Stable Output-Tracking Control of Interconnected Systems via Soft-Computing Approach: Lyapunov Method Is Crucial Tool

Tatjana D. Kolemisevska-Gugulovska, Georgi M. Dimirovski, and Mile J. Stankovski

Abstract—**Crucial role of Lyapunov second method in designing the output-tracking fuzzy adaptive control for a class of composite non-linear systems is studied. The realistic presumption that this class of composite plant systems can be represented solely by models on the grounds of its inputs and measurable outputs is observed. It is further assumed that controlled plants belong to a class of composite non-linear systems, and that its non-linear system functions can efficiently be approximated by fuzzy logic control to be designed using adjustable parameters. The crucial role of Lyapunov second method is demonstrated by the new theorem which guarantees the synthesized indirect fuzzy adaptive control does ensure stable output tracking.**

***Keywords*—Adaptive control, composite systems, fuzzy control, MIMO non-linear systems, Lyapunov design, output tracking.**

I. Introduction

THIS paper studies the design of a kind of fuzzy adaptive output-tracking control for a class of composite non-linear systems, i.e., a class of complex systems [1] - [2]. The composite system is characterized by unknown functions, and by presumption its model can only be represented by means of using inputs and outputs. In the control system synthesised, a novel parameter non-linearized fuzzy logic system that approximates the unknown plant functions is employed [6]. The stability property is ensured by means of the actually performed Lyapunov design synthesis [1]-[3].

It is known that during passed decades adaptive control theory has attracted great research attention by numerous researchers, and in due time has become a powerful methodology in resolving problems of feedback control of non-linear systems with parameter uncertainties. There exist considerable many math-analytical adaptive control algorithms for non-linear systems with unknown functions; for instance, see [2], [5]. Recently, on the other hand, fuzzy control has paved its way in industrial control systems [10], and considerable development of fuzzy adaptive control has also taken place [6]-[11]. On the grounds of the theory of universal fuzzy approximators [7], several methods of direct and indirect fuzzy adaptive control have been derived [3], [6]. Most of them, however, are applied to SISO system control problems [8]-[10], and only few are used for controlling complex or MIMO systems [10], [11]. In this paper we discuss a new kind of fuzzy adaptive control tailored to fit poorly known composite MIMO plants [1].

The paper is organized as follows. The preliminary analysis and problem statement are presented in Section II. In Section III thereafter, the novel design synthesis for the adaptive output tracking control problem and its stability issues are studied by means of fuzzy systems and Laypunov second method. The derived additional assumptions representing some realistic conditions on the system maps are also given along with the new theorem. Conclusion and references follow thereafter.

II. Problem Statement and Preliminary Analysis

A. System Representation Model

Consider a class of composite non-linear systems, which consist of p subsystems S_i, that is represented by the following set of differential equations $(i = 1,\cdots,p)$:

$$y_i^{(n_i-1)} = f_i(x_i) + g_i(x_i)u_i + m_i(x) \qquad (1)$$

In this representation model, vector-valued variable u_i denotes the control input of *i*-th subsystem, while variable $x_i = [y_i,, y_i^{(n_i-1)}]$ denotes the output of the *i*-th subsystem. Hence, $y_i \in R^{n_i}$ and $u_i \in R^{r_i}$, where n_i, r_i are positive integers. The vector $x = [x_1,\cdots,x_p]$ denotes the measurable outputs of the composite system.

B. Assumptions on System Representation and Objectives

In conjunction with the system class (1), the following realistic assumptions are satisfied. Functions $f_i(x_i)$ and $g_i(x_i)$ are assumed to be known. Function $m_i(x)$ is unknown. Functions $f_i(x_i)$, $g_i(x_i)$ and $m_i(x)$ are

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supposed to be smooth function. Thus basic consideration of plant systems with unknown models of system maps is fully consistent with the systems science setting, which is summarised in the illustration depicted in Figure 1, in terms of the input-output approach. The problem of concern is the design of controlling infrastructure so as to achieve stable output tracking of the given desired reference output.

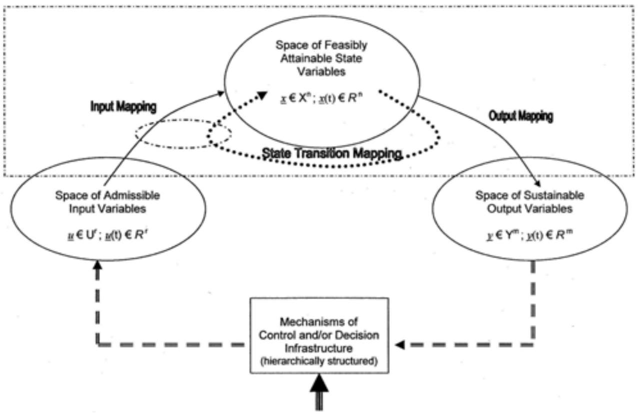

Fig. 1. The setting of control of general systems science setting in engineering terms [1], [4]; note the freedom to choose both mathematical representation formalism and design methodology and technique.

In here, the control design objective is to design a fuzzy adaptive controller for this poorly modelled class of composite systems such that:

(a) It overcomes the missing information on system functions by approximating them on the grounds of measurable inputs and outputs.

(b) It guarantees stable output tracking of a given required vector-valued reference signal $y_r = [y_{r1}, , y_{rp}]$ by the overall output vector of the plant system.

For this purpose, let take into consideration the set of the respective variables of the tracking errors as follows:

$$\begin{aligned} e_{i1} &= y_i - y_{ri}, \\ e_{i2} &= \dot{y}_i - \dot{y}_{ri}, \\ &\vdots \\ e_{in_i} &= y^{(n_i-1)}{}_i - y^{(n_i-1)}{}_{ri}, \\ e_i &= (e_{i1}, \ldots, e_{in_i})^T. \end{aligned} \tag{2}$$

Then one can derive that the system representation (1) can be expressed in terms of the error-tracking dynamics $(i = 1, \cdots, p)$ as follows

$$\dot{e}_i = A_i e_i + b_i \{ f_i(\cdot) + g_i(\cdot) u_i + m_i(x) - y_{ri}^{(n_i)} \}, \tag{3}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 \\ 0 & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix},$$

$$b_i = (0, 0, \cdots, 0, 1)^T.$$

In the next section, we present our design solution for the indirect adaptive controller of the considered class of non-linear composite systems and the relevant stability theorem.

III. Lyapunov Design of Fuzzy Adaptive Stabilising Tracking Control and Main Result

A. Some Preliminary Design Considerations

In the tracking problem of concern, observe carefully the above system representation. Then, let choose [1], [2] a set of matrices K_i such that matrices $A_{m_i} = A_i - b_i K_i$ are Hurwitz matrix. That is, all eigenvalues of the latter are in the left open plane of complex numbers. In turn, we can rewrite (3) as

$$\begin{aligned} \dot{e}_i = A_{mi} e + b_i \{ & f_i(\cdot) + g_i(\cdot) u_i + \\ & + m_i(x) + K_i e_i - y_{ri}^{(n)} \}. \end{aligned} \tag{4}$$

Secondly, since the system interconnection maps $m_i(x)$ are unknown these have to be approximated somehow in the course of control operation. Therefore, let suppose that a few fuzzy IF-THEN rules about the unknown function $m_i(x)$ are available as follows:

$R_{m_i}^{(r_i)}$: IF y_1 is $A_{1,1}^{r_i}$, …, $y_1^{(n_1-1)}$ is $A_{1,n_1}^{r_i}$, …, y_p is $A_{p,1}^{r_i}$, …, $y_p^{(n_p-1)}$ is $A_{p,n_p}^{r_i}$,

THEN $m_i(x)$ is $C_i^{r_i}$, $r_i = 1, \cdots, M_i$.

It should be noted some a-priory qualitative knowledge on the plant to be controlled is available always. Then, should we manage to synthesise a fuzzy logic system emulating $m_i(x/\theta_i)$ to approximate $m_i(x)$ (in fact, a set of approximators), the tracking control problem becomes tractable [1]. Then, we can make use of this fuzzy system in order to design the overall controller for the class (1).

B. On the Design of Approximating Fuzzy System

Following Wang [10] this fuzzy logic system actually performs a mapping of the type $U \subseteq R^{(p*n)} \to R$. Therefore, let take $U = U_{1,1} \times \cdots \times U_{n,p}$, $(i = 1, \cdots p$, $j = 1, \cdots n)$. In order to synthesise this novel fuzzy system within the controlling infrastructure so that enables the reduced set of fuzzy rules, we select the fuzzy model in which the parameters are not to be linearized. Its model representation can be given by the following formula [6]

$$m_i(x/\theta_i) = \frac{\sum_{l=1}^{M_i} \bar{y}_i^l \left[\prod_{i=1}^{p} \prod_{j=1}^{n_i} \exp\left(-\left(\frac{x_{i,j} - \bar{x}_{i,j}^l}{\sigma_{i,j}^l}\right)^2\right) \right]}{\sum_{l=1}^{M_i} \left[\prod_{i=1}^{p} \prod_{j=1}^{n_i} \exp\left(-\left(\frac{x_{i,j} - \bar{x}_{i,j}^l}{\sigma_{i,j}^l}\right)^2\right) \right]}, \quad (i = 1, \cdots, p) \tag{5}$$

where $\bar{y}^l, \bar{x}_{i,j}^l, \sigma_{i,j}^l$ are parameters. Quantity $x_{i,j}$ stands for $y_i^{(j-1)}$, and θ_i stands for the sum of adjustable parameters $\bar{y}^l, \bar{x}_{i,j}^l, \sigma_{i,j}^l$, while M_i is the number of fuzzy rules of *i*-th subsystem. Then, as in [6], we can take

$$\theta_i^* = \operatorname{argmin}_{\theta_i \in \Omega_i} \left[\sup_{x \in U_c} \left| m_i(x/\theta) - m_i(x) \right| \right]$$

where Ω_i is the obligation aggregation of θ_i. Note that it is needed θ_i to be bounded and $\sigma_{i,}^l$ to be positive. Hence, Ω_i is represented as follows:

$$\Omega_i = \left\{ \theta_i : |\theta_i| \le Ma_i, \sigma_{i,j}^l \ge \sigma_i \right\}.$$

Furthermore, we can take

$$U_c = U_{c,1,1} \times \cdots \times U_{c,i,j} \times \cdots \times U_{c,p,n_i}$$

be the obligation aggregation of the vector x, where $U_{c,i,j}$ is the obligation aggregation of $x_{i,j}$ [10]. With the choice of U_c we have the design of approximating fuzzy system at hand [3].

Also, let introduce the projection $\operatorname{Proj}(\Theta, \Phi)$ by means of the following definition

$$\operatorname{Proj}(\Theta, \Phi) = \begin{cases} \Phi - \dfrac{(\|\Phi\| - \beta)\Phi^{\mathrm{T}}\Theta}{\delta \|\Phi\|^2} \Phi, \\ \qquad \Phi^T \Theta \ge 0 \qquad if \qquad \|\Phi\|^2 = \beta, \\ \Theta, \; otherwise. \end{cases} \quad (6)$$

Note for the time being that the role and importance of this projection formula shall become clear in the next subsection.

B. On the Stable Output Tracking Control Design

Having derived the above results (4) and (5), we can rewrite equations (4) as

$$\begin{aligned} \dot{e}_i = A_{mi} e_i + b_i \{ f_i + g_i u_i + m_i(x/\theta_i^*) - \\ - y_{ri}^n + K_i e + w_i \} \end{aligned} \quad (7a)$$

where

$$w_i = m_i(x) - m_i(x/\theta_i^*), (i = 1, \cdots, p) \quad (7b)$$

represents the error that takes place when using the fuzzy system to approximate the unknown functions $m_i(x)$.

Now, with regard to Eqs. (7), let adopt a control law

$$u_{i1} = 1/g_i(-f_i + y_{ri}^{(n)} - Ke - m_i(x/\hat{\theta}_i)) \quad (8)$$

where $\hat{\theta}_i$ is the estimation of θ_i^*. Next, let substitute $u_i = u_{i1}$ into (7) and obtain

$$\dot{e}_i = A_{mi} e_i + b_i \{ m_i(x/\theta_i^*) - m(x/\hat{\theta}_i) + w_i \}. \quad (9)$$

By using Taylor's formula, one can derive that

$$\begin{aligned} m_i(x/\theta_i^*) - m_i(x/\hat{\theta}_i) = \tilde{\theta}_i^T \left(\frac{\partial m_i(x/\hat{\theta}_i)}{\partial \hat{\theta}_i} \right) + \\ + \mathrm{O}(\left| \tilde{\theta}_i \right|^2), \end{aligned} \quad (10)$$

where $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$. Therefore we can express Eq. (9) as follows:

$$\dot{e}_i = A_{mi} e_i + b_i \{ \tilde{\theta}_i^T \left(\frac{\partial m_i(x/\theta_i)}{\partial \theta_i} \right) + \mathrm{O}(\left| \tilde{\theta}_i \right|^2) + w_i \} =$$

$$= A_{mi} e_i + b_i \{ \tilde{\theta}_i^T \left(\frac{\partial m_i(x/\theta_i)}{\partial \theta_i} \right) + v_i \}, \quad (11)$$

$$v_i = w_i + \mathrm{O}\left(\left| \tilde{\theta}_i \right|^2 \right). \quad (12)$$

In addition, following the results of Wang [8], [9], it is pointed out that for the algorithm of $\dfrac{\partial m_i(x/\hat{\theta}_i)}{\partial \theta_i}$ we can derive the following set of equations:

$$\frac{\partial m_i(x/\hat{\theta}_i)}{\partial \bar{y}_i^l} = \frac{b_{mi}^l}{\sum_{l=1}^{M_i} b_{mi}^l}, \quad (16a)$$

$$\frac{\partial m_i(x/\hat{\theta}_i)}{\partial \bar{x}_{i,j}^l} = \frac{\bar{y}_i^l - m_i(x/\hat{\theta}_i)}{\sum_{l=1}^{M_i} b_{mi}^l} b_{mi} \frac{-2(x_{i,j} - \bar{x}_{i,j})}{(\sigma_{i,j})^2}, \quad (16b)$$

$$\frac{\partial m_i(x/\hat{\theta}_i)}{\partial \sigma_{i,j}^l} = \frac{\bar{y}_i^l - m_i(x/\hat{\theta}_i)}{\sum_{l=1}^{M_i} b_{mi}^l} b_{mi} \frac{-2(x_{i,j} - \bar{x}_{i,j})^2}{(\sigma_{i,j})^3}, \quad (16c)$$

where

$$b_{mi}^l = \prod_{i=1}^{p} \prod_{j=1}^{n_i} \exp\left(-\left(\frac{x_{i,j} - \bar{x}_{i,j}^l}{\sigma_{i,j}^l}\right)^2\right). \quad (16d)$$

C. The New Theoretical Result on System Stability

On the grounds of the above design considerations and the additional, but realistic, assumptions adopted in the course of Lyapunov stability synthesis, the main result is stated in terms of Theorem 1 below. In the Appendix, we give the expressions for the synthesised Lyapunov function and its first derivative along error state trajectories with respect to time, while the proof is found in [6]. Now, note first the set of these additional assumptions.

Assumption 1. All v_i $(i = 1, \cdots, p)$ are bounded, and $|v_i| \le M_{ei}$.

Assumption 2. $g_i(x_i) \neq 0$.

Assumption 3. y_i is known, and derivative of y_i, of all orders, are known.

Assumption 4. Equation $P_i A_m + A_m^T P_i + Q_i = 0$, in which Q_i is positive definite matrix, has the positive definite matrix solution P_i.

It is readily seen that these assumptions are realistic and do not impose some severe constraints on control design derived.

***Theorem 1*:** Consider system (1) along with Assumptions 1 to 4 satisfied and with the initial condition $\theta_i(0) \in \Omega_i$, and apply the designed controller given below

$$u_i = u_{i1} + u_{i2} \ , \tag{13}$$

$$u_{i1} = 1/g_i(-f_i + y_{ri}^{(n)} - Ke - m_i(x/\hat{\theta}_i)), \tag{14}$$

where $\hat{\theta}_i$ is the estimation of θ_i^*, and

$$u_{i2} = -\frac{1}{g_i} M_{ei} \operatorname{sgn}(e_i^T pb), \tag{15}$$

along with the adaptive approximation law

$$\dot{\hat{\theta}}_i = \Pr oj(\hat{\theta}_i, \eta_i P_i b_i \frac{\partial m_i(x/\hat{\theta}_i)}{\partial \hat{\theta}_i}), \ i = 1, \cdots, p \ . \tag{16}$$

Then the control system with the above composite fuzzy adaptive control law ensures that system output performs stable tracking of the given desired reference signal.

The enclosed appendix presents some detail of the appropriate Lyapunov function and first derivative with respect to time, which may well serve the purpose of a hint for proving the stability.

IV. Conclusion

In this paper we studied the output-tracking problem of a class of non-linear composite systems by applying a novel fuzzy adaptive control synthesised and Laypunov stability design. The designed overall composite controller guarantees stable adaptive output tracking.

Lyapunov stability based design was instrumental to achieve this stable tracking property of the overall system, hence the crucial role of Lyapunov's method. For, the problem solved is the one of non-linear control systems.

It should be also noted that the adaptive fuzzy system based algorithm has the advantage of reducing the number of fuzzy rules, which are needed within the controller's subsystem, which is also important in cases of when plants are complex and of high-order.

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Appendix

The Lyapunov function chosen is

$$V = \sum_{i=1}^{p} (\frac{1}{2} e_i^T P_i \ e_i + \frac{1}{2\eta i} \tilde{\theta}_i^T \ \tilde{\theta}_i),$$

where η_i is a positive real number, which cane be conveniently altered. It can be shown that its first time-derivative is given by the formula

$$\dot{V} = \sum_{i=1}^{p} \{ \tfrac{1}{2} e_i^T (A_{mi}^T P_i + P_i A_{mi}) e_i + (\tilde{\theta}_i^T \left(\frac{\partial m_i(x/\theta_i)}{\partial \theta_i} \right) +$$

$$+ u_{i2} + v_i)^T b_i^T P_i e_i + \tfrac{1}{\eta_i} \dot{\tilde{\theta}}_i^T \ \tilde{\theta}_i \} = \sum_{i=1}^{p} \{ -\tfrac{1}{2} e_i^T Q_i e_i +$$

$$\frac{1}{2\eta_1} \tilde{\theta}_i^T (\dot{\theta}_i + \eta_i e_i^T P_i b_i \left(\frac{\partial m_i(x/\theta_i)}{\partial \theta_i} \right)) +$$

$$+ (u_{i2} + v_i) b_i^T P_i e_i \} ,$$

and then shown to satisfy

$$\dot{V} \leq \sum_{i=1}^{p} -\tfrac{1}{2} e_i^T Q_i e_i \ \leq 0 \, .$$