Optimal Control of a Two-Wheeled Mobile Robot: Simulation for Selecting of the Motors

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Abstract— **This paper presents how a novel simulation package for optimal control based on dynamic programming can be used for selecting the drives once the constraints are known: range of speeds, trajectory (minimum radius for turns), load that will be carried by the mobile robot and its position on the platform (inertial properties of the mobile robot with the load). We calculate the necessary driving torques at the wheels of the mobile robot for various trajectories having a shape of the figure eight within a given time. The simulation uses fully customized dynamic model of the mobile robot that is propelled by two independent wheels and has third non-powered wheel that freely rotates around the vertical shaft to ensure three degrees of freedom. Dynamic programming and the discrete mathematic model allow simulation of the nonholonomic system. We presented in this paper only one possible application, that is, the analysis of three different loads carried along the same trajectory. The simulation clearly shows the relation between the tracking error and required diving torque; thereby, allow selection of the adequate driving motors for a given load and vice versa.**

Keywords— **Mobile robot, dynamic programing, optimal tracking**

I. Introduction

The design of a mobile robot propelled by two independent wheels is of interest for automation of production lines. A two-wheel-mobil robot is a nonholonomic system. Many studies considered the kinematics of mobile robots [1, 2, 3]. These studies provide important information for dynamic analysis and synthesis of controllers. It is known that stabilization of nonholonomic mobile robots is quite difficult [4, 5, 6]. The problem becomes even more complicated if one wants to eliminate the restriction of mobility to only equilibrium state [7].

In our earlier work we developed a simulation method for optimal control that relies on dynamic programming [8, 9, 10, 11]. This presentation is dedicated to show how this simulation method can be used for designing the drives upon the known range of speeds, trajectories (minimum radius), load that will be carried by the mobile robot, and needed velocity .We discuss here the necessary driving torques at the wheels of the mobile robot for various loads along the same trajectory that should be covered in a given time. The simulation uses the dynamic model of the mobile robot with two powered wheels, one free rotational wheel, platform, and the load that can be positioned at various posts at the platform. The discrete mathematical model and dynamic programming allow simulation of this nonholonomic system [8, 9]. The optimal control minimizes the cost function that is a sum of the LMS tracking error and the square of the actuation voltages at both wheels. We selected to analyze DC motors, yet the algorithm allows the analysis of other electrical motors. The simulation was used in an iterative procedure, that is, that is, the mass (load) on the platform was varied and tracking and driving torques calculated. . In this way it was possible to select the motors that secure tracking for a range of trajectories, loads positioned at different posts at the platform of the robot, and various maximum speeds. The analysis allows selection of the most appropriate components when designing the mobile robot.

II. Dynamic model of mobile robot

We start the presentation with the constraints that have been introduced in this study. We analyze a planar model of a mobile robot with two actuated wheels. The mobile robot shown in Fig. 1 is a typical example of a nonholonomic mechanical system. It consist of vehicle with two actuated wheels mounted on opposite sides of the platform, platform, and a front free running and rotating wheel. The motion and orientation are achieved by independent actuators, e.g., DC motors providing the necessary torques to the rear wheels. The motion can be described in Cartesian coordinates xOy, and the system has three degrees of freedom.

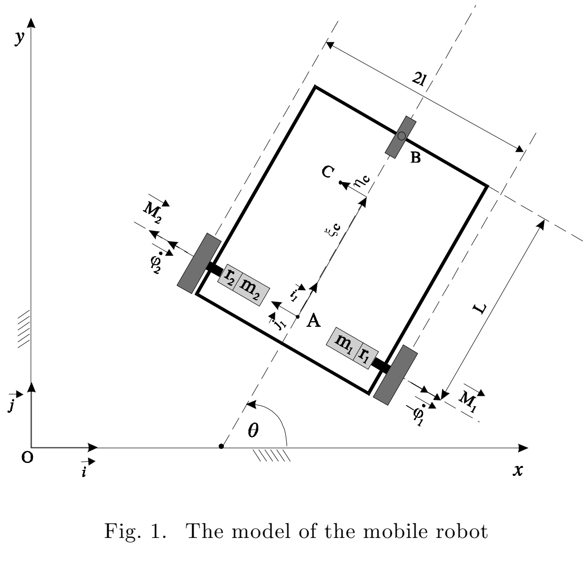

Fig. 1. The model of the mobile robot

The following notations are used (Fig. 1): $2l$ is the width of the mobile robot, a is the radius of the wheel, $O - \mathbf{ij}$ is the world coordinate system, and $A - \mathbf{i_1 j_1}$ is the coordinate

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system fixed to the mobile robot. A is the origin of the coordinate system A-$\mathbf{i_1 j_1}$ and the middle between the right and left driving wheels. The center of mass of the body and wheel with motor is C, which is distance from A description with ξ_c and η_c along $\mathbf{i_1}$ and $\mathbf{j_1}$ respectively.

The following other notations will be used in the derivation of the constraint equations and dynamic equations:

- $A(x, y)$: the intersection of the axis of symmetry with the driving wheel axis, where (x_d, y_d) is coordinates of point A in the inertia frame $O - \mathbf{ij}$;
- $C(\xi_c, \eta_c)$: the center of mass of the mobile robots,where (ξ_c, η_c) is coordinates of point C in the inertia frame A-$\mathbf{i_1 j_1}$;
- l: the distance between the driving wheels and the axis of symmetry;
- a: the radius of each driving wheel;
- m: the mass of the mobile robot;
- J_A: the moment of inertia of the mobile robot about vertical axis through A;

We considered $\dot{\varphi}$ as a control input and construct the control system for the following kinematic model[9]

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{a}{2}\cos\theta & \frac{a}{2}\cos\theta \\ \frac{a}{2}\sin\theta & \frac{a}{2}\sin\theta \\ \frac{a}{2l} & -\frac{a}{2l} \end{bmatrix}\begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} \tag{1}$$

where $\dot{\varphi}_1$ and $\dot{\varphi}_2$ represent the angular velocities of right and left wheel.

The Lagrange formalism was used to derive the dynamic equations of the mobile robot. Since the trajectory of the mobile base is constrained to the horizontal plane, i.e. since the system cannot change its vertical position, its potential energy U remains constant.

The following system of differential equations describes the dynamic [8], [9]:

$$\begin{gathered} C_1\ddot{\varphi}_1 + C_2\ddot{\varphi}_2 + C_3\left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\dot{\varphi}_2 = M_1 \\ C_4\ddot{\varphi}_1 + C_5\ddot{\varphi} - C_6\left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\dot{\varphi}_2 = M_2 \end{gathered} \tag{2}$$

where are M_1 and M_2 torques on the shafts right and left wheel respectively, and

$$\begin{gathered} C_1 = \frac{ma^2}{4}\left(1 - \frac{2\eta_c}{l} + \frac{J_A}{ml^2}\right); C_2 = \frac{ma^2}{4}\left(1 - \frac{J_A}{ml^2}\right) = C_4; \\ C_3 = \frac{ma^3}{4l^2}\xi_c = C_6;\; C_5 = \frac{ma^2}{4}\left(1 + \frac{2\eta_c}{l} + \frac{J_A}{ml^2}\right) \end{gathered}$$

We adopted for this study the dynamic model of the actuators in the form

$$\begin{gathered} \bar{M}_{m1} = A_1 u_1 - A_2\ddot{\varphi}_1 - A_3\dot{\varphi}_1 \\ \bar{M}_{m2} = A_1 u_2 - A_2\ddot{\varphi}_2 - A_3\dot{\varphi}_2 \end{gathered} \tag{3}$$

where are

$$A_1 = \frac{C_m N_M}{R_r};\; A_2 = J_m N_v N_M;\; A_3 = B_c - \frac{C_e N_v C_m N_M}{R_r}$$

The following notations apply for the Eq. (3):

- $u_{1,}u_2$ - voltages of the DC motors;
- C$_m$- torque constant;
- R$_r$- armature resistance;
- J$_m$- the moment of inertia of rotor DC motor's;
- B$_c$- the viscous friction coefficient;
- C$_e$- back emf-constant
- $\bar{M}_{m1}, \bar{M}_{m2}$- the total torque on the shaft of reduction gearheads
- N_v, N_M- the transmission ratio(velocity and torque)

After simple manipulation of the Eqs. (1) - (3), the differential equations get the following form :

$$\begin{gathered} C_1\ddot{\varphi}_1 + C_2\ddot{\varphi}_2 + C_3\left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\dot{\varphi}_2 = A_1 u_2 - A_2\ddot{\varphi}_2 - A_3\dot{\varphi}_2 \\ C_4\ddot{\varphi}_1 + C_5\ddot{\varphi}_2 - C_6\left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\dot{\varphi}_2 = A_1 u_2 - A_2\ddot{\varphi}_2 - A_3\dot{\varphi}_2 \end{gathered} \tag{4}$$

The vector of state variables is $x = (x_1, x_2, x_3, x_4)$, where $x_1 = \varphi_1, x_2 = \dot{\varphi}_1, x_3 = \varphi_2, x_4 = \dot{\varphi}_2$. By solving the system.(4) we can describe the dynamics of the mobile robot in the form[10], [11]:

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = P_2 + \sum_{j=1}^{2} G_{2j}u_j \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = P_4 + \sum_{j=1}^{2} G_{4j}u_j \end{gathered} \tag{5}$$

The terms $P_2, P_{4,}G_{2j}, G_{4j}, j = 1,2$ are nonlinear functions obtained as result of a series of linear transformations of the system.4 In order to chose an admissible control $\mathbf{u} = \mathbf{u(t)}$ in such a way that the actual trajectory $X = X(t)$ will be close as possible to desired trajectory $Z = Z(t)$, and constraining the voltages level of motors, we introduce the following *cost function* :

$$\begin{aligned} R(u) = \int_{t_0}^{t_0+T} & \{[x_1(t) - z_1(t)]^2 + [x_3(t) - z_3(t)]^2 \\ & + \lambda[u_1^2(t) + u_2^2(t)]\}dt \end{aligned} \tag{6}$$

III. Simulation results

In the simulation, the dynamic equations of the mobile robot derived in Section 2 were used to obtain the motion data of two wheels at all times. The parameters used for simulation are presented in Tables I and II. For the simulation we used the parameters provided by Faulhaber DC-Micromotors 3557-012 CS. The simulation was performed by using the MatLab and Simluink.

Table I.
Parameter values of the vehicle

Explanation	Notation	Value
mass of the platform	m	$5 \div 100kg$
length of the platform	L	$1m$
distance between wheels and the axis of symmetry	l	$0.5m$
radius of driving wheel	a	$0.1m$
coordinate center of mass	ξ_c	$-0.3 \div 0.3m$
coordinate center of mass	η_c	$0 \div 0.7m$

TABLE.II.
PARAMETER VALUES OF THE DC MOTORS

Explanation	Notation	Value
nominal voltage	U_n	12 (V)
recommended speed	n_e	$\leq$5000 (rpm)
recommended torque	M_e	$\leq$50(mNm)
terminal resistance	R	1,34$(\pm 12\%)$ (Ω)
back EMF constant	C_e	0,02.076(mV/rpm)
torque constant	C_M	19.82(mNm/A)
rotor inertia	J_M	4,693 10^{-6} (kgm^2)
weight	m_m	270(g)
angular acceleration,max	$\beta_{\max}$	74 10^{-3} $(rad\ s^{-2})$
reduction ratio	N_v, N_M	159

Fig. 2 illustrates the test path. An “8”¨-shaped trajectory was employed, where c is a constant relating the velocity and $2a$ and b represent the long and short axis, respectively. Accordingly, the desired angular position for the driving wheel at every sampling time can be computed by the path generator using the kinematic equations related to the structure of driving wheels of mobile robots. In the simulation, $a = 1m$, $b = 0.4m$ and $c = 0.02$.

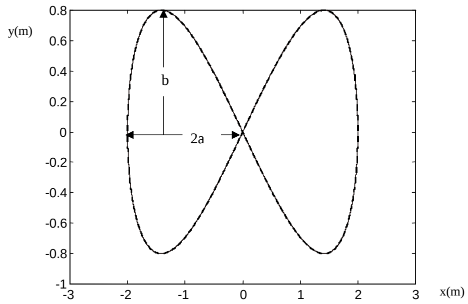


Fig. 2. Test path (full line) and actual path (dashed line)

The trajectory is defined by: $x = 2a\cos(c\omega t)$ and $y = b\sin(2c\omega t)$, , where $\omega = 2\pi/cT$, and t belongs to the interval $0 - T$, $T = 60s$. This 60 seconds interval is the time during which the mobile robot should complete a full trajectory. The dashed line in Fig. 2 shows the actual path obtained from simulation.

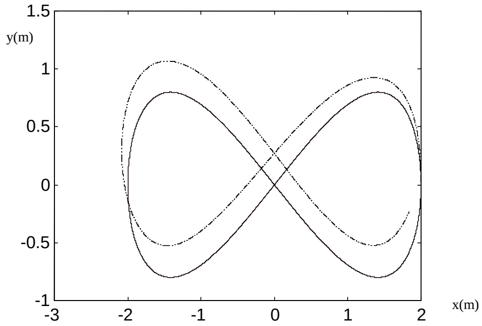
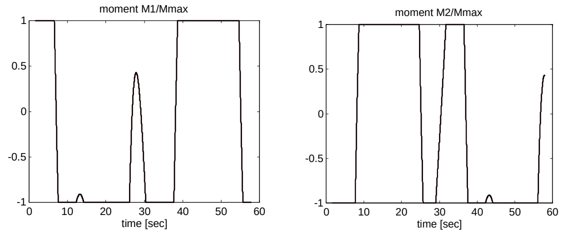


Fig. 3. Desired trajectory (full line) and actual path (dashed line) in the case m=50kg



(a) (b)

Fig. 4. Normalized driving torques in the case m=50kg

Full line in Fig. 3 shows the desired trajectory, and the dashed line the path obtained when optimal control was applied for the nominal loading of the platform with the mass $m = 50\ kg$.

Fig. 4 presents the normalized driving torques during the described motion.

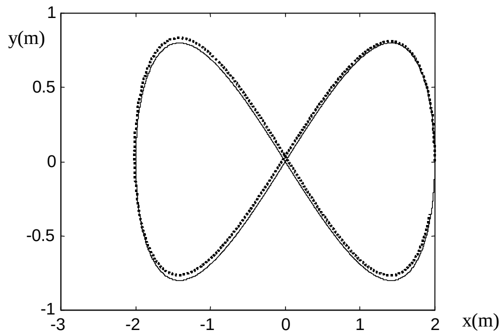
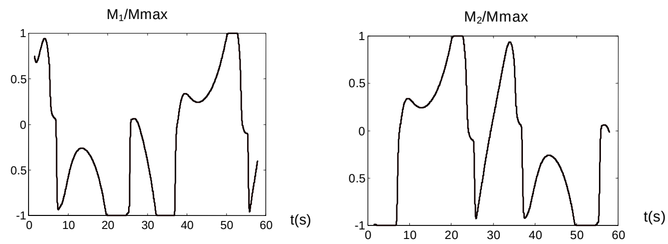


Fig. 5. Desired path (full line) and actual path (dashed line) during the described motion in the case m = 15 kg



(a) (b)

Fig. 6. Normalized driving torques in the case m=15kg

Fig. 5 shows the desired trajectory and superimposed path obtained with optimal control for the case where the loading was decreased form 50 kg to mass of $m = 15\ kg$. Fig. 6 presents the normalized driving torques during the described motion in this case..

Finally in Figs. 7 to 8 the desired trajectory, realized path and normalized torques are for the loading of platform with the mass $m = 8\ kg$.

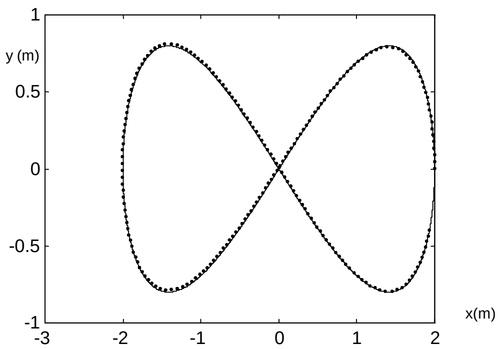
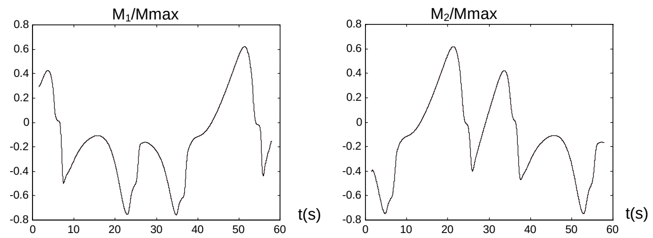


Fig. 7. Desired path (full line) and actual path (dashed line) during the described motion in the case m=8 kg



(a) (b)

Fig. 8. Normalized driving torques in the case m=8kg

IV. Discussion and conclusion

We presented results for three different loads (Figs. 3 - 8). There is a characteristic discrepancy between the presented Figures. In the latest set (Figs. 7 - 8) when the smallest load (8 kg) positioned at the platform was analyzed the tracking error was very small, and the normalized torques were within the allowed limits, that is, achievable by the motors used in simulation. In the second set, when the load was 15 kg, the error was substantially bigger yet tolerable, and the driving torques occasionally reached the allowed maximum. This suggests that this load, at the selected trajectory and selected speed (determined with the interval T = 60 seconds in this case) was still achievable. The first presented case (Figs 3 - 4) demonstrates that the error was bigger compared to the later two, and that the motors were basically running at maximum, still not providing adequate power. This suggests, that for a load of 50 kg other more powerful motors must be selected in order to ensure good tracking. This analysis can be repeated for other trajectories, as well as other speeds (by changing the time T).

The simulation runs very fast (10 seconds) on a PC computer with 400MHz and 32Mb of RAM memory. This allows that a series of simulation can be performed and data base formed that defines the limitations and requirements for selection of drives for a mobile robots. The specific value of the simulation is that the dialog windows within Simulink allow direct change mass, position of the load, dimensions of the mobile robot, and all other parameters. The simulation allows in addition the analysis of the variation of the orientation of the mobile robot with respect to the desired orientation pointing tangential to the trajectory. This might be of interest for analysis of the space needed for the mobile robot when operating in constrained space.

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