SIMULATING CHROMATIC DISPERSION IN OPTICAL FIBER CHANNELS USING SPLIT STEP FOURIER METHOD

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Abstract

The paper describes a method for simulating the chromatic dispersion in the optical fiber caused by nonlinear effects. This effects are dependent by optical wavelength of the propagated light in the fiber. The method proposed is a numerical solution of the Non Linear Schrodinger equation. Chromatic dispersion is one of the main contributors for degrading of performance in optical transmission systems and shortening the maximum transmission span in the optical line. The method proposed in this paper enables the researchers to evaluate and dimension high capacity optical systems in near future. SSFM enables the possibility to evaluate chromatic dispersion and attenuation in optical fiber versus the optical transmission distance. For simulating beta coefficients to 3rd order are taken into account. The proposed algorithm is written on Matlab simulation platform.

1. INTRODUCTION

Optical fiber properties and their impact to optical transmission systems are essential for developing next generation telecommunication systems [1]. Data volume traffic in the world is rising in exponential rate which increases the demand for creating optical transmission lines with increased capacity and higher throughputs while reducing the operational and maintenance cost of the existing optical transmission lines.

Legacy optical transmission lines are based on an On-Off keying optical modulation schemes [2]. With this modulation increasing the capacity and throughputs is based on shortening the pulses propagated in the optical fiber. The current optical telecommunication systems are at point where the period of the pulse Ts ~ 10ns which increases the impact of nonlinear effects of the silica of the optical fibers and distorting the performance of the telecommunication system [3]. The nonlinear effects in the fiber are shrinking the maximum length of the transmission system which must be evaluated carefully when developing long-haul optical transmission systems.

One of the main factors for limitation of the maximum reach of the transmission line is the chromatic dispersion. The chromatic dispersion is related to the material properties of the optical fiber.

This paper is focused on a simulation model for simulating nonlinear effects in the optical fiber.

2. FIBER TRANSMISSION DYNAMIC

With increasing the baud rate in the optical fiber the nonlinear effects are contributing significantly for the overall performance of the transmission system. Chromatic and polarization dispersion are the main contributors for degrading the BER and SNR at the receiver side.

2.1. Chromatic dispersion

The initial point when mentioning to the chromatic dispersion is the expansion of the mode propagation constant or "wave number" parameter, β, using the Taylor series:

$$\beta(w) = \frac{wn(w)}{c} = \beta_0 + \beta_1 \Delta w + \frac{1}{2}\beta_2 \Delta w^2 + + \frac{1}{3}\beta_3 \Delta w^3 \qquad (1)$$

where ω is the angular optical frequency, n(ω) is the frequency-dependent refractive index of the fiber. The parameters

$$\beta_n = \left(\frac{\partial^n \beta}{\partial \omega^n}\right) \Big|_{\omega=\omega 0} \qquad (2)$$

have different physical meaning as:

β_0 is involved in the phase velocity of the optical carrier which is defined as

$$v_p = \frac{\omega_0}{\beta_0} = \frac{C_0}{n(w_0)} \qquad (3)$$

β_1 determines the group velocity of v_g which is related to the mode propagation constant β of the guided mode

$$v_g = \frac{1}{\beta} = (\frac{\partial \ \beta}{\partial \omega})^{-1} \Big|_{\omega=\omega 0} \quad (4)$$

and β_2 is the derivative of group velocity with respect to frequency. Hence, it clearly shows the frequency dependence of the group velocity. This means that different frequency components of an optical pulse travel at different velocities, hence leading to the spreading of the pulse or known as the dispersion [1].

The parameter β_2 is know as GVD (Group Velocity Dispersion). The fiber is said to exibit normal dispersion for $\beta_2 > 0$ or anomalous dispersion if $\beta_2 < 0$.

A pulse having the spectral width of $\Delta\omega$ is broadened by $\Delta T = \beta_2 L \, \Delta\omega$. In practice, a more commonly used factor to represent the chromatic dispersion of a single mode optical fiber is known as D (ps/nm.km). The dispersion factor is closely related to the GVD $\beta 2$ and given by:

$$D = -2\pi c \beta_2 / \lambda^2 \quad (5)$$

At the operating wavelength λ where $\beta_3 = d\beta_2/dw$ contributes to the calculation of the dispersion slope S(λ) .

This parameter can be obtained by higher order derivatives of the propagation constant.

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda^2}\right)\beta_2 + \left(\frac{4\pi c}{\lambda^3}\right)\beta_3 \quad (6)$$

$$L_d = \frac{10^5}{D.B^2} \quad (7)$$

where B is the bit rate (Gb/s), D is the dispersion factor (ps/nm km) and *LD* is in km [2].

This provides a reasonable approximation even though the accurate computation of this limit that depends the modulation format, the pulse shaping and the optical receiver design. Thus, for 10 Gb/s OC-192 optical transmission on a standard single mode fiber (SSMF) medium which has a dispersion of about ±17 ps/nm.km, the dispersion length *LD* has a value of approximately 60 km i.e corresponding to a residual dispersion of about ±1000 ps/nm and less than 4 km or equivalently to about ± 60 ps/nm in the case of 40Gb/s OC-768 optical systems. These lengths are a great deal smaller than the length limited by ASE noise accumulation. The chromatic dispersion therefore, be-comes the one of the most critical constraints for the modern high capacity and long haul transmission optical systems.

$$\delta\omega = -\frac{\delta\varphi_{NL}}{\delta T} = -\gamma \frac{\delta P}{\delta T} L_{eff} \quad (8)$$

From (8), the amount of $\delta\omega$ is proportional to the time derivative of the signal power P. Correspondingly, the generation of new spectral components may mainly occur the rising and falling edges of the optical pulse shapes, i.e. the amount of generated chirp is larger for an increased steepness of the pulse edges.

3. MODELING OF PROPAGATION

Modeling of the optical complex envelope of optical pulses can be described by the well known Nonlinear Schroedinger Equetion (NLSE).

$$\frac{\partial \ A(z,t)}{\partial z} + \frac{\alpha}{2} A(z,t) + \beta_1 \frac{\partial \ A(z,t)}{\partial t} + \frac{j}{2}\beta_2 \frac{\partial^2 A(z,t)}{\partial t^2} - \frac{1}{6}\beta_3 \frac{\partial^3 A(z,t)}{\partial t^3} = -j\gamma |A(z,t)|^2 A(z,t) \quad (9)$$

where z is the spatial longitudinal coordinate, α accounts for fiber attenuation, β_1 indicates the differential group delay (DGD), β_2 and β_3 represent the second and third order factors of GVD and γ is the nonlinear coefficient. Equation (11) involves the following effects in a single-channel transmission fiber:

1) the attenuation, 2) chromatic dispersion, 3) 3rd order dispersion factor i.e the dispersion slope, and 4) self phase modulation nonlinearity.

Other critical degradation factors such as the nonlinear phase noise due to the fluctuation of the optical intensity caused by ASE noise via Gordon-Mollenauer effect is mutually included in the equation.

Thus, in SSFM, the linear operator representing the effects of fiber dispersion and attenuation and the nonlinearity operator taking into account fiber nonlinearities are defined separately as

$$D = \frac{i\beta_2 \partial^2}{2\partial \ \ T^2} + \frac{\beta_3 \partial^3}{6\partial T^3} - \frac{\alpha}{2} \quad (10)$$

$$N = i\gamma |A|^2 \quad (11)$$

where *A* replaces A(z,T) for simpler notation and *T=t-z/vg* is the reference time frame moving at the group velocity. The NLSE equation can be rewritten as:

$$\frac{\partial \ \ A}{\partial z} = (D + N)A \tag{12}$$

and the complex amplitudes of optical pulses propagating from z to z+ is calculated using the approximation is given:

$$A(z + h, T) \approx e^{(hD)}\, e^{(hD)}\, A(z, T) \tag{13}$$

Equation (13) is accurate to second order in the step size *z*. The accuracy of SSFM can be improved by including the effect of the nonlinearity in the middle of the segment rather than at the segment boundary as illustrated in Equation (14) can now modified as:

$$A(z + \delta z) \approx \exp\left(\frac{\delta z D}{2}\right) \exp\left(\int_{z}^{z+\delta z} N(z)dz\right) \exp\left(\frac{\delta z D}{2}\right) A(z, T) \tag{14}$$

This method is accurate to third order in the step size *z*. The optical pulse is propagated down segment from segment in two stages at each step. First, the optical pulse propagates through the first linear operator (step of *z*/2) with dispersion effects taken into account only. The nonlinearity is calculated in the middle of the segment. It is noted that the nonlinearity effects is considered as over the whole segment. Then at z+*z*/2, the pulse propagates through the remaining *z*/2 distance of the linear operator. The process continues repetitively in executive segments *z* until the end of the fiber. This method requires the careful selection of step sizes *z* to reserve the required accuracy.

The Simulink model of the lightwave signals propagation through optical fiber is shown in Figure 1.

4. RESULTS

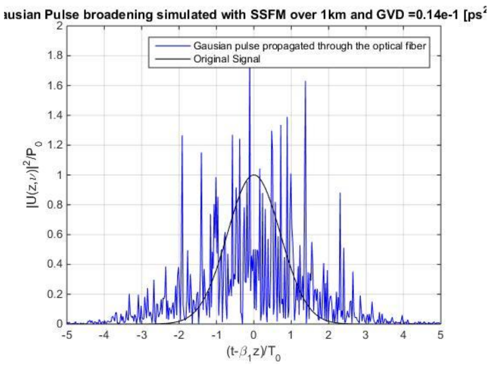

Figure 1. Gausian pulse propagated over fiber with GVD = 0.014 [ps^2/nm].

In figure 2 is showed the broadening of the gausian impulse propagated through optical fiber. On Y axis is showed the nominal Amplitude of the signal. On X axis is showed the broadening of the signal related to time.

After certain distance the broadening of the optical impulse increases the initial period of the signal. This effects are caused by the chromatic dispersion of the optical fiber. This broadening of the optical impulse can cause ISI in high speed transmission links which can degrade the overall performance of the system which reduces the throughput of the optical link.

Figure 3 shows the broadening of the signal related to the nominal distance of the link which spans in the range of Z/L=[0,1];

From this figure it is shown that after certain distance the chromatic dispersion cannot be compensated. This defines and active length of the link beyond that chromatic dispersion must be compensated with other active or passive systems.

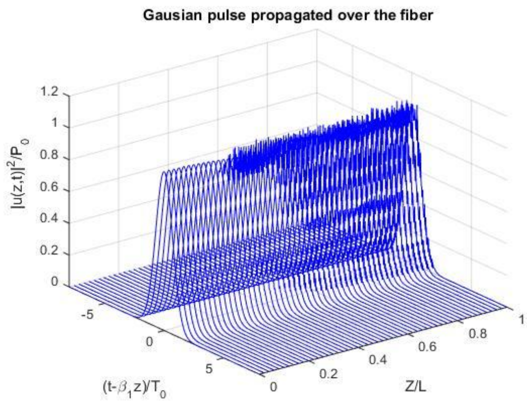

Figure 2. 3D presentation of the optical signal, correlated with distance, broadening and nominal amplitude of the signal.

5. CONCLUSIONS

We have demonstrated a way for simulating a chromatic dispersion in optical fiber using Matlab simulation. The Split Step Fourier Method is a good approximation of the Nonlinear effects in the optical fiber. The mathematical solution gives the possibility to simulate and evaluate the pulse broadening caused by chromatic dispersion in the optical fiber. Also the current mathematical solution gives the possibility to simulate high speed optical links and the performance in the optical fiber. This solution unlocks the possibility to simulate higher order modulation schemes and the impact to overall performance.

References

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