The Probability Stability Estimation of Discrete Systems with Random Parameters

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Abstract - **The method for determining the probability stability of discrete systems whose parameters are random, normally distributed values is presented in this paper. The system with random parameters is often encountered in industry of plastic materials and rubber industry. Presented method enables the analysis of the system consisting of two and more random parameters.**

Keywords – **Probability, Discrete times systems, Multidimensional normal distribution.**

I. INTRODUCTION

In the process industry, the industry of chemical products and especially in industry of plastic materials treatment and the rubber industry, the systems in which some parameters couldn't be calculated and measured precisely are encountered. The reason is the fact that these parameters depend on values, which have stochastic character, such as plasticity, elasticity, intensity and compactness of material. The consequence of this is that the performances of the whole system differ from wanted, i.e., projective values. In certain cases, those deviations are so large that with no adjustment the system can be bringing into the normal working state. Because of that is necessary to estimate the influence of stochastic parameters on system performances in advance. Especially, this estimation is very important in relation to the system stability and the quality of system work.

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The basic methods for probability stability estimation are given in [1], [2], [3], [4]. These methods are connected to the continual systems of automatic control. In this paper the method for stability estimation of discrete systems will be presented. The discrete systems of the second and higher order will be discussed, also. Some theorems from the theory of the random processes [5], as the basic conditions of the discrete systems stability [6] are used. Let us notice that for the effective analysis of this problem the D - decomposition in the parametric space for discrete systems can be used.

II. THE DETERMINING OF PROBABILITY OF DISCRETE SYSTEMS STABILITY

Let the discrete system is given by:

$$\sum_{i=0}^{n} l_i \, x(k+n-i) = 0, \;\; l_0 = 1 \tag{1}$$

From the condition of the discrete system stability [6], the stability area of the system given by Eq. (1) in the parametric space is determined. It is important to determine the stability area in the following way. The z - transformation is applied to the system (1) and bilinear transformation, $z = \frac{s+1}{s-1}$, too. In that way, the stability area is determined similar as at continual systems (using Hurwitz criterion). Let the stability area of the system (1) in parametric space $l_1, \ldots, l_n$ is S_n.

Let the parameters $l_1, \ldots, l_n$ have normal distribution:

$$p_i = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(l_i - \bar{l}_i)^2}{2\sigma_i^2}} \tag{2}$$

where $\bar{l}_i$ are mathematical expectations and σ_i standard deviations.

If the parameters are independent random variables, then the multidimensional distribution density is given by:

$$p(l_1,\ldots,l_n)=\frac{1}{\left(\sqrt{2\pi}\right)^n\prod_{i=1}^{n}\sigma_i}e^{-\sum_{i=1}^{n}\frac{\left(l_i-\bar{l}_i\right)^2}{2\sigma_i^2}} \tag{3}$$

The probability stability of the system (1) whose parameters is random and has densities of probability distribution (2), [2], [3], is given by:

$$P=\int_{S_n}\cdots\int p(l_1,\ldots,l_n)\,dl_1\cdots dl_n \tag{4}$$

A. The Second Order Discrete System

Let the second order discrete system is given by:

$$x(k+2)+l_1x(k+1)+l_2x(k)=0 \tag{5}$$

Using the z - transformation, firstly, the following equation is obtained:

$$z^2+l_1z+l_2=0 \tag{6}$$

and then applying the bilinear transformation, $z=\frac{s+1}{s-1}$, the discrete system is transformed into the s - domain in the following way:

$$s^2(1+l_1+l_2)+2s(1-l_2)+1-l_1+l_2=0 \tag{7}$$

As it was said, it is necessary to determine the stability area of this system in the parametric plane. From the stability conditions, using, for example, the Hurwitz criterion, the stability area S_2 is given with the next set of relations:

$$\begin{aligned}&1-l_1+l_2\geq 0\\&1+l_1+l_2\geq 0\\&l_2\leq 1\end{aligned} \tag{8}$$

The stability area S_2 of the second order discrete system is given on Fig.1 where N_2 presents the non stability area.

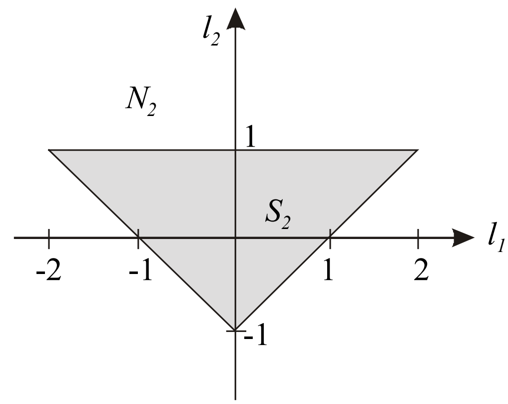

Fig.1. The stability area S_2 of the second order discrete system

In the case of the second order discrete system, the area stability is the triangle. Parameters are independent, random values and the distribution density is:

$$p(l_1,l_2)=\frac{1}{2\pi\sigma_1\sigma_2}e^{-\frac{\left(l_1-\bar{l}_1\right)^2}{2\sigma_1^2}-\frac{\left(l_2-\bar{l}_2\right)^2}{2\sigma_2^2}} \tag{9}$$

The probability of the second order discrete system stability is given by:

$$P=\iint_{S_2}p(l_1,l_2)\,dl_1dl_2 \tag{10}$$

This probability is determined using *MATHEMATICA*. Because of the complexity of calculating the integral, the exponential function is presented using the Taylor series of the ninth order:

$$e^x=\sum_{i=0}^{n-1}\frac{x^n}{n!}+R_n \tag{11}$$

where $R_n=\frac{x^ne^{\Theta x}}{n!}$ is the error, and $0<\Theta<1$.

For the optional values of mathematical expectations $\bar{l}_1=0.4$, $\bar{l}_2=0.5$ and standard deviations $\sigma_1=0.67$, $\sigma_2=0.47$, the probability stability of the second order discrete system is P=0.9377.

B. The Third Order Discrete System

Let the third order discrete system is given by:

$$x(k+3)+l_1x(k+2)+l_2x(k+1)+l_3x(k)=0 \tag{12}$$

The stability area is obtained in the same way as at the second order discrete system and is given by the following relations:

$$\begin{aligned} &l_1+l_2+l_3>-1\\ &l_1-l_2+l_3<1\\ &l_1l_3+1>l_2+l_3^2 \end{aligned} \tag{13}$$

This area is given on Fig.2.

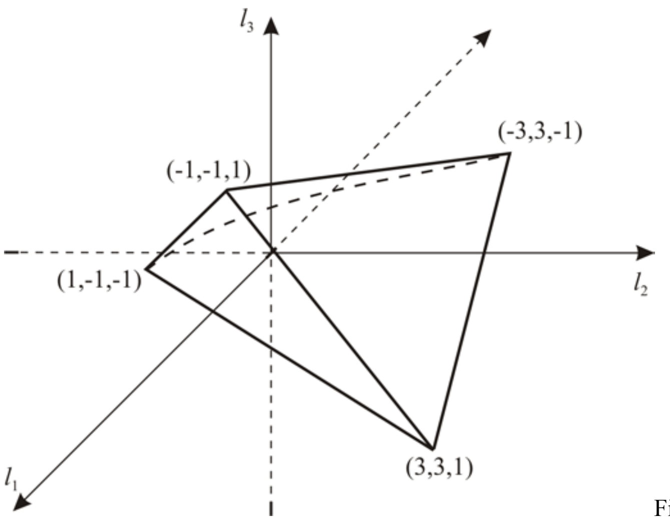

Fig. 2. The stability area S_3 of the third order discrete system

The probability of stability is determined by :

$$P=\iiint_{S_3} p(l_1,l_2,l_3)dl_1dl_2dl_3 \tag{14}$$

where:

$$p(l_1,l_2,l_3)=\frac{1}{2\pi\sigma}e^{-\frac{(l_1-\bar{l}_1)^2}{2\sigma_1^2}-\frac{(l_2-\bar{l}_2)^2}{2\sigma_2^2}-\frac{(l_3-\bar{l}_3)^2}{2\sigma_3^2}} \tag{15}$$

and $\sigma=\sigma_1\sigma_2\sigma_3$.

In the case when the stability area is determined in relation to more than two parameters, the problem is far too complex. Above all, the surfaces which present the area stability limits are usually expressed by very complicated mathematical relations. Besides, at the probability stability estimation of the system, it is necessary to integrate by the area S_n, which is analytically practically impossible in consideration to the complex functions of random parameters distribution densities.

In our case, for the optional values of mathematical expectations $\bar{l}_1=0.2,\ \bar{l}_2=0.3,\ \bar{l}_3=0.4$ and standard deviations $\sigma_1=0.7,\ \sigma_2=0.8,\ \sigma_3=0.9$, the probability stability of the second order discrete system is *P*=0.9456.

In the case of the n–th order system, the area stability is determined by known conditions for the stability of the discrete systems of the n-th order using bilinear transformation, also. In that way the closed body in the parametric plane of the n-th order, which includes the origin, is obtained. This body is very complicated so the calculation of the probability stability is very difficult. That is why, for the practical applications, is important to estimate the probability stability. For that purpose two theorems, [7], can be used.

The first theorem states that the stability area is included in hyper parallelepiped:

$$P_n \Leftrightarrow |l_i| \le \binom{n}{i}, \quad i=1,2,\ldots,n \tag{16}$$

The second theorem states that the stability area includes the simplex polyhedron:

$$O_n \Leftrightarrow |l_1|+|l_2|+\cdots+|l_n| \le 1 \tag{17}$$

Using these theorems, the probability stability can be estimated in the following way:

$$\iint_{O_n}\cdots\int \frac{1}{(\sqrt{2\pi})^n\prod_{i=1}^{n}\sigma_i} e^{-\sum_{i=1}^{n}\frac{(l_i-\bar{l}_i)^2}{2\sigma_i^2}} dl_1\cdots dl_n <$$

$$< P \le \iint_{P_n}\cdots\int \frac{1}{(\sqrt{2\pi})^n\prod_{i=1}^{n}\sigma_i} e^{-\sum_{i=1}^{n}\frac{(l_i-\bar{l}_i)^2}{2\sigma_i^2}} dl_1\cdots dl_n \tag{18}$$

The more precisely estimation can be performed if the limits of the stability area are approximated with hyper planes in the parametric space.

III. CONCLUSION

The method presented in this paper enables the probability stability estimation of the discrete system with more than two random parameters. For the systems with more random parameters, as for the systems for which the limits of the stability area in the parametric space can be hardly approximated with linear functions, probability of stability calculation is complex and the applications of computer is recommended. For its simplicity, this method can be used in practice, for example, in the industry of chemical products, industry of plastic materials treatment and the rubber industry where some technological parameters have stochastic character, i.e., they present random, normally distributed values.

REFERENCES

[1] Ajsagaliev S. A, Cerenskij G. S, "Ocenka verojatnosti ustojcivosti linejnih sistem so slucajnimi parametrami", *Tehniceskaja kibernetika*, 1981, No. 5, pp. 119-202.
[2] Mihajlicenko A. M, "K Vyboru optimaljnogo metoda ocenki kacestva sistemy pri dejstvii parametriceskih vozmoscenij", Institut matematiki an USSR, Kiev, 1989.
[3] B. Dankovic, " The probability stability estimation of the systems with more random parameters", *Hipnef*, pp. 300-307, 1988 (in serbian).
[4] B. Dankovic, M. Jevtic, "On the estimation of working capability of the automatic control system", *Hipnef*, Belgrade, pp. 233-238, 1990 (in serbian).
[5] J. Malisic, "Random processes", *Gradjevinska knjiga*, Belgrade, 1989 (in serbian).
[6] M. Stojic, "Digital control system ", *NAUKA*, Belgrade, 1990 (in serbian).
[7] B. Dankovic, B. Vidojkovic, "On the necessary conditions for the existence of limit sets in high order nonlinear discrete systems", *ETRAN*, Zlatibor, pp. 432-435, 1997 (in serbian).