IMAGE SEGMENTATION ALGORITHM BASED ON MULTIFRACTAL SPECTRUM COMPUTING

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Abstract

In the image analysis problem segmentation algorithms are very important. We can use classical algorithms based on morphological analysis. But it is well known that for image having complex structure, e.g high resolution images, classical approach may fail to achieve good results. The solution is to use multifractal analysis that gives an information about the geometric distribution of the singularities of the image. In this work we apply the algorithm of direct calculation of multifractal spectrum for image segmentation. Fractal dimensions for level sets are calculated by fractal signature method. Experimental results for 2 classes of biomedical preparations images are given.

1. INTRODUCTION

Image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels). The goal of segmentation is to simplify the representation of an image so the obtained image may be easier to analyze. This method is used to locate objects and boundaries in images. In each segment all the pixels are similar with respect to some characteristics or computed property such as color or intensity.

Image segmentation is widely used in machine vision, medical imaging, object detection, recognition tasks. In order to effectively solve the domain's segmentation problems this technique must be combined with specific knowledge in the subject area.

Classical methods applied in image segmentation are methods of mathematical morphology: thresholding, edge detection (line, curve) [6,7].

For digital images with very high spatial resolution classical segmentation methods fail to give homogeneous segments and usually give very sparse results. As it was mentioned in [9], texture approach and fractal analysis are not appropriate methods, but "multifractal analysis is the perfect tool to analyze a highly varying regularity from point to point".

Multifractal analysis methods are based on a partition of the image on the set such that every of them has its own fractal dimension. The set of these dimensions characterizes the image and is called multifractal spectrum. To obtain such a partition we should factorize the image points in accordance with a sign. As this sign one can use density function [10], or Holder exponents [9]. The most common approach to calculate fractal dimensions is to use capacity dimension. When using multifractal methods we match to the image a measure and suppose that it is distributed according to exponential law: the measure of a partition cell is approximately equal to the size of the cell in a power. Multifractal spectrum may be obtained by calculation of the Regny spectrum and the following Legendre transform [4], by using generalized statistical sum [4] or by direct calculation by using density function [10].

The choice of the method depends on the problem. If the task is to obtain a spectrum then it is sufficient to compute the Regny spectrum. If our purpose is to use the partition on the sets for segmentation problem we have to devote attention to the separating sign choosing.

In [2] we considered the algorithm of direct calculation of multifractal spectrum based on the calculation of a density function at each point of the image.

In this work we reveal a dependence of the density function (and *ipso facto* the result of segmentation) from the measure and use fractal signature method [1,3] which allows us to make calculations more effective comparing capacity dimensions computing. Experimental results for 2 classes of biomedical preparations images are given.

2. METHOD DESCRIPTION

Let μ be a measure defined through pixel intensities. For $\square \in \square^2$ we denote $\square(\square,\square)$ a square of length r with center x. We describe $\mu(\square(\square,\square)) = \square\square^{\square(\square)}(\square)$ with $\square(\square)$ the density function and *k* some constant. The local density function of *x* is defined as

$$\square(\square) = \lim_{\square\to 0} \frac{\log \mu(\square(\square,\square))}{\log \square} \quad (1)$$

The set of all image points x with local density α is a level set

$$E_\alpha = \{x \in R^2 : d(x) = \alpha\} \ . \quad (2)$$

Thus we obtain a point categorization $\{E_\alpha : \alpha \in R\}$ of the image with a multifractal spectrum defined as

$$\{f(\alpha) : \alpha \in R\} = \{\dim(E_\alpha) : \alpha \in R\} \ . \quad (3)$$

The density function describes how locally the measurement μ satisfiers the power law behavior. It measures the non-uniformity of the intensity distribution in the square $B(x,r)$.

We note that using (2) may lead to considerable increasing the number of level sets and segmentation may be useless. So in practice one consider sets

$$E(\alpha,\varepsilon) = \{x \in R^2, d(x) \in [\alpha, \alpha+\varepsilon)\} \quad (4)$$

The measure $\mu(B(x,r))$ may be calculated by several ways. We used the sum of intensity pixels in the square and the sum of Laplacians in the square.

1. $\mu(B(x,r))$ — the sum of intensity pixels in the square withy side r and center x. $\mu(B(x,r)) = \int\limits_{\mu(B(x,r))}\int I(y)dy$,

where *I(y)* — is the pixel intensity in $y \in B(x,r)$.

In discrete case

$$\mu(B(x,r)) = \sum_{y\in B(x,r)} I(y) \ . \quad (5)$$

2. $\mu(B(x,r)) = \int\limits_{\mu(B(x,r))}\int \nabla^2(I(y))dy$, where ∇^2 — the Laplace operator.

In discrete case in accordance with [6] we have

$$\mu(B(x,r)) = \sum_{y\in B(x,r)} L(y) \ , \quad (6)$$

where $L(y) = 8I(y) - \sum_{z\in B(y,2)\setminus y} I(z)$.

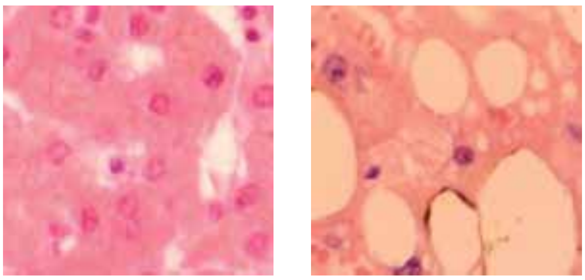
It was shown in [4] that namely using Laplacian resulted in the separation of classes of connective tissues in accordance with obtained spectra.

The density $d(x)$ is obtained as the slope of the line fitted to the data $\{\log r, \log \mu(B(x,r))\}$ by the least square method. Then we take a discrete set $\{\alpha_i\}$ from an interval (1,2) and find for each α_i the point set $E(\alpha_i,\varepsilon)$ (ε=0.1) according to (4).This set contains all the pixels whose densities are close to α_i.

The fractal dimension $f(\alpha_i)$ is computed as the Minkovsky dimension by the fractal signature method.

3. NUMERICAL EXPERIMENTS

The experiments were performed for two classes of liver: healthy (a) and fatty liver disease (b). The examples of these images are shown on Fig.1.



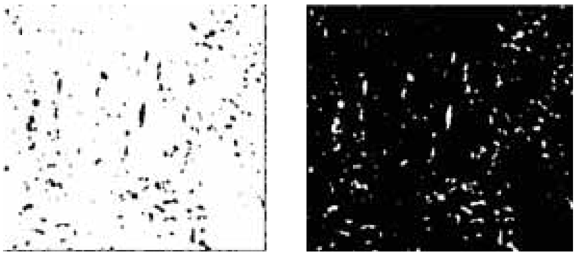
a b

Fig. 1. Liver images: a — healthy liver, b — fatty liver disease

We consider the dependence of segmentation results on the measure choice.

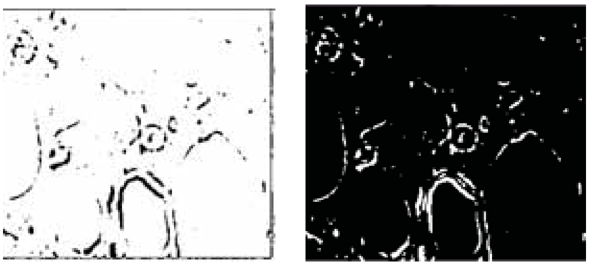
3.1. Measure is the sum of pixel intensities

For given images the results of calculation of $E(\alpha,\varepsilon)$ were the following. The sets containing the most number of points was obtained for α = 1.6. For α = 1.5 the set was not so dense. The rest of sets were almost empty. This situation is illustrated on Fig.2, where black color means the point belongs to the set.



a b

Fig. 2. $E(\alpha,\varepsilon)$ for healthy liver: α = 1.5 (a) and α = 1.6(b)



a b

Fig. 3. $E(\alpha,\varepsilon)$ for FLD: α = 1.5 (a) and α = 1.6(b)

Multifractal spectrum:

α	Healthy liver	Fatty liver desease
1.0	0	0
1.1	0.4352	0.9009
1.2	0.4352	0.9009
1.3	0.474	0.493
1.4	0.474	0.493
1.5	0.4988	0.5024
1.6	1.0538	0.8488
1.7	0	0.0591
1.8	0	0.1042
1.9	0	0

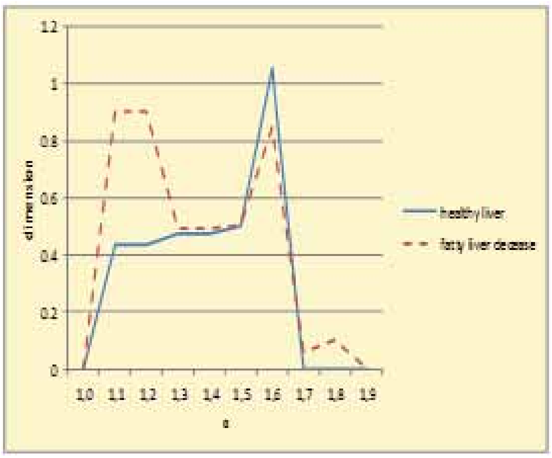
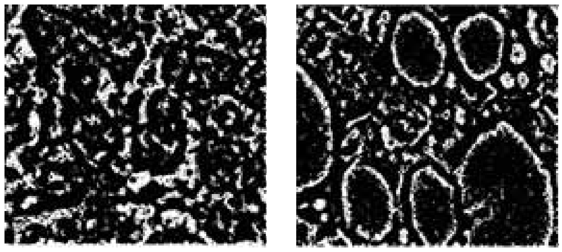


Fig. 4. Graphics of spectra for the measure as the sum of pixel intensities

3.2. Measure is the sum of Laplacians

In this case the density function (that depends on the measure) generates another partition on level sets and, as a consequence, another spectra. The most part of the image pixels belong to $E(\alpha,\varepsilon)$ with α = 1.0/



a b

Fig. 5. $E(\alpha,\varepsilon)$ for α = 1.0:(a) –healthy liver,(b)– FLD

Multifractal spectrum:

α	Healthy liver	Fatty liver desease
1.0	0.7291	0.7509
1.1	0.4376	0.499
1.2	0.4376	0.499
1.3	0.3986	0.4804
1.4	0.3986	0.4804
1.5	0.3932	0.4878
1.6	0.4032	0.5122
1.7	0.3962	0.5172
1.8	0.3921	0.4787
1.9	0.396	0.5122

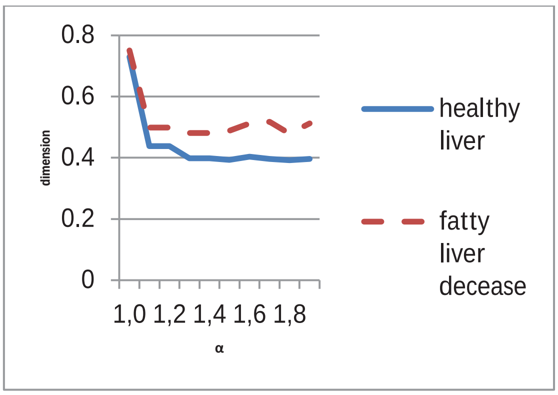


Fig. 6. Graphics of spectra for the measure with Laplacian

CONCLUSION

We presented a segmentation method based on a direct calculation multifractal spectrum. A categorization of the image points is performed by using density function that characterizes intensity changes in the neighbourhood of a given point. Two methods of the measure calculation were considered —- the sum of pixel intensities and the measure using Laplacian.

The experiments show that image segmentation based on using density function is a practical tool for analysis. Moreover, using the Laplacians leads to better segmentation and separability of multifractal spectra.

Acknowledgements

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