

# ANALYSIS OF TWO-STAGE LOW NOISE AMPLIFIER WITH IMPROVED INPUT MATCHING BY USING NON-UNILATERAL CHARACTERISTICS OF TRANSISTORS

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## Abstract

Improving input matching of low noise amplifiers (LNA) is general task for microwave engineers. Known methods for solving this problem and their advantages and disadvantages are discussed in [1] and [2]. In these papers is also presented one novel method which improves input matching by using non-unilateral characteristics of transistors. A detailed analysis of single stage amplifier is presented in [2]. It was shown that using this method costs loss in insertion gain of amplifier due to poor output matching. One possible decision of this problem is to add second stage of amplifier. This will be the topic of this paper. A detailed analysis and mathematical model of two stages LNA with improved input matching will be presented here. Furthermore the real S - parameters of transistor will be used to evaluate performance of theoretically designed amplifier.

## 1. INTRODUCTION

When single stage amplifiers cannot provide sufficient gain, we can either cascade stages already impedance matched or continuously match the output of one stage to the input of next stage until the desired gain is reached. As was shown in [2] improving input matching of single stage low noise amplifier by using non-unilateral characteristics of transistors costs poor output matching. For this reason we will try to match directly input of second stage to required output impedance of the first stage in order to obtain good input matching. Generalized block diagram with all specific impedances and reflection coefficient of two-stage amplifier which use this type of matching is shown in fig.1.

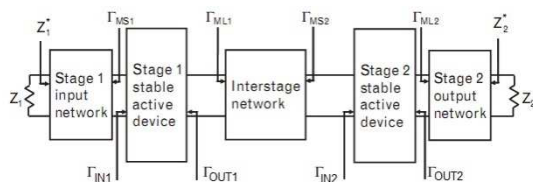


Fig.1. Block diagram of two-stage amplifier with all specific reflection coefficients

Analyzed amplifier in this paper use transistors NE3210 manufactured by NEC corporation for both stages. Its S-matrix and noise parameters for 12 GHz are:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.44 \angle -166.2^\circ & 0.10 \angle 2.1^\circ \\ 3.91 \angle 23.5^\circ & 0.21 \angle -123.1^\circ \end{bmatrix}$$

$$\Gamma_{opt} = 0.29 \angle 133^\circ$$

$$R_n = 4.5$$

$$F_{min} = 1.08$$

Where  $\Gamma_{opt}$  is optimal reflection coefficient,  $R_n$  and  $F_{min}$  are noise resistance and minimum noise figure. We will use designed single stage amplifier in [2] and upgrade it with second stage. In this paper was explained in details requirements for reflections at the input and the output of transistor in order to obtain low noise figure and conjugate matched input. So we will use synthesized in [2] input matching network, which is show in fig.2.

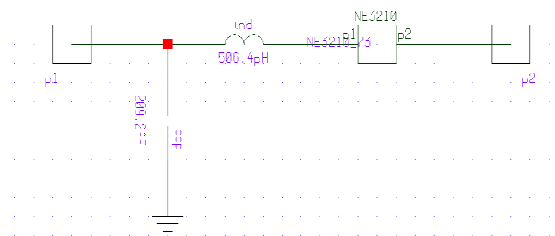


Fig. 2. Input noise matching network

The interstate and output matching circuits will be designed in order to be obtained conjugate matched input and output of amplifier and maximize insertion gain while noise figure remains unchanged.

## 2. ANALYSIS OF TWO-STAGE LOW NOISE AMPLIFIER WITH IMPROVED INPUT MATCHING

To analyse how this method for improving input matching influence to the other parameters of two-stage LNA is necessary the amplifier to be described mathematically. In this chapter will be presented mathematical model that find S – matrix and noise figure of this amplifier.

In general when we design multi- stage amplifier we cascade two port networks which are not conjugate matched to characteristic impedance. For analysing such circuit are used ABCD parameters of connected networks. Relationship between ABCD and S parameters is:

$$(1) \quad A = \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}$$

$$(2) \quad B = \frac{(1 + S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}} Z_0$$

$$(3) \quad C = \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}Z_0}$$

$$(4) \quad D = \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}}$$

Using (1), (2), (3), (4) and S parameters given above we can find ABCD matrix of used transistor at 12GHz:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.44 \angle -166.2^\circ & 0.10 \angle 2.1^\circ \\ 3.91 \angle 23.5^\circ & 0.21 \angle -123.1^\circ \end{bmatrix}$$

They will be used further when designed LNA is analyzed.

The mathematical model of amplifier can be obtained by multiplying ABCD matrices of active components and matching networks. From this model can be obtained reflection coefficient and insertion gain in every point of this circuit.

$$(5) \quad \begin{bmatrix} A_{LNA} & B_{LNA} \\ C_{LNA} & D_{LNA} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}$$

Where:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} - \text{ABCD matrix of input matching circuit.}$$

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} - \text{ABCD matrix of inter - stage matching circuit.}$$

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} - \text{ABCD matrix of output matching circuit.}$$

By using S-parameters and equations (1), (2), (3), (4) we find ABCD matrix of input matching network

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1.001 \angle -0.126^\circ & 38.189 \angle 90.149^\circ \\ 0.016 \angle 90.144^\circ & 0.397 \angle -0.318^\circ \end{bmatrix}$$

The inter-stage and output matching networks will be designed later by using interactive smith chart and linear optimization procedure in "Ansoft Sere-nade circuit simulator". Their ABCD parameters will be calculated with (1), (2), (3) and (4). When we have ABCD matrix of two stages amplifier we can calculate its S – parameters with equations:

$$(6) \quad S_{11} = \frac{A + \frac{B}{Z_0} - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$(7) \quad S_{12} = \frac{2(AD - BC)}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$(8) \quad S_{21} = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$(9) \quad S_{22} = \frac{-A + \frac{B}{Z_0} - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

After S – parameters of amplifier are found is necessary to be analysed influence of second stage to noise figure. For this analysis is necessary to be known separately gain and noise figure of every stage. Total noise figure of amplifier can be calculated using equation:

$$(10) \quad F = F_1 + \frac{F_2 - 1}{|S_{21\_A}|^2}$$

Where: F is total noise figure of two stages LNA,  $F_1$  is noise figure of the first stage,  $F_2$  is noise figure of second stage and  $S_{21\_A}$  is gain of the first stage with input matching circuit.

The first stage is noise matched:

$$(11) \quad F_1 = F_{\min}$$

The  $S_{21\_A}$  can be found using ABCD matrices and equation (8).

$$(12) \quad \begin{bmatrix} A_{-A} & B_{-A} \\ C_{-A} & D_{-A} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

To finish analysis of noise figure we need to know noise figure of the second stage. It can be calculated using equation:

$$(13) \quad F = F_{\min} + \frac{4R_n |\Gamma_{MS2} - \Gamma_{opt}|^2}{Z_0 |1 + \Gamma_{opt}|^2 (1 - |\Gamma_{MS2}|^2)}$$

The  $\Gamma_{MS2}$  is reflection coefficient at the output of inter-stage matching circuit. We find  $\Gamma_{MS2}$  by multiplying ABCD matrices of input matching circuit, transistor and output matching circuit. And then use equation (6) to derive it.

$$(14) \quad \begin{bmatrix} A_{-B} & B_{-B} \\ C_{-B} & D_{-B} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

### 3. DESIGN INTER-STAGE AND OUTPUT MATCHING CIRCUITS OF TWO STAGE LNA WITH IMPROVED INPUT MATCHING

As was explained in [2], for obtaining conjugate matched input is necessary to have a specific reflection coefficient of output matching network. In case of two stage amplifier this reflection coefficient will be obtained from interstate matching network as shown in fig.1. From [2] we take:

$$\begin{aligned} \Gamma_{ML1} &= 0.789 \angle -48.89 \\ Z_{ML1} &= 104.6 \angle -67.46 \end{aligned}$$

We need to design matching circuit which transform input impedance of transistor to required impedance  $Z_{ML1}$ . Furthermore the output matching circuit have to provide conjugate matching at the output of amplifier:

$$(15) \quad \Gamma_{ML2} = \overline{\Gamma_{OUT2}}$$

As was explained transistors do not have perfect isolation between output and input. For this reason inter-stage and output matching networks have to be designed simultaneously. This can be expressed

mathematically by solving system of equations but here will be shown simplified method for design matching networks. The full mathematical analysis of the matching circuits is not subject of this paper. First will be designed interstate matching network. We use interactive smith chart to synthesize it as shown in fig.2. This network have to transform input impedance of the second stage to required output impedance of the first stage in order to be obtained conjugate matched input of amplifier. Synthesized matching network contains parallel capacitor and series inductance.

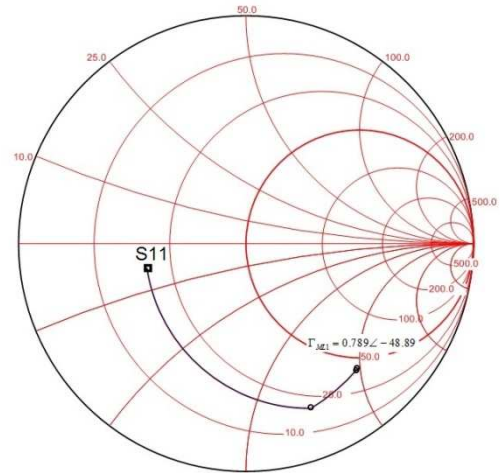


Fig. 3. Design inter - stage matching network using interactive smith chart

After inter-stage matching network is designed it will be connected to the input of transistor and  $\Gamma_{OUT2}$  will be calculated. The output matching network will be designed in order to satisfy equation (15). Again will be used interactive smith chart as shown in fig. 4.

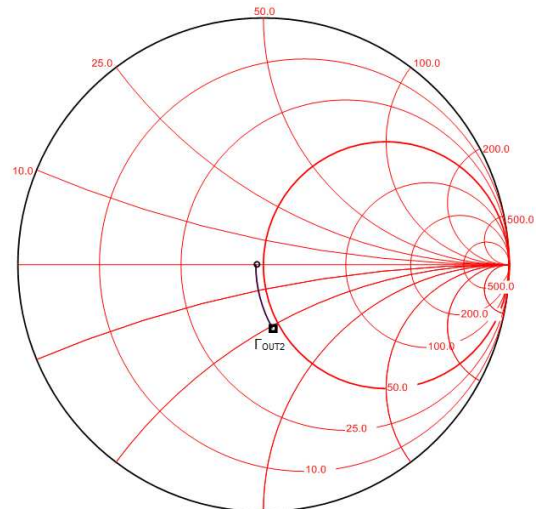


Fig. 4. Design output matching network using interactive smith chart

After that both matching networks will be connected to corresponding port of transistor. There will be some mismatch due to not perfect isolation and it will be compensated by using optimization. The ABCD parameters of optimized networks are:

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ & 68.54\angle -90^\circ \\ 0.005\angle -90^\circ & 0.65\angle 0^\circ \end{bmatrix}$$

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ & 30.16\angle 90^\circ \\ 0\angle 0^\circ & 1\angle 0^\circ \end{bmatrix}$$

When all matching networks are available, the ABCD and S parameters of two stages amplifier can be calculated using equation (5), (6), (7), (8), (9). The S-matrix of designed LNA us:

$$\begin{bmatrix} S_{11LNA} & S_{12LNA} \\ S_{21LNA} & S_{22LNA} \end{bmatrix} = \begin{bmatrix} 0.002\angle -146.7^\circ & 0.008\angle -9.5^\circ \\ 10.64\angle 33.26^\circ & 0.001\angle 48.2^\circ \end{bmatrix}$$

The noise figure can be found using equation (10). To solve it is necessary to calculate gain of the first stage with input matching network. We use equation (12) and then substitute in equation (8) and find:

$$S_{21\_A} = 4.19\angle -28.96^\circ$$

The other parameter that we need to find noise figure of LNA is noise figure of second stage by itself. It can be calculated using (13) but first must be found reflection coefficient at the input of second stage. To do this we use (14) and then substitute in (6) and find:

$$\Gamma_{MS2} = 0.67\angle -62.64^\circ$$

The calculated noise figure of second stage is:

$$F_2 = 1.95$$

Now we have all parameters needed to find noise figure of designed two-stage LNA.

$$F = 1.13$$

Practically parameters of amplifiers are expressed in decibels:

$$G = 20\log|S_{21LNA}| = 20.5dB$$

$$RL_{IN} = 20\log|S_{11LNA}| = -53.8dB$$

$$RL_{OUT} = 20\log|S_{22LNA}| = -58.8dB$$

$$NF = 10\log|F| = 0.55dB$$

Where G is gain of amplifier,  $RL_{IN}$  is input return loss,  $RL_{OUT}$  is output return loss and NF is noise figure. Theoretically lowest possible noise figure will be obtained when inputs of both stages are noise matched. Then total noise figure of amplifier will be 0.36dB. So we can conclude that conjugate matched input and output of two stages LNA will increase noise figure with about 0.2dB. This will be due to contribution of second stage to total noise figure of amplifier. But as was explained in [2] conjugate matching is just theoretical. In practice will never be required amplifier to be conjugate-matched. Return loss of -20dB is often sufficient for microwave amplifiers. So some trade-offs between matching, gain and noise figure can be done in order to be satisfied practical requirements to LNA.

## CONCLUSION

Theoretical analysis of two stages LNA with improved input matching by using non-unilateral characteristics of transistor was presented in this paper. The results of this analysis shows that in two stage configuration can be reached simultaneously noise matched input and conjugate matched ports of amplifier but contribution of second stage to total noise figure can be significant. Total gain of amplifier will also be reduced by this type of matching. Some tradeoffs between input matching, gain and noise figure can be done and amplifier will meet most of the practical requirements. Analysis of this trade-offs will be subject of further work.

## References

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