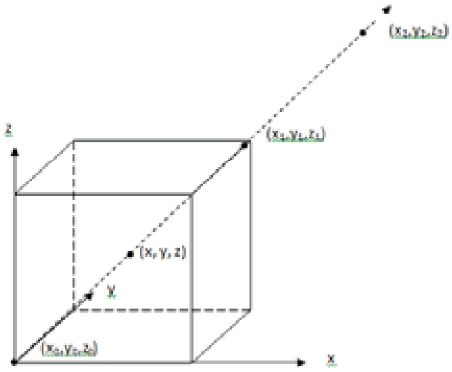


Fig. 2. Base nodes and interpolation point for one-dimensional Lagrange interpolation

The base polynomial has the form

$$l_n(x,y,z) = \prod_{n=0,n\neq i}^{2}\frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{\sqrt{(x_n-x_i)^2+(y_n-y_i)^2+(z_n-z_i)^2}} \qquad (3)$$

and satisfies the conditions

$$l_n(x_i, y_i, z_i) = 0, i \neq n$$

$$l_i(x_i, y_i, z_i) = 1.$$

The interpolation polynomial is represented as

$$L(x,y,z) = \sum_{n=0}^{2} f_n \cdot l_n(x,y,z), \qquad (4)$$

where где f_n is the value in the base node (x_n, y_n, z_n).

As in the previous case (4) means that the interpolation is performed by coordinate-wise.

2.3. Cubic spline

Let a function $f(x)$ be defined on a segment $[a,b]$. Consider a partition of the segment $[x_i, x_{i+1}]$, where $x_0 = a, x_n = b, i = 0,\ldots n-1$. A function $S(x)$ is said to be cubic spline for $f(x)$ if it satisfies the conditions:

1. on every element of the partition $S(x)$ is the polynomial with power not greater than 3;
2. the first and the second derivatives of $S(x)$ are continuous on $[a,b]$;
3. $f(x_i) = S(x_i), i = 0,..n$;
4. $S''(a) = S''(b) = 0$.

As it is known [2], the function $S(x)$ is defined by the conditions uniquely.

The polynomial $S(x)$ on the segment $[a,b]$ is constructed by the following way:on every element of the partition $[x_i, x_{i+1}]$ we write interpolation polynomial in the form

$$S_i(x) = a_i + b_i(x - x_i) + \frac{c_i}{2}(x - x_i)^2 + \frac{d_i}{6}(x - x_i)^3.$$

By writing the above conditions we obtain a system to find the coefficients a_i, b_i, c_i, d_i.

The coefficients are obtained by sweep method applied to the matrix of the system.

One can obtain a complexity estimation for the described algorithm [2]. Let *n* be the common number of points and *k* be the number of point where we know values of the function. Time complexity of sweep method for 3-diagonal matrix is $O(n)$. Time complexity of the interpolation in every from *n-k* points is $O(k)$. So time complexity of the algorithm is $O(n)+(n-k)\ O(k)$. If *k* depends on *n* linearly, then time complexity is $O(n^2)$. If k is a constant we obtain $O(k)$.

The method of cubic spline is used coordinatewise,i.e coordinates of magnetic induction vectors. Supposing that parameters of parallelepiped bounding the area where electromagnetic induction is calculated are *l* (length), *m* (width), and *h* (height), the asymptotic estimation for time complexity is $O(lmh)$.

3. RESULTS OF EXPERIMENTS FOR 3-DIMENSIONAL LINEAR AND ONE-DIMENSIONAL QUADRATIC INTERPOLATION

We analyzed run-time for programs implementing the algorithms by the following way. For a given base lattice electromagnetic induction vectors are calculated in the nodes. The interpolation is performed in the centers of cubes of the lattice. The number of nodes in the base lattice (n) is comparably with the number of interpolation points (n/8), that allows us to compare run-times of calculation and interpolation objectively.

To estimate miscalculation we find norms (module maxima) of two 3-dimensional arrays: obtained by calculation (B) and by linear interpolation (L) and square one (S). The interpolation coefficients are defined as L/B and S/B. The table 1 shows that miscalculation is not greater than 8 % for linear interpolation and not greater than 12% for quadratic one. As 3-dimensional linear interpolation (3d I) uses 8 nodes, it has the greater accuracy than quadratic one. In experiments we considered the area 600x500x232 mm. In the table the number of base nodes is given depending on the step by every dimension.

Table 1. Comparing run-times of algorithms

Lattice size		Run time(sec)			Interpolation coefficient	
n	step(mm)	Base algorithm	3d I	1d I	3d I (L/B)	1d I n (S/B)
70000	10	6	1	<1	0.92	1.03
200000	7	17	2	1	0.93	1.10
550000	5	49	8	5	0.96	1.10
2550000	3	228	38	25	0.97	1.12

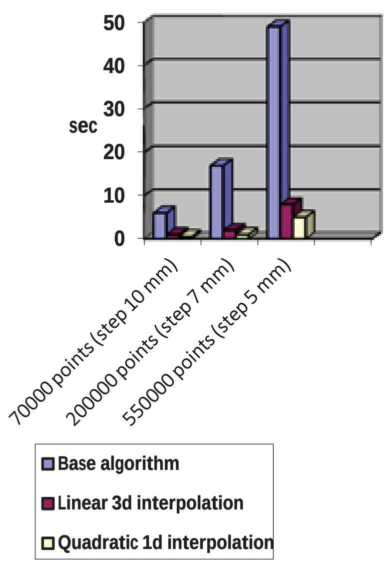


Fig. 3. Diagram of the algorithms run times

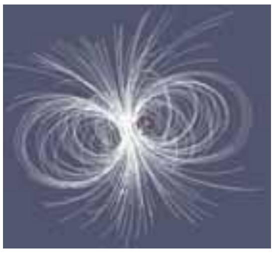
4. RESULTS OF INTERPOLATION BY CUBIC SPLINE

In experiments we used the data obtained by calculation in the points of integer lattice with size 100x100x100 (module1). Then we constructed sets of initial data for interpolation as all the points of the base lattice with coordinates: a) by module 2; b) by module 4; c) by module 8 and d) by module 16. Interpolation by cubic spline method was performed in the inter lattice points by module 1. The results were compared with given data. In the table run times (in sec) of the methods are given.

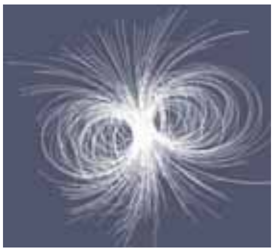
Config.	a	b	c	d
Run time	6.912	4.896	4.541	4.356

The following figure shows the initial magnetic field (obtained by calculation) and results of visualization for configurations a), b), c), d). The most reliable results were obtained for configurations a) and b), i.e. for interpolation nodes being on the distance not greater than 4 in the integer lattice.

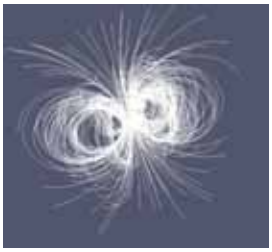
Initial data



a



b



c

d

5. CONCLUSION

The interpolation of electromagnetic field in magnetotherapy devices is very important because it reduces runtime considerably. The results of experiments show that all the described interpolation methods give reliable results.

Acknowledgements

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