PHASE NOISE OF THE EFFECTIVE LOCAL OSCILLATOR WAVEFORM IN HARMONIC REJECTION MIXERS

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Abstract

In this paper the relationship between the phase noise of the effective local oscillator waveform (ELOW) of harmonic rejection mixers (HRMs) and the phase noise of their clock oscillators is investigated. HRMs and conventional mixers are compared with respect to the phase noise level of their ELOWs. It was found that HRMs and conventional mixers have nearly equal phase noise levels especially at low frequency offsets.

1. INTRODUCTION

The noise sources in the oscillator circuits cause short-term, random fluctuations in the amplitude and the phase of the generated waveforms. They are known as amplitude and phase noise respectively. However, practical oscillators have some inherent amplitude-limiting mechanism, which strongly suppresses the amplitude noise. Therefore, the phase noise (PN) is the dominating one [1]. The PN can be seen as a phase modulation of the generated waveform and therefore, produces corresponding noise sidebands.

The oscillator PN causes harmful effects in wireless communications, such as reciprocal mixing, caused by the local oscillator PN in receivers [2] and interference in neighboring channels caused by the transmitter PN sidebands [2]. Therefore, the PN level is among the most important characteristics of RF oscillators. Usually the PN level is characterized by the power spectral density (PSD) of the noise sidebands at a particular offset from the oscillation frequency, related to the carrier level and is measured in dBc/Hz.

In the last decade, harmonic rejection mixers (HRMs) have gained in popularity because they greatly relax preselect filtering requirements in wireless receivers. An HRM is a complex mixer, consisting of several parallelly operating elementary hard-switching mixers, driven by respective rectangular pulse trains, derived from a common clock oscillator [3]. An HRM can be seen as a single multiplier, multiplying the input RF signal by an *effective* local oscillator waveform (ELOW) from which some harmonics are excluded. Obviously, the ELOW has its own PN, caused by the clock oscillator PN and other noise sources in the HRM circuit. Numerous publications on HRMs appeared, but to the best our knowledge, the phase noise of the HRM ELOW has not yet been investigated.

In this paper the phase noise transfer mechanism from the clock oscillator to the ELOW is examined and the relationship between the corresponding PN PSDs is expressed. The PN contribution of the rest of the HRM circuit will be subject of further research.

Next, it is useful to compare HRMs with conventional mixers (CMs), i. e. mixers not having harmonic rejection with respect to PN. Nowadays most of the CMs are hard-switching [2]. Due to abrupt switching, the actual local oscillator waveform (which is nearly sinusoidal) and the waveform, which effectively multiplies the input RF signal, are different. Therefore, it is suitable to use the term ELOW for CMs also. The PN level of the HRM ELOW should be compared with the PN level of CM ELOW and not with the PN level of its local oscillator itself.

The rest of the paper is organized as follows: In the next section, some preparatory considerations regarding phase noise are made. In Sections 3 and 4, the phase noise transfer from the clock oscillator to the ELOW of HRMs and conventional mixers, is examined, respectively. The phase noise PSDs of the ELOWs are derived in Section 5 and HRMs are compared with CMs with respect to PN level.

2. PHASE NOISE OF NON-SINUSOIDAL WAVEFORMS

ELOWs are non-sinusoidal; therefore it is important to specify what we understand by PN of a non-sinusoidal waveform.

A sinusoidal oscillation with PN can be expressed as $s(t) = A\sin[2\pi f_0 t + \phi(t)]$, where $\phi(t)$ represents the PN. (It happened to be more convenient to use sine instead of the widely accepted cosine.) Taking into account that $\phi(t)$ is small, we obtain:

$$s(t) \approx A[\sin(2\pi f_0 t) + \phi(t)\cos(2\pi f_0 t)]. \quad (1)$$

Let us consider a periodic non-sinusoidal noiseless waveform expressed by:

$$\bar{s}(t) = A\sin(2\pi f_0 t) + C_0 + \sum_{n=2}^{\infty} \dot{C}_n \cos(2\pi n f_0 t) \quad (2)$$

where the fundamental is the component of interest. If the corresponding noisy waveform can be expressed as

$$s(t) = \bar{s}(t) + A\phi(t)\cos(2\pi f_0 t), \quad (3)$$

then $\phi(t)$ can be interpreted as phase noise of $s(t)$.

3. HRM PHASE NOISE DERIVATION

There are numerous HRM implementation alternatives. To be specific we will examine a passive current commutating HRM as in [3], but the results will be applicable for most HRMs. The conceptual diagram of the HRM is given in Fig. 1, which is self-explanatory.

The ELOW can be easily found if a DC voltage of 1 V is imaginarily applied to the RF input. Then the ELOW will appear at the output. The ELOW is by nature a sinusoid uniformly sampled at a rate of Nf_{LO}, where N is the number of commutations per LO cycle. Therefore, ELOW contains nonzero harmonics only at frequencies of $kNf_{LO} \pm 1$, where $k = 1, 2, \ldots$.

In practical implementations, in order to avoid large voltage swings at the transconductance amplifier (TCA) outputs, they should be switched to ground (GND) in the time intervals when they are not connected to the transimpedance amplifier (TIA) input.

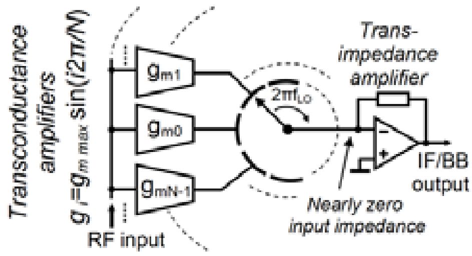

Figure 1. HRM operation principle

However, it is more rational to use the currently unneeded TCAs to form additional IF/baseband (BB) outputs. This makes possible the use of techniques for improved harmonic rejection, as in [3].

Naturally, the rotary switch is implemented by separate transistor switches. They are controlled by respective pulse trains V_{Ci}, derived by logic circuits from a clock oscillator (CO) running at $f_{CLK} = Nf_{LO}$.

The following assumptions were adopted:

1. MOS field-effect transistors (FETs) are used as switches. They operate in the deep triode region and their drain-source conductance is:

$g_{SW} = \beta(V_{DC} + V_G)$, where β is a MOSFET parameter, V_{DC} incorporates the FET threshold voltage and bias voltages, and V_G is the control voltage applied to the gate.

2. The duration t_{COM} of a commutation is at least an order of magnitude shorter than the clock period $T_{CLK} = 1/f_{CLK}$.

3. The phase disturbance $\phi(t)$ of the CO in nearly constant in time intervals $\Delta t \le \phi(t)/(2\pi f_{CLK})$

4. The time shifts $\tau = \phi(t)/(2\pi f_{CLK})$ are orders of magnitude shorter than the commutation interval.

5. Without loss of generality, a TIA gain $G_{TIA} = 1$ V/A and maximum a TCA transconductance $g_{m\max} = 1$ A/V can be assumed.

6. The edge forming of the pulses controlling the switches can be adequately modeled by passing of the clock oscillator sine wave via limiting amplifiers with unsaturated voltage gain G_V.

In the ideal case, taking into account Assumption 5, ELOW can be easily expressed as:

$$LO_{eff}(t) = v_{out}(t)/v_{RF}(t) = \sum_{i=-\infty}^{\infty} \sin\left(i\frac{2\pi}{N}\right)\mathrm{rect}\left(\frac{t - iT_{CLK}}{T_{CLK}}\right). \quad (4)$$

It can be established that the first harmonic amplitude of ELOW is $(N/\pi)\sin(\pi/N)$.

The clock oscillator phase noise can affect the ELOW only during the commutations. Outside the commutation intervals, the TCA outputs are firmly connected to the TIA inputs. This can be modeled by multiplying the phase noise by corresponding time shifted pulses $p(t)$ with unity amplitude and duration t_{COM}.

The clock oscillator waveform is given by

$$v_{OSC} = \sin(2\pi f_{CLK} t) + \phi_{OSC}(t)\cos(2\pi f_{CLK} t) \tag{5}$$

where $\phi_{OSC}(t)$ represents the CO phase noise.

In order to have a zero initial phase of the ELOW fundamental, the commutations take place around the time instants $iT_{CLK} + T_{CLK}/2$ (Fig. 2). Taking into account Assumption 2, the following approximation of (5) can be used in the commutation intervals:

$$v_{OSC} \approx -2\pi f_{CLK}(t - iT_{CLK} - T_{CLK}/2) - \phi_{OSC}(t). \tag{6}$$

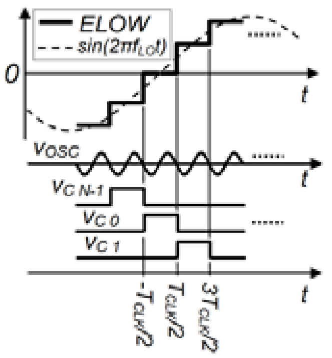

Figure 2. HRM timing

For the controlling pulses *in the commutation intervals* $v_{Ci} = \mp G_V v_{OSC}$ is fulfilled (Fig. 3). The sign alternates for rising and falling edges.

Let us consider the commutation of the j^{th} TIA input from the i^{th} to the $i+1^{st}$ TCA output. The corresponding HRM fragment with the four switches involved is given in Fig. 4. The switch conductances $g_{i,j+1}$ and $g_{i+1,j}$ increase from zero to their "on" values, and $g_{i,j}$ and $g_{i+1,j-1}$ decrease from their "on" values to zero. The commutation begins just when the conductance of the two initially open switches is no longer zero and ends when the conductance of the two initially closed switches becomes zero.

In the commutation intervals, V_C changes from $-V_{DC}/2$ to $V_{DC}/2$ for the closing switches and from $V_{DC}/2$ to $-V_{DC}/2$ for the opening switches, as can be deduced taking into account Assumption 1 (Fig. 3).

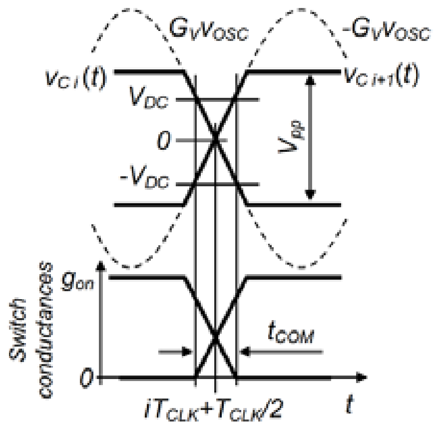

Figure 3. Switch conductances in the i-th commutation

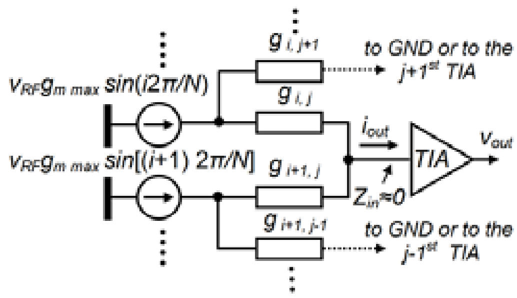

Figure 4. Commutation from the i-th to the i+1-st TCA output

For the considered commutation can be written:

$$\frac{i_{out}}{v_{RF}} = \sin\left(i\frac{2\pi}{N}\right)\frac{g_{SW\,i,j}}{g_{SW\,i,j} + g_{SW\,i,j+1}} + \sin\left[(i+1)\frac{2\pi}{N}\right]\frac{g_{SW\,i+1,j}}{g_{SW\,i+1,j} + g_{SW\,i+1,j-1}}. \tag{7}$$

Taking into account Assumption 5, we have $i_{out} = v_{out}/G_{TIA} = v_{out}$; therefore, (7) gives indeed v_{out}/v_{RF}, which is $LO_{eff}(t)$. Substituting in (7) the switch conductances according to Assumption 1, we obtain:

$$LO_{eff}(t) = \sin\left(i\frac{2\pi}{N}\right)\left(\frac{1}{2} + \frac{G_V v_{OSC}}{2V_{DC}}\right) + \sin\left[(i+1)\frac{2\pi}{N}\right]\left(\frac{1}{2} - \frac{G_V v_{OSC}}{2V_{DC}}\right) \tag{8}$$

After substituting of v_{OSC} in (8) and rearranging, we separate only the noise component $n_i(t)$ for the i^{th} commutation. We obtain the total ELOW noise $n(t)$ by summing all $n_i(t)$, multiplied by the corresponding pulses:

$$n(t) \approx \frac{G_V}{V_{DC}} \sin\left(\frac{\pi}{N}\right) \cos(2\pi f_{LO} t) \times \phi_{OSC}(t) \sum_{i=-\infty}^{\infty} p\left[t - \left(\frac{T_{CLK}}{2} + iT_{CLK}\right)\right] \quad (9)$$

After inspecting (9) and comparing it with the noise term in (3), we can recognize that the noise of ELOW is only a phase noise and it is the phase noise of the clock oscillator, non-ideally sampled at rate f_{CLK} and scaled by magnitude.

To obtain the ELOW phase noise $\phi_{ELOW}(t)$, (9) should be expressed in the form $n(t) = A\cos(2\pi f_{LO} t)\phi_{ELOW}(t)$,were $A = (N/\pi)\sin(\pi/N)$ is the fundamental amplitude of the ELOW. After multiplying and dividing (9) by (N/π), and realizing that $V_{DC} = G_V 2\pi f_{CLK} t_{COM}/2$, we obtain:

$$\phi_{ELOW}(t) \approx \frac{1}{N}\frac{T_{CLK}}{t_{COM}} \times \phi_{OSC}(t) \sum_{i=-\infty}^{\infty} p\left[t - \frac{T_{CLK}}{2} - iT_{CLK}\right] \quad (10)$$

Eq. (10) was verified by Matlab simulations.

A similar expression for $\phi_{ELOW}(t)$ was also derived after replacing Assumption 6 with the following assumption: The switching of the logic gates producing the control pulses is extremely abrupt and the CO PN affects the ELOW instantaneously only at its zero-crossing instants, shifting the ELOW transitions by $\phi(iT_{CLK} + T_{CLK}/2)/\omega_{CLK}$. In this case, the PSD of ϕ_{ELOW} for the frequencies of interest was the same as the PSD obtained here.

4. CONVENTIONAL MIXER PN

The equivalent circuit diagram of a conventional current commutating double balanced mixer is given in Fig. 5. Note that usually two commutations take place in one cycle of the local oscillator.

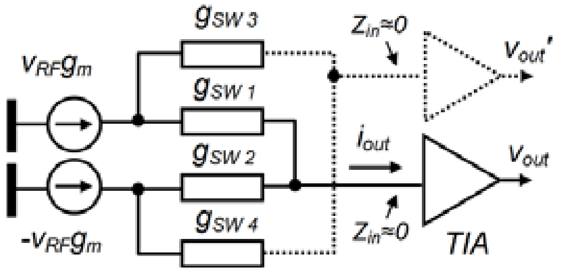

Figure 5. Equivalent circuit of a CM

Without loss of generality we assume $g_m = 1$ and $G_{TIA} = 1$. In a similar way as in the previous section we obtain for the ELOW in the commutation interval:

$$LO_{eff} = \frac{g_{SW1}}{g_{SW1} + g_{SW3}} - \frac{g_{SW2}}{g_{SW2} + g_{SW,4}} = \frac{G_V v_{OSC}}{V_{DC}}. \quad (11)$$

The noise in the ELOW is:

$$n(t) = \frac{G_V}{V_{DC}} \cos(2\pi f_{LO} t) \times \phi_{OSC}(t) \sum_{i=-\infty}^{\infty} p\left(t - i\frac{T_{LO}}{2}\right). \quad (12)$$

Taking into account that the amplitude of the first harmonic of the ELOW is $4/\pi$, we express (12) in the following form:

$$n(t) = \frac{4}{\pi} \cos(2\pi f_{LO} t)\phi_{ELOW}(t), \quad (13)$$

where

$$\phi_{ELOW}(t) = \frac{T_{LO}}{4t_p} \phi_{OSC}(t) \sum_{i=-\infty}^{\infty} p\left(t - i\frac{T_{LO}}{2}\right) \quad (14)$$

represents the phase noise of the CM ELOW It is the PN of the actual local oscillator, non-ideally sampled at rate $2f_{LO}$ and scaled by magnitude.

5. PHASE NOISE PSD

The PN PSD is found as

$$S_{HRM}(f) = \lim_{T\to\infty} \frac{\mathrm{E}\left[\left|\dot{\Phi}_{ELOW}(f)_T\right|^2\right]}{T} \quad (15)$$

where $\dot{\Phi}_{ELOW}(f)_T$ is the Fourier transform of $\phi_{ELOW}(t)$; the subscript T and $\mathrm{E}[X]$ denote a truncation to a finite time interval T and expectation, respectively [4].

$$\dot{\Phi}_{ELOW}(f)_T = \\ = \frac{1}{Nt_{COM}} \sum_{i=-\infty}^{\infty} \dot{P}(iNf_{LO}) \dot{\Phi}_{OSC}(f - iNf_{LO})_T \quad (16)$$

where $\dot{P}(f)$ and $\dot{\Phi}_{OSC}(f)_T$ are the Fourier transforms of $p(t)$ and $\phi_{OSC}(t)$, respectively. Strictly speaking, the CO PN is correlated in frequency domain, but this can be neglected in most cases. Then for the PN PSD, we obtain:

$$S_{HRM} = \left(\frac{1}{Nt_{COM}}\right)^2 \\ \times \sum_i P(iNf_{LO})^2 S_{OSC}(f - if_{CLK}) \quad (17)$$

where S_{OSC} denotes the PN PSD of the CO and $P(f) = |\dot{P}(f)|$.

For a given ELOW frequency, an HRM requires an oscillator whose frequency is N times higher than that one required by a conventional mixer. Under equal other conditions, the PN PSD of an oscillator running at an N times higher frequency, is expected to be nearly N^2 times higher, as can be deduced from [1]. Therefore, in order to make a fair comparison between HRMs and conventional mixers, we choose $S_{OSC} = N^2 S_{LO}$, where S_{LO} is the PN PSD of the conventional mixer local oscillator. Then

$$S_{HRM} = (1/t_{COM})^2 \\ \times \sum_i P^2(iNf_{LO}) S_{LO}(f - iNf_{LO}). \quad (18)$$

In a similar way, we obtain the PN PSD of the conventional mixer as:

$$S_{CM} = (1/4)(1/t_{COM})^2 \\ \times \sum_i P^2(i2f_{LO}) S_{LO}(f - i2f_{LO}) \quad (19)$$

In both cases, the PN PSD is a sum of scaled frequency shifted replicas of the oscillator PN PSD. In typical cases, the PN PSD of oscillators falls rapidly with the frequency offset, so the close-in PN PSD of the ELOW will be a scaled replica of the oscillator PN PSD. Then it can be seen that S_{HRM} is 4 times higher than S_{CM}.

At this point, work is almost done, but we need to know the PSD of the sidebands caused by PN rather than the PN PSD itself. It is commonly assumed that the PSD of the noise sidebands are a frequency translated and scaled by $1/4$ replica of the PN PSD. This is an acceptable approximation if the PN PSD is low enough at $f \geq f_0$. However, this is not the case when the PN is sampled. Indeed, the multiplication of the PN by the cosine (see Eq. 3) produces a sum of two PN spectrum replicas, shifted by $\pm f_{LO}$, or shifted by $2f_{LO}$ to each other. Since the PN in conventional mixers has a sampling rate of $2f_{LO}$, the PN spectrum has a periodicity of $2f_{LO}$. Therefore, the summation results in a doubled noise voltage or the resultant PSD becomes 4 times higher. In contrast, in HRMs the PN sampling frequency is Nf_{LO}, and the frequency shift by $2f_{LO}$ leads to summation of uncorrelated components. Therefore, the resultant PSD is a sum of two shifted replicas of S_{HRM}. In addition, for small offset frequencies, in practical cases, the PDS replica, which is shifted by $+f_{LO}$, dominates and practically fully determines the resultant PSD. Hence, the PN levels of HRMs and CMs are the same at relatively low frequency offsets. This statement was verified by Matlab simulations.

6. CONCLUSION

In general, the level of the PN transferred from the CO to HRM ELOWs is the same as in CMs. In contrary to some intuitive guesses, the CO PN causes only PN in HRM ELOW (and not amplitude noise or/and harmonic suppression degradation). The investigations confirmed the expectation that the ELOW PN is by nature sampled at a rate of f_{CLK} clock oscillator PN.

Further research should be done to evaluate the PN contribution of the rest of the HRM circuit. This, along with the results presented here, will allow us to decide conclusively whether the use of HRMs instead of CMs in wireless devices is at the expense of phase noise performance degradation.

References

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