ON DIGITAL IMAGES SEGMENTATION BASED ON FRACTAL AND MULTIFRACTAL METHODS

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Abstract

Segmentation as an important step in image processing is widely used to analyze medical images, faces, images of the Earth surface and many others. For complex textures and high resolution images fractal and multifractal methods often lead to better results than classical ones, because they seem to be more sensitive to intensity changes. In this work we apply fractal and multifractal methods to analyze biomedical preparation images. Both methods allow us to obtain numerical characteristics and perform the based on them segmentation.

1. INTRODUCTION

The common practice in image analysis is to apply several methods and compare results. The applicability of a given segmentation method depends on the problem under investigation. For complex texture images researches segmentation algorithms are based on fractal and multifractal analysis. Such an approach is suitable for high resolution images containing large number of fine details.

Textures are often may be considered as fractal (or multifractal) sets. Fractal sets have a self-similarity property that may be characterized by a scaling exponent. These exponents are base to define a fractal dimension (or several dimensions) of the set. In practice the class of so called box-computing dimensions (e.g. capacity dimension) is widely used [5]. A set M is covered by $N(\varepsilon)$ boxes with a side ε. It is assumed that the number of boxes is proportional to $\varepsilon^{-\alpha}$. Decreasing ε and taking the limit of $\frac{\ln N(\varepsilon)}{\ln\frac{1}{\varepsilon}}$ we obtain the value of α, which is called capacity dimension of M. For greyscale (or monochrome) images the Minkovsky dimension is more appropriable [5].

A union of several fractals, which of them has its own fractal dimension, is called multifractal. Such sets may be characterized not one by several scaling exponents. As components of multifractals are usually arranged by a complex way, segmentation problem by classical methods may be not very easy. Hence we may use scaling components as characteristics by which we combine image pixels in subsets: namely, we assume that elements (cells of an image partition) with close exponents (or dimensions) belong to the same set.

In fractal methods it is convenient to calculate the fractal signature or the Minkovsky dimension for each cell of a given partition. Multifractal methods are based on the assumption that there is a measure defined on the image. Such a measure is given in terms of pixel intensities. Measure of a cell may be defined as the sum of pixel intensities or as the relation of the number of pixels having a given intensity to the common number of pixels. For multifractal methods one also assume that the measure of i-th cell $P_i(l) \sim l^{\alpha_i}$, where α_i are called singularity (scaling) exponents. Fractal dimensions of subsets of cells with close values of exponents form multifractal spectrum.

But for segmentation it is sufficient to use only singularity exponents.

In this work we consider two methods for image segmentation: modified fractal signature and multifractal analysis ones. The first method is based on the construction of the grey level function graphics for a greyscale image and calculation of the surface area for this graphic. This method is based on the blanket technic and was described in detail in [5, 10]. Using the area value we obtain a special characteristic — fractal signature and the Minkovsky dimension. The method was successfully applied to analyze text documents [11], ISAR images [6, 8, 9] and images of biomedical preparations [1-3]. The segmentation may be performed by a partition of the image on cells, calculation fractal signature (or the Minkovsky dimension) and marking the cells by a color in accordance with the obtained value.

The second method is a modification of calculation of multifractal spectrum by using so called density function [13]. The image is divided on subsets of points that have close values of density function (level sets). The set of fractal dimensions of level sets is multifractal spectrum. This method was used to obtain level sets for different biomedical images [4]. We propose to optimize this approach and calculate density function for a small base cell. The size of cell is a parameter. Experiments show that such an optimization speeds up run time considerably and does not affect visibly on segmentation results.

2. FRACTAL SIGNATURE METHOD

The idea of the method is to calculate fractal dimension of the surface formed by the graphic of the grey level function for a given image. To do it we calculate the approximate value of the surface area, and this area may be obtained by using the volume of a special object.

Let $F = \{X_{ij}, i = 0,1,\dots,K, j = 0,1,\dots,L\}$be an image with multigray level and X_{ij} be the gray level of the (i,j)-th pixel. In image processing the gray level function F is a nonempty bounded set in R^3. The surface area A_δ may be calculated using the volume Vol_δ of a special δ-parallel body — blanket with the thickness 2δ [5].

$$A_\delta = \frac{Vol_\delta - Vol_{\delta-1}}{2}.$$

The expression $S_\delta = \frac{\log_2 A_\delta}{\log_2 \delta}$ is called fractal signature. Fractal dimension of the surface is defined by the formula

$$D \approx 2 - \frac{\log_2 A_\delta}{\log_2 \delta}.$$

In practice the approximate value of S_δ is obtained by the least square method as the slope of the line in axes $(\log_2 \delta, \log_2 A_\delta)$.

For segmentation we do the following. Partition the image into n cells by size $N \times N$ and then for $k = 1, 2, \dots, n$ calculate grey level functions F^k, ${u_0}^k$ and ${b_0}^k$, ${u_\delta}^k$ and ${b_\delta}^k$ (for $\delta = 1, 2, 3$), ${Vol_\delta}^k$ and ${A_\delta}^k$.

By the least square method calculate approximate values

$$S^k = \lim_{\delta\to 0} \frac{\log_2 {A_\delta}^k}{\log_2 \delta}.$$

Mark the cell having close values S^k by the same color.

3. CALCULATION OF DENSITY FUNCTION

3.1. Base method

In what follows we use the definition given in [13]. Let μ be a measure defined through pixel intensities. For $x \in R^2$ we denote $B(x,r)$ a square of length r with center x. Let $\mu(B(x,r)) = kr^{d(x)}(x)$ where $d(x)$ is the local density function of x and k some constant. Then

$$d(x) = \lim_{r\to 0} \frac{\log \mu(B(x,r))}{\log r}.$$

The density function measures the non-uniformity of the intensity distribution in the square $B(x,r)$. The set of all points x with local density α is a level set

$$E_\alpha = \{x \in R^2 : d(x) = \alpha\}.$$

In practice, not to increase the number of level sets, one really consider sets

$$E(\alpha,\varepsilon) = \{x \in R^2, d(x) \in [\alpha, \alpha+\varepsilon)\}.$$

It is clear that showing level sets results in a segmentation of the image. This approach was successfully applied in [4] to analyze biomedical preparation images.

3.2. Modification

It is easy to note that in segmentation problem we do not need to calculate multifractal spectrum. Hence we can use density functions (that are a sort of singularity exponents for one point). As the computing this function for every point of the image may be time-consuming we calculate it for a small base cell.

Let l be a side of a base cell, the measure (the sum of pixel intensities) of i-th cell be $\mu_i(l)$, and density function — α_i .

In this case for segmentation we partition the image into n base cells by size $l \times l$ and for $i = 1, 2, \dots, n$ calculate $\mu_i(l)$, select neighbourhoods of base cells by size $(l+2) \times (l+2)$ and

$(l+4)\times(l+4)$, calculate $\mu_i(l+2), \mu_i(l+4)$. By the least square method using points $(\ln l, \ln \mu_i(l))$, $(\ln(l+2), \ln \mu_i(l+2))$,

$(\ln(l+4), \ln \mu_i(l+4))$ calculate approximate values α_i. Mark base cells with close values of α_i by the same color.

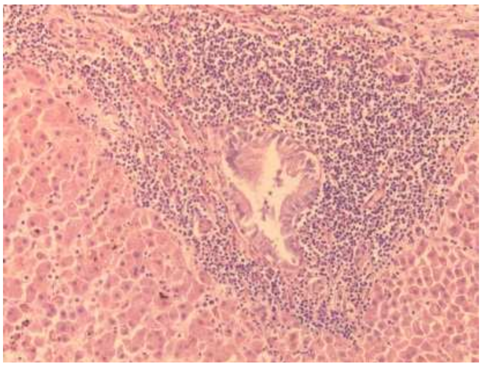
The number of colors depends on the range of exponent values.

4. NUMERICAL EXPERIMENTS

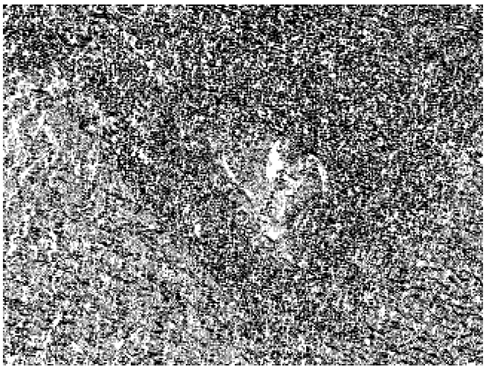
The experiments were performed for biomedical preparations images including high resolution ones. For color images numerical characteristics were obtained for all components of RGB palette. The number of colors used for segmentation was selected in accordance with the range of fractal signatures (the method of fractal signature) and the range of density function for base cells (the method of multifractal spectrum calculation). The results show that for complex textures multifractal method leads to better results.

4.1. Liver tissue

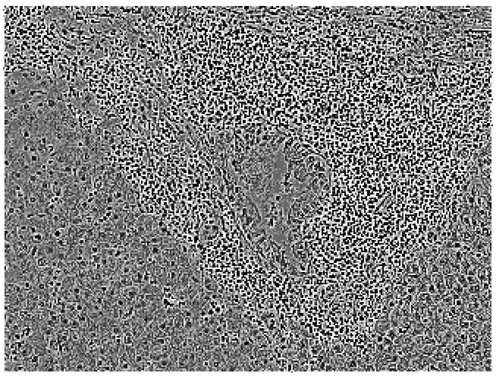
We considered high resolution image of liver tissue (chronic cholangitis). The image size is 2584× 1936. The results are shown for R component of RGB. We use the following notations: l — a cell side, nc — the number of colors. The fractal signature method localizes objects more clearly, whereas the multifractal one shows an averaged common structure. Both the results are important for analysis. We note that the selection of small l may not result in a finer segmentation. This depends on the image structure and the size of homogeneous segments. The most optimal values of parameters (l, nc) are selected experimentally.



a



b

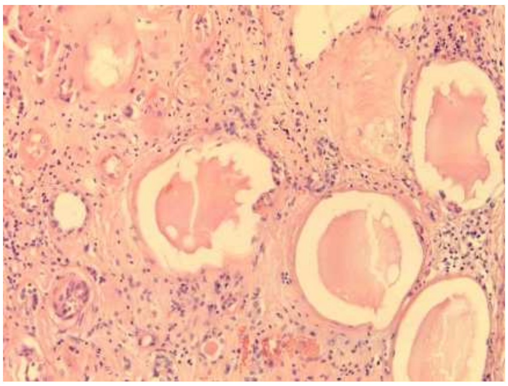


c

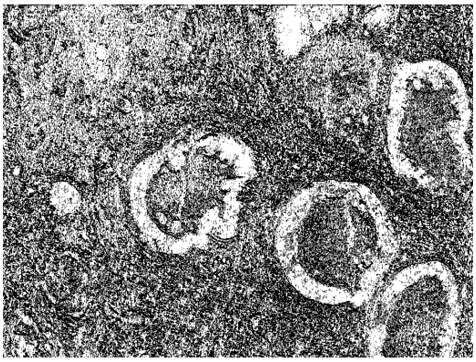
Fig.1a – the initial image; b – application of fractal signature method (l = 8, nc=5); c – application of multifractal method (l = 8, nc=6)

4.2. Kidney tissue

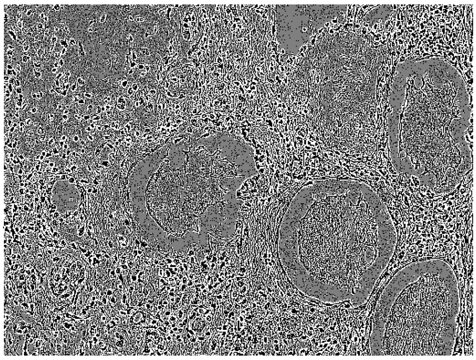
The image of kidney with chronic pyelonephritis was analyzed. The experiments were performed for all components of RGB. The results are shown for R component. The both methods give clear localization of main objects, but multifractal one reveals fine details more clearly.



a



b



c

Fig. 2a – the initial image; b – application of fractal signature method (l = 5, nc=3); c – application of multifractal method (l = 5, nc=3)

5. CONCLUSION

Complex textures and high resolution images often have fractal structure. Hence fractal and multifractal methods may be used for their analysis. As a result we obtain a set of numerical characteristics that may be used as classification signs. Moreover, these characteristics are may be a base for natural image segmentation. Experiments show that these methods are often more preferable because they are very sensitive to pixel intensity changes.

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