# FSO AVAILABILITY DEPENDING ON THE METEOROLOGICAL CONDITIONS

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***Abstract***

*This paper studies the availability of FSO systems. For the purpose meteorological data from different regions is used to obtain statistical model of the atmospheric visibility $S_M$. Statistical method, that utilizes the cumulative distribution function (CDF) of $S_M$ and the mechanical vibrations of the transmitting antenna (Δρ) for calculating FSO availability, is proposed. Based on the derived analytical expressions numerical results are obtained and FSO availability is analyzed.*

## 1. INTRODUCTION

The commercialization of FSO systems in the past years imposes the need for improving their channel capacity and reliable work (availability). This poses the question for selecting the optimal parameters of the free space laser communication system [1].

One of the main factors for unreliable work of FSO is the atmospheric visibility [3]. Unbearable drawbacks in $S_M$ can cause system outage.

Another cause for the decrease of FSO reliable work is the random mechanical vibrations of the transmitting aperture [2], which result in linear shifts of the laser beam axis from its original direction. These shifts significantly decrease the quality of the transmitted data and FSO availability.

In this paper the FSO availability under different distributions of $S_M$ is studied. The availability of FSO systems working with fixed divergence angle, $\theta_t$, and working with variable (optimal) beam divergence angle, $\theta_{t,opt}$, (FSO employing a device for controlling $\theta_t$) is made. Also a graphical representation of the outage probability, due to unbearable aperture misalignment, is shown.

## 2. THEORY

### 2.1. Statistical model of the atmospheric visibility

A real yearly measurements of the atmospheric visibility [3] have been used for creating this generalized version of the statistical model of atmospheric visibility. The steps for generalization of the measured data are:

- Calculating the arithmetic mean value, $\bar{x}$, of each $P(S_M \geq S_{M,min})$
- Calculate the standard deviation σ for the given statistical data
- For each $S_M \geq S_{M,min}$ column we create an interval $[\bar{x} - \sigma;\ \bar{x} + \sigma]$
- Plot the results as a graphic and approximate the resulting curves using the simple trend line tool of Excel software

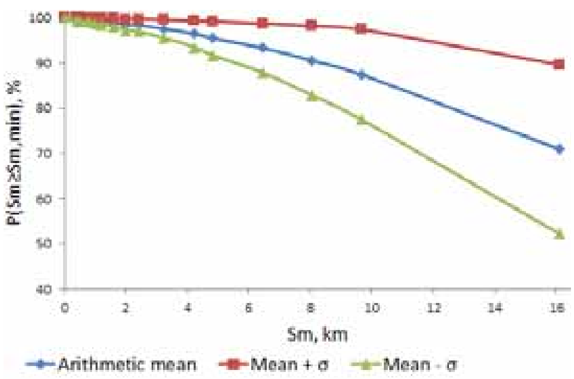


Fig. 1. Generalized model of the atmospheric visibility

The trend lines that approximate the three models of $P(S_M \geq S_{M,min})$ are:

- worst case "Mean - σ":

$$P(S_M \geq S_{M,min}) = [illegible]\,x^{[illegible]} - [illegible]\,x^{[illegible]} - [illegible]\,x + [illegible] \quad (1)$$

- arithmetic mean:

$$P(S_M \geq S_{M,min}) = [illegible]\,x^{[illegible]} - [illegible]\,x^{[illegible]} - [illegible]\,x^{[illegible]} - [illegible]\,x + [illegible] \quad (2)$$

- best case "Mean + σ":

$$P(S_M \geq S_{M,min}) = -\square x^{\square} + \square x + \\ + \square \tag{3}$$

For complete statistical model, the cumulative distribution function (CDF) and the probability density function (pdf) of $S_M$ are needed. By definition CDF is equal to the probability $P(S_M \leq S_{M,min})$, hence for each of the models in Fig.1:

$$CDF = P(S_M \leq S_{M,min}) = \\ = 1 - P(S_M \geq S_{M,min}) \tag{4}$$

Also

$$P(S_M \leq S_{M,min}) = \int_0^{S_{M,min}} f(S_M)\,dS_M \tag{5}$$

Hence the probability density function $f(S_M)$ for each of the models (Arithmetic mean, Mean + σ and Mean – σ) is equal to

$$f(S_M) = \frac{dP(S_M \leq S_{M,min})}{dS_M} \tag{6}$$

As defined in probability theory the pdf is the probability for the random variable X falling within the infinitesimal interval [x,x+dx], and it is a derivative of CDF.

Having all the functions $P(S_M \geq S_{M,min})$, $CDF(S_M)$ and $f(S_M)$ defined, the statistical model for the atmospheric visibility is complete.

## 2.2. Statistical model of the mechanical vibrations of the transmitter

In order to define a statistical model for FSO availability, it is also needed to have the model of the mechanical vibrations of the transmitting aperture. Consider the case illustrated on Fig. 2

When the transmitting aperture (TA) is randomly shifted in any direction with a random angle γ, the receiving aperture (RA), with radius $R_r$ is displaced from the axis (z) of the optical beam. This displacement Δρ is also random, because, when z=Z:

$$\Delta\rho = \gamma Z \tag{7}$$

Where γ represents the angular misalignments defined on Fig. 2.

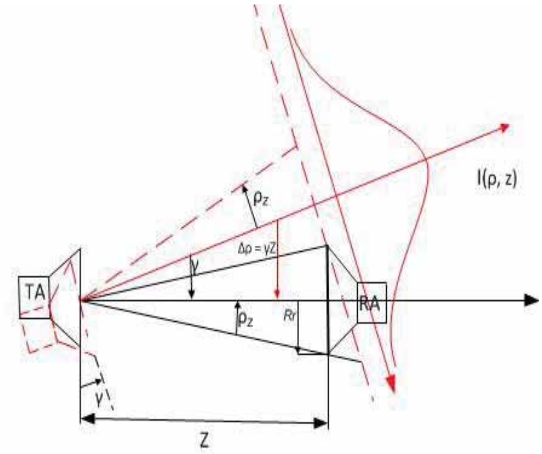


Fig. 2. Misalignment between transmitter and receiver

Having no statistical data in the literature, it is practical to assume a Gaussian distribution of the angular misalignments of the transmitting aperture [2], which considering (7) makes the linear displacements, Δρ, of RA in the plane z=Z, also normally distributed:

$$f(\Delta\rho) = \frac{1}{\sqrt{2\pi}\sigma_{\Delta\rho}} \exp\left(-\frac{\Delta\rho^2}{2\sigma^2_{\Delta\rho}}\right) \tag{8}$$

In accordance to (7), it can be concluded that

$$\sigma^2_{\Delta\rho} = Z^2\sigma^2_{\gamma} \tag{9}$$

The cumulative density function is:

$$CDF = F(\rho_{max}) = \int_{-\infty}^{\rho_{max}} f(\Delta\rho)\,d\Delta\rho = \\ = \frac{1}{2}\left[1 + erf\left(1 + erf\frac{\Delta\rho}{\sqrt{2}\sigma_{\Delta\rho}}\right)\right] \tag{10}$$

## 2.3. Mathematical model of FSO system's availability

To have an FSO system working reliably, it is required that the power of the optical radiation, at the receiver's site (plane z=Z), is greater than or equal to the minimal acceptable power. So it is required that $I(\rho, z) \geq I_{min}$ [1].

When FSO is set according to particular channel parameters ($S_M$), that is beam divergence angle ($\theta_t$), beam radius ($\rho_z$) and maximum acceptable misalignments ($\Delta\rho = \rho_{max}$) are calculated according to $S_{M,min}$, the system will be in outage if:

1) $S_M$ falls below critical value $S_{M,min}$, for which system parameters are calculated,

or

2) Linear misalignments Δρ exceed the maximum acceptable value $\rho_{max}$, defined by $S_{M,min}$.

It is considered that $S_M$ and Δρ are two independent random variables. Therefore $f$ ($S_M$) and $f$ (Δρ) are also independent.

From condition 1) follows that the probability for reliable work is $P(S_M \geq S_{M,min})$, which was already defined in equations (1) through (3).

From condition 2) FSO will work reliably if mechanical vibrations are less than $\rho_{max}$ in both directions X > 0 and X < 0

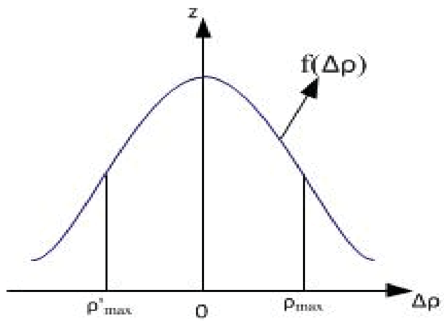


Fig. 3. Interval of maximum acceptable mechanical vibrations Δρ

This means reliable work of FSO, depending on Δρ is equal to:

$$\int_{\rho'^{\square}_{\square\square}}^{\rho_{\square\square}} f\square\Delta\rho\square d\square\Delta\rho\square = \square\int_{\square}^{\rho_{\square\square}} f\square\Delta\rho\square d\square\Delta\rho\square = \tag{11}$$

$$= \square\square F\square\rho_{\square\square}\square - F\square\square\square\square$$

Hence availability is equal to the joint distribution of the two random variables (Δρ and $S_M$):

$$Availability =$$
$$P\square S_{\square} \geq S_{\square\square\square}\square\rho^{\square}_{\square\square} \leq \Delta\rho \leq \rho_{\square\square}\square \tag{12}$$

Which considering the assumption that $f$ ($S_M$) and $f$ (Δρ) are independent and (11) is equal to

$$P\square S_{\square} \geq S_{\square\square\square}\square\rho^{\square}_{\square\square} \leq \Delta\rho \leq \rho_{\square\square}\square =$$

$$= \int_{S_{\square\square\square}}^{\infty}\int_{\rho'^{\square}_{\square\square}}^{\rho_{\square\square}} f\square S_M\square\Delta\rho\square dS_{\square}\, d\Delta\rho = \tag{13}$$

$$= \square\int_{S_{\square\square\square}}^{\infty} f\square S_{\square}\square dS_{\square}\int_{\square}^{\rho_{\square\square}} f\square\Delta\rho\square d\Delta\rho$$

Needed system parameters are calculated as proposed in [1]:

$$\rho_{\square\square\square\square} = Z\theta_{\square\square\square\square} \tag{14}$$

Where

$$\theta_{\square\square\square\square} = \frac{\square}{Z}\sqrt{\frac{\square\tau_{\square}\tau_{\square}\tau_{\square}R_{\square}\Phi_{\square}}{\square\Phi_{\square\square}\square_{\square\square=\square\square\square\square}}}\square e = \square\square\square\square\square \tag{15}$$

And

$$\rho_{\square\square} = \frac{\square}{\sqrt{\square}}\rho_{\square}\sqrt{\square\square\frac{\square\tau_{\square}\tau_{\square}\Phi_{\square}}{\pi\rho_z^{\square}\square\square - e^{-\square}\square I_{\square\square}}} \tag{16}$$

Hence, considering (11) through (13), FSO availability can be easily calculated with the following analytical expression:

$$Availability =$$
$$= \square\square F\square\rho_{\square\square}\square - F\square\square\square P\square S_{\square} \geq S_{\square\square\square}\square \tag{17}$$

## 3. SIMULATION RESULTS

To make the calculations for FSO availability simulations, the following system parameters were used:

Channel capacity, $C_l$ = 100 Mbps; quantum efficiency of the photodetector material, $\eta$ ($\lambda_0$) = 0,7; SNR = 11,2 (corresponds to BER = $10^{-8}$);central wavelength $\lambda_0$ = 0,85 and 1,55 μm; T = 300 K; A = 5; value of the resistor in the feedback of the preamplifier, $R_{Fb}$ = 1 kΩ; $\tau_r = \tau_t$ = 0,85; $R_r$ = 5,5 cm; transmission wavelength of the interface filter before the photodetector $\Delta_{\lambda,F}$ = 10 nm; background radiation, $L_{\lambda,B} = 10^{-2}$ (corresponds to bright day); angular width of the receiving antenna $\theta_r$ = 5 mrad. System parameters are derived using [1] and formulas (14) through (16). Availability is calculated from (17).

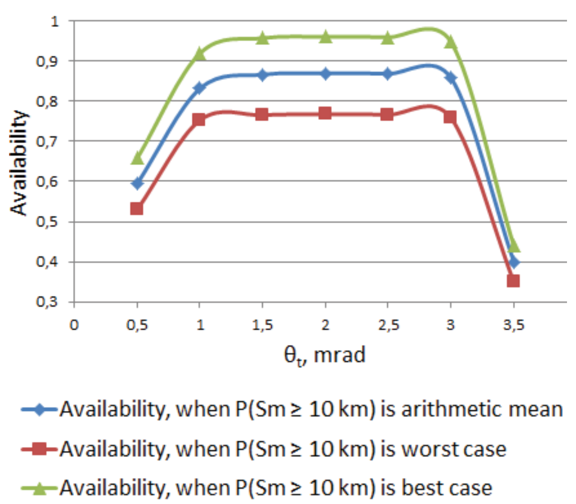


Fig. 4. FSO Availability depending on $\theta_t$, z=2km, $\Phi_L$=10 mW

Figures 4 through 6 represent the change of systems reliable work (FSO availability) when different parameters of FSO are varying.

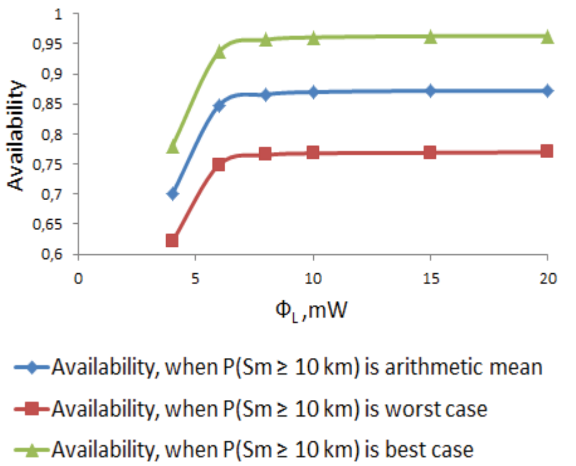


Fig. 5. FSO Availability depending on $\Phi_L$, z=2km, $\theta_t$ = 2 mrad

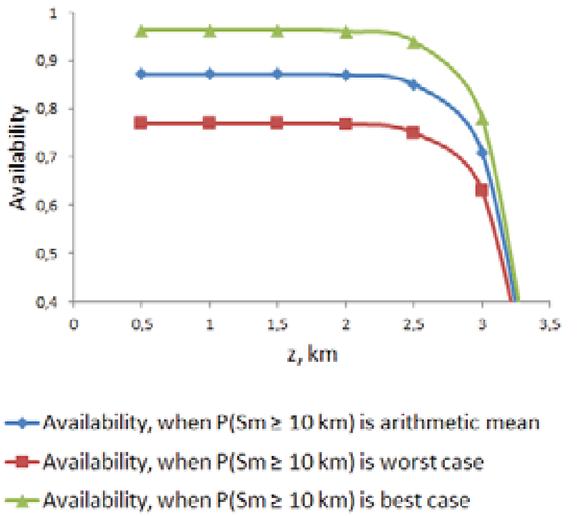


Fig. 6. FSO Availability depending on z, $\theta_t$ = 2 mrad, $\Phi_L$=10 mW

Simulations are performed using all three models for P($S_M$≥10 km) (Arithmetic mean, Best case-"Mean + σ" and Worst case – "Mean - σ"). For calculation of P($0 \le \Delta\rho \le \rho_{max}$) $\sigma_\gamma$ = 0.5mrad, which corresponds to $\sigma_{\Delta\rho}$ = 1 m is used, see (9).

It is observed that in extreme cases, when high or very low values of $\theta_t$ (above 3 mrad or below 1 mrad) or low power lasers (bellow 10 mW) are used there is a significant drop in availability. That is because when using low beam divergence or low power lasers, the acceptable mechanical vibrations of the transmitter ($\rho_{max}$) are lower and can be often exceeded.

Following figures depict the comparison of two FSO systems. One is using fixed value of $\theta_t$; the other one always employs optimal value of $\theta_t$ =*var*= $\theta_{t,\ opt}$, as derived from (15), thus allowing for absolute maximal values of Δρ. Simulations are done as follows: $\sigma_{\Delta\rho}$ =1 m, arithmetic mean for P($S_M$ ≥ $S_{M,min}$) is used. FSOs with $\lambda_0$ = 0,85 and 1,55 μm are compared.

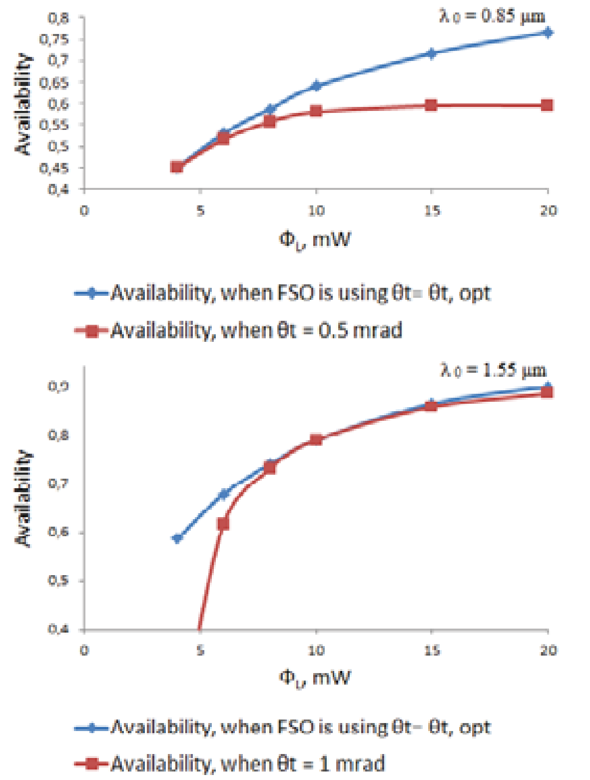


Fig. 7. Comparison between Availability of two FSO systems ($\Phi_L$ = var, z = 3km)

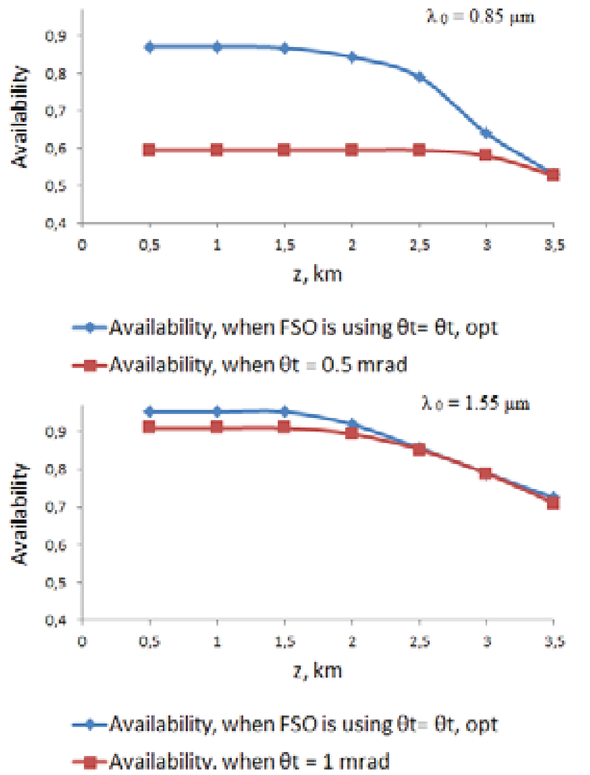


Fig. 8. Comparison between Availability of two FSO systems (z = var, $\Phi_L$ = 10 mW)

Calculations of Availability for $\lambda_0$ = 0.85 μm are made with $P(S_M \geq 10km)$ and calculations for $\lambda_0$ = 1.55 μm are done for $P(S_M \geq 5km)$. It is observed that using $\theta_{t,opt}$ increases the system availability. The results are better, when using smaller λ.

## 4. CONCLUSION

In this paper we've presented a generalised statistical model for atmospheric visibility. A mathematical model for evaluating the FSO systems availability was proposed. Using that model we've analysed the availability of FSOs with different system parameters. Also we've compared the availability of FSO system using fixed $\theta_t$ for different conditions and FSO using $\theta_t = \theta_{t,\,opt}$. Comparison is made for two wavelengths $\lambda_0$ = 0.85 and 1.55 μm. The results clearly show that using optimal values of the FSO parameters could result in significant increase of system's availability.

## 5. ACKNOWLEDGEMENT

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