FRACTAL CHARACTERISTICS OF DIGITAL IMAGES AND THEIR WAVELET TRANSFORMS

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Abstract

We consider two methods to obtain classification signs for some classes of biomedical preparation images. The first method is based on the computation of the fractal signatures vector. The second one calculates fractal signatures for a set of wavelet transforms of the initial image. The images are represented both in gray scale and HSV (Hue, Saturation, Value) palettes. Experiment results show that the best separation for given classes is achieved by application of the second method and using H-component of HSV.

1. INTRODUCTION

When analyzing and classifying digital images we have to choose an appropriate method of investigation which is oriented on special range of application and based on mathematical and computer models. To analyze textures one can use both statistical and morphological methods, fractal and multifractal ones lead to good results for biomedical preparations images. [2,9].

We consider two based on fractal characteristics approaches to image analysis and classification. The both methods use fractal signature of a color image transformed to the gray scale palette (G) or presented by the component H in HSV palette. In both cases the image is considered as a function F of integer coordinates, i.e. a two-dimensional surface [6].

The first method uses a vector of fractal signatures for a given image, where fractal signatures are obtained in accordance with [8] (Fractal signature method). The second one is based on obtaining fractal characteristics of the image wavelet transforms. Fractal signature method consists in the calculation of the area of so called δ-parallel body for the surface of the graphic of a function F. By δ-parallel body one mean the set of points being at the distance not greater than δ from the surface of graphics of F; these points form a blanket» with width 2δ. Using the “blanket” volume we may obtain the approximate value A_δ of the surface area of the F graphics. The ratio $\log A_\delta / \log \delta$ is called fractal signature. By changing δ in an interval [1,N] one can obtain a fractal signatures vector which we consider as the image characteristics.

To perform wavelet transform we use Gauss function and its second order partial derivatives, because they have good localizing properties [1].

We apply such a transform to a given digital image and then calculate the fractal signature of the obtained wavelet transform that we consider as an image – wavelet image. By changing the scale value in empirically selected range we obtain a set of wavelet images and the vector of corresponding them fractal signatures. In this case fractal signatures are calculated for $\delta = 1,2$.

Thereby, in each method we obtain a vector of fractal signatures: in the first method — by changing δ in the second one — by calculating fractal signature for every wavelet image. To compare images from different classes we compare their fractal signature vectors. The less the distance between vectors corresponding images A and B, the more probability that A and B are in the same class. Experiments allows us to select a method that gives a better separability for given classes of images. We also compare the results for gray scale and HSV (H component) representations of an image. For almost all of the images under investigation the using H component gives better classification results.

2. METHODS DESCRIPTION

The given image is presented by a discrete function F (in gray scale or HSV (H component) palette: F =

$\{X_{ij},\ i=0,1,\ldots,K,\ j=0,1,\ldots,L\}$, where X_{ij} —is (i,j) pixel intensity, K, L define the image size. In points with real coordinates F is redefined by its value on the left end of the integer interval. Hence one can consider a surface of the graphic of F and calculate its area.

2.1. Fractal signature method

The method is based on the B. Mandelbrot idea about the approximate calculation of the length *L* of a coastline which has complex fractal structure [7].

Consider all the points with distances to the coastline of no more than δ. They form a strip of width 2δ. Then the strip area divided by 2δ is an approximation to $L(\delta)$. The length increases as δ decreases. At the same time Mandelbrot noted that there is an interval for δ in which the value $L(\delta)$ becomes stable.

In [8] athors applied this idea to calculate the approximate value of the surface area of the graphic of the function presenting a digital image. They construct a special neighbourhood of the surface (δ-parallel body) calculated the body volume and divided it on the height. Changing δ one can obtain several approximate values for the surface area.

We define δ-parallel body [5] as the set of points being at the distance not greater than δ from the surface of graphics of F. They form a "blanket» with width 2δ having upper (u_δ) and bottom (b_δ) bounds which are calculated in each point of the image on its intensity and the intensity of neighbours. The formulas are given in [8,10].

The volume is calculated as the following

$$Vol_\delta = \sum_{i,j} u_\delta(i,j) - b_\delta(i,j) \qquad (1)$$

To calculate approximations to the surface area we may use two variants:

$$A_\delta = Vol_\delta / 2\delta\,, \qquad (2)$$

or

$$A_\delta = \frac{Vol_\delta - Vol_{\delta-1}}{2}\,. \qquad (3)$$

As for fractal surfaces (3) is more preferable, we use it. Fractal signature S_δ is defined as

$$S_\delta = \frac{\log A_\delta}{\log \delta}\,.$$

For $\delta \in [1, N]$ the sequence S_δ is calculated. The approximate value of S_δ is obtained by the least square method and is the slope of the line (in log-log scale) fitted to

$(\log(\delta-1), \log A_{\delta-1}), (\log\delta, \log A_\delta)$ and $(\log(\delta+1), \log A_{\delta+1})$

To compare two images one should to find the distance between their fractal signature vectors.

2.2. Wavelet transform method

We apply a wavelet transform [3,4] to the discrete function presenting the image. The transform reads as

$$W(a,b_1,b_2) = \frac{1}{a}\sum_{x=0}^{K-1}\sum_{y=0}^{L-1}\varphi\left(\frac{x-b_1}{a},\frac{y-b_2}{a}\right)F(x,y)\,, \qquad (4)$$

where a – scale parameter, b_1, b_2 – shifts on coordinate axis, φ – wavelet.

As a wavelet we use the sum of the second order partial derivatives of Gauss function:

$$\varphi(x,y) = \exp\left(-\frac{x^2}{2}-\frac{y^2}{2}\right)*(x^2+y^2+2xy-2). \qquad (5)$$

Such a wavelet has narrow energy spectrum and seems to be preferable when higher order singularities are analyzed [1]. As experiments show, though higher order derivatives allows obtaining more exact results than the initial function, the using derivatives higher than the second order complicates calculations without the accuracy increasing.

For the function presenting given image we fix *a* parameter and perform wavelet transform in accordance with (4). Here b_1, b_2 change from 1 to K, L respectively, where K and L define the image size. The obtained set we consider as a new "image" (in coordinates (b_1, b_2) that has the same size as the initial one.

By changing a in an interval (comparable with the interval of δ) we construct a series of images corresponding to the initial one. Then for every image from the set we calculate its fractal signature. The vector of fractal signatures is the image characteristic.

3. NUMERICAL EXPERIMENTS

We considered the biomedical preparation images of two different classes: healthy liver and Fatty Liver Disease (FLD) (Figure 1 and Figure 2). The size of all images is 300x300. In each class 5 images were selected.

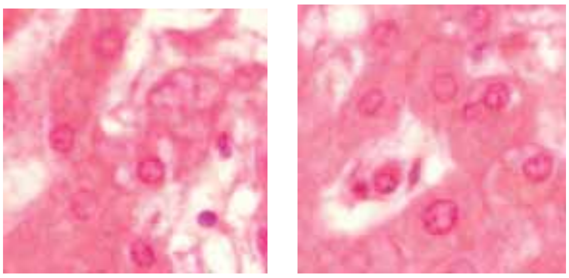

Fig. 1. Healthy liver images

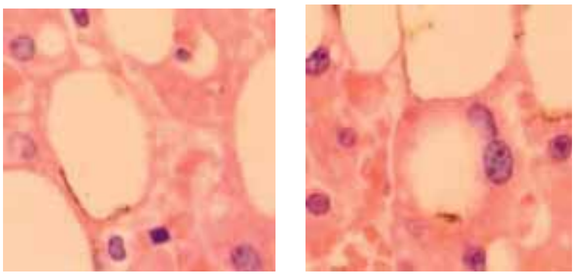

Fig. 2. Images of liver with FLD

Note that the images were beforehand classified by an expert.

The application of the first method when using HSV palette did not lead to a separation of images (Fig. 3)

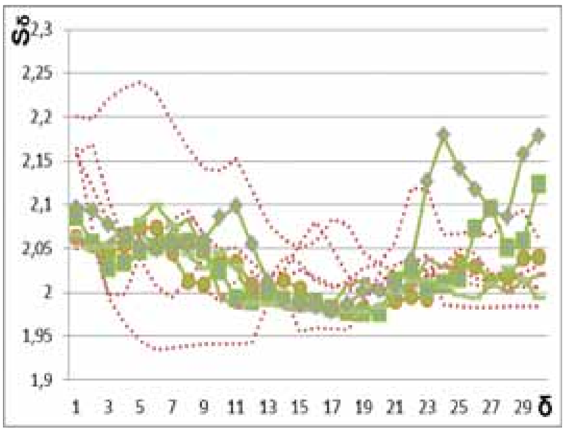

Fig. 3. The result of the first method in HSV palette

The vector size (30) was selected empirically: for δ >30 fractal signature values changed insignificantly. For the second method empirical estimation for scaling parameter was 10.

The application of the second method when using HSV palette demonstrated good separability of fractal signature vectors, and ipso facto two classes of images.

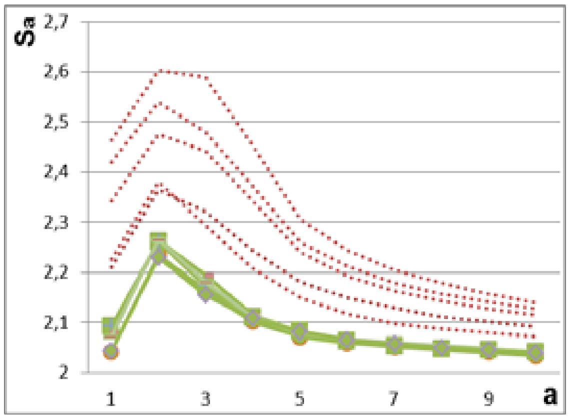

Fig. 4. Separation of vectors for two classes of images in the second method in HSV palette

The praphic illustrated the dependence fractal signature on the scale parameter *a*. Dotted lines denote the graphics for healthy liver, solid lines – graphics for FLD images. Hence wavelet transform with the following application of fractal signature method allowed us to separate two given classes of images.

The first method with using gray scale palette did not result in the separation of two classes. For obtained vectors we also considered graphics of their average, maximum and minimum values. As it is shown for maximal values on Fig. 5, this experiment did not improve the situation

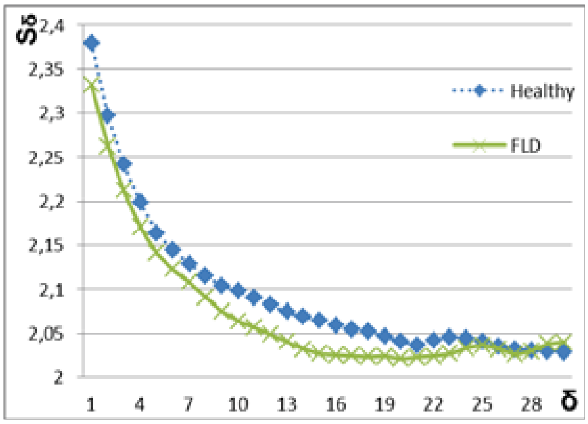

Fig. 5. Closeness of vectors of maximum for the first method in gray scale palette

The second method in grayscale palette was not successful (Figure 6).

We also considered the class of images of the liver with plephora and applied the described methods by the above scheme. The results of the experiments for this class seemed to be close to the results for two considered classes: the combination of the second method and HSV palette (H component) led to the separation of the images by the chosen characteristic. The results are summed up in the following table, where "+" means the success of experiments.

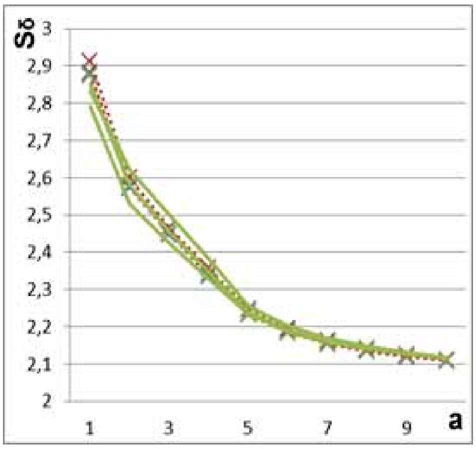

Fig. 6. Closeness of vectors of maximal values for the second method in gray scale palette

palette	method 1	method 2
HSV	-	+
grayscale	-	-

4. CONCLUSION

We performed the comparison of two methods of analysis of some classes of digital images. Both methods use fractal signature vector as a classifying sign. The first method forms the vector of fractal signatures obtained for a sequence of δ-parallel bodies when $\delta \in [1, N]$. The second one at first forms a sequence of wavelet transforms of the image by changing the scale parameter and then calculates fractal signatures for this sequence when $\delta \in [1,2]$. For both methods the results were more exact when using HSV palette (component H). Experiments show that wavelet transform may successfully applied to distinguish images with similar texture.

Acknowledgements

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