Continuous-Time Sliding Mode Based Minimum Variance Control - A Chattering Alleviation

Darko Mitic[1]

Abstract - **A combination of a minimum variance control and a variable structure control is considered in this paper. The main result of such an approach in the control design is the alleviation of the chattering, the undesired phenomenon inherent in variable structure systems with sliding mode, arising when the unmodeled system dynamics is excited by a high frequency discontinuous control signal. The proposed control law is verified by means of digital simulation results.**

Keywords – **variable structure control, sliding mode, minimum variance control, chattering reduction**

I. INTRODUCTION

The variable structure systems (VSSs) are the well known and the well studied class of the nonlinear control systems [1,2,3]. They are characterized by a control signal which switches when a system state crosses the hypersurface *a priori* defined by the rule known as the switching function. Sliding mode is of particular interest in VSSs. It occurs when the control forces the system state to move along the hypersurface described by the equation obtained when the switching function is equalized with zero. Thereby, the system dynamics is utterly defined by the equation of the system motion in the sliding mode, which is, by the way, of the lower order, and the system becomes robust to the parameter perturbations and the external disturbances. Due to the presence of the system nonidealities (hysteresis, time delays and small time constants), the switching frequency of the control signal is not infinite, so that the ideal sliding mode does not exist in VSS. The result is the so-called *zig-zag* motion of the system state in the vicinity of a sliding hypersurface, which produces the high frequency component of the control signal. Such the control signal excites the unmodeled system dynamics and generates the chattering. In the last years, the significant attention is dedicated to the reduction [4,5] and to the elimination [6,7] of this unwished phenomenon in VSSs with sliding mode.

The minimum variance (MV) control is also the well known control method which ensures the minimum variance of the variable representing the system coordinate or the function of the system coordinates. In the deterministic case, it means that the zero value of the variable should be provided by the MV control. However, if the system parameters vary, the MV control should be used both with the known parameter identification methods, while in the case when the external disturbances act on the system, it is recommended to introduce the integral term into the MV control [8].

[1]Darko Mitic is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, E-mail: darkom@elfak.ni.ac.yu

The combination of the sliding mode and the MV control endeavor to renounce the shortcomings possessed by each one. Namely, the introduction of the SM control component in the existing MV control rejects the need for the implementation of the parameter identification algorithms, as well as for the introduction of the integral term in order to eliminate the constant and the slow-varying external disturbances.

On the other hand, the component of the sliding mode control enables the application of variable structure control to the minimum phase plants (the plants whose transfer functions have stable zeros), and, as we will see later, the alleviation of the chattering.

The paper is organized as follows. In Section II, the problem to be solved is settled. Section III gives the synthesis of the proposed control law, the stability proof, as well as the recommendation for the choice of the controller parameters in order to reduce chattering. The digital simulation results are presented and discussed in Section IV. Section V contains the conclusive remarks.

II. PROBLEM STATEMENT

Let the single-input-single-output plant be described by the continuous-time analog of the discrete-time ML model [9]:

$$y(t) = \frac{B(s)}{A(s)}u(t) + \frac{D(s)}{A(s)}f(t), \qquad (1)$$

where $A(s)$, $B(s)$ and $D(s)$ are the polynomials of n-th, m-th and k-th order, respectively, $y(t)$ is an output signal, $u(t)$ is a plant input and $f(t)$ is a disturbance representing, for the sake of simplicity, both the parameter perturbations and the external disturbances acting on the input and the output of a plant. s is the differential operator i.e. $s \equiv \mathrm{d}/\mathrm{d}t$.

The goal is to design the control $u(t)$ which ensures the minimum variance, i.e. the zero value in the deterministic case, of the system state coordinates function:

$$g(t) = \frac{C(s)}{P(s)}\left(y(t) - r(t)\right), \qquad (2)$$

which, as we will see later, represents the switching function of the sliding mode control. The polynomial C*(s)* is Hurwitz, whilst $P(s) = p_1 s + p_0$, $p_0, p_1 > 0$. *r(t)* is a reference input signal. Note that if $g(t) = 0$ is provided, the error signal $e(t) = y(t) - r(t)$ asymptotically tends to zero regardless of the parameter perturbations and the external disturbances.

III. CONTROL DESIGN

In order to attain the zero value of the function $g(t)$, the control law is chosen in the following form:

$$u(t) = -\frac{1}{E(s)B(s)}\left[F(s)y(t) - C(s)r(t) + \frac{P(s)}{s}u_{sm}(t)\right], \quad (3)$$

where $E(s)$ and $F(s)$ are the polynomials with the coefficients obtained as the solutions of:

$$E(s)A(s) + F(s) = C(s). \quad (4)$$

The polynomial $F(s)$ is of the (n-1)-th order. $u_{sm}(t)$ is the sliding mode control component given in the relay form as:

$$u_{sm}(t) = K\,\mathrm{sgn}(g(t)). \quad (5)$$

Note that if $u_{sm}(t)=0$ and $P(s)=1$, the control law (3) represents the traditional MV control. With $P(s)=1$ and sliding mode control component (5), Eq. (3) represents the continuous-time sliding mode based MV control free of chattering, respecting that the integral of high-frequency discontinuous signal is continuous one, what as the result has the smooth control which does not induce the chattering phenomenon [7]. The similar approach is given in [10]. However, since the polynomial $C(s)$ is of the n-th order, we need the n–th time-derivative of the output signal $y(t)$, which is not available, in order to calculate the switching function. It can be estimated by using the available system state coordinates obtained by direct measuring or by using an observer. Unfortunately, the n-th time-derivative of $y(t)$ depends then on the plant parameters. When the plant parameters change, the error appears in the estimation of the unavailable system state. The practical implementation of the control algorithm [11] shows that this error can be neglected if the most significant coefficient of the polynomial $C(s)$ is chosen to be as small as possible. The mentioned approach is also possible when the n-th time-derivative is available, and that is the case when we can omit the fast system dynamics and use the system model of the lower order (i.e. the model of *dc* motor with the neglected electrical time constant)

The implementation of traditional MV control demands the plant of the relative order one. As it is necessary to evaluate the switching function (2) in the process of control design, i.e. to measure or to observe system state coordinates, the latter limitation is not important, since the first term in (3) can be obtained by virtue of the estimated time-derivatives of the system output. That means that the relative order of the plant could be greater than one. Moreover, in order to realize the control (3), the presumption is committed that the higher time-derivatives of the reference input signal are known.

If $P(s) = p_1 s + p_0$, $p_0, p_1 > 0$, the switching function (2) can be formed since all coordinates necessary for its realization are available. Dividing $C(s)$ by $P(s)$ yields the (n-1)-th order polynomial and the first order transfer function. Hence, in order to form the switching function (2) we utilize n-1 time-derivatives of the output signal $y(t)$ and the reference input signal $r(t)$, as well as the coordinate representing the output of the filter $1/P(s)$ with the error signal $e(t) = y(t) - r(t)$ as its input. Unfortunately, we can not speak about the control free of chattering, but only about the control with substantial reduction of the chattering

The next Theorem gives the stability conditions of the system with the proposed control algorithm.

Theorem: The system (1) with the sliding mode based MV control (2)-(5) is asymptotically stable if the inequality:

$$K > F_{\max}, \quad (6)$$

is valid where $F_{\max} = \max\left|\frac{E(s)D(s)}{P(s)}\dot{f}(t)\right|$.

Proof: By substituting (3) in (1), taking into account (4), (5) and (2), the following equation can be easily obtained:

$$\dot{g}(t) = -K\,\mathrm{sgn}(g(t)) + \frac{E(s)D(s)}{P(s)}\dot{f}(t). \quad (7)$$

Since the sufficient reaching and existence conditions of the sliding mode defined by $g(t)\dot{g}(t) < 0$ are satisfied if the inequality (6) is fulfilled and $g(t) = 0$ is in sliding mode, on the basis of (2) we have that $y(t)$ tends to $r(t)$ asymptotically, and consequently, the system is asymptotically stable. □

The chattering alleviation by the proposed control algorithm is done by the proper choice of the coefficient p_1 and p_0 of the polynomial $P(s)$. As $P(s)/s$ represents the well known transfer function of the proportional-plus-integral control law, by choosing the coefficient p_1 to be as small as possible, the integral term of $P(s)/s$ becomes dominant, resulting in the continuous control signal with the superposed high frequency signal of the small amplitude. Theoretically speaking, if $p_1 \to 0$ the system with the continuous-time sliding mode based MV control becomes free of chattering.

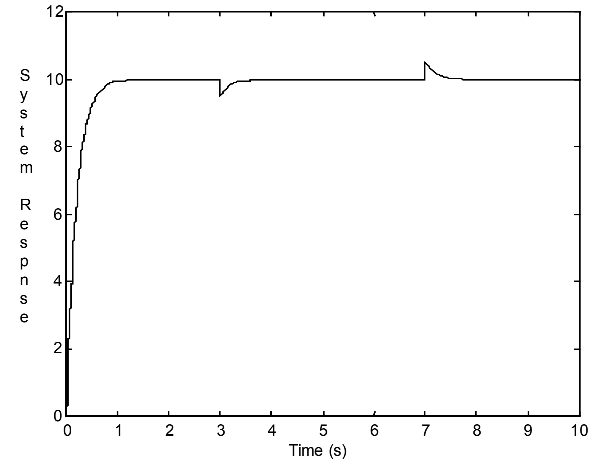
IV. Digital Simulation Results

The good characteristics of the proposed control law are verified by means of the digital simulation results. The polynomials of the plant model are chosen in the following manner: $A(s) = s(s+14)$, $B(s) = 595$ and $D(s) = 1$ ($D(s) = B(s)$), when the external disturbance acts on the plant output (input). The switching function $g(t)$ is formed by the use of the following polynomials $C(s) = 0{,}001s^2 + 0{,}2s + 1$ and $P(s) = 0{,}001s + 0{,}15$. On the basis of the chosen polynomials $A(s)$ and $C(s)$, the polynomials $E(s)$ and $F(s)$ become: $E(s) = 0{,}001$ and $F(s) = 0{,}186s + 1$. The gain of the sliding mode control component is K=1. The external disturbance is defined by $0{,}5(h(t-3) - h(t-7))$.

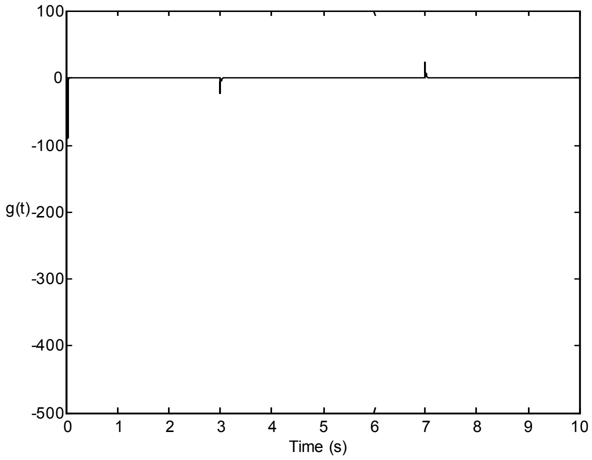
In Figs. 1(a), 1(b) and 1(c), the system response with the elimination of the disturbance acting on the plant output, the switching function $g(t)$ response and the control signal u(t) are shown, respectively. Likewise, the above mentioned responses are given in the same order in Figs 2(a), 2(b) and 2(c), but for the case when the external disturbance acts on the plant input. We can see in Figs 1(a) and 2(a) that the elimination of the disturbances is complete i.e. that the zero offset of the system output is provided in the steady-state. The chattering alleviation is evident since the control signals, presented in Figs 1(c) and 2(c), are the smooth functions of time.

V. Conclusion

The combination of the traditional minimum variance (MV) control and variable structure control (VSC) with sliding mode, which is the subject of the analysis and the design in this paper, ensures, above all, the elimination of the main drawbacks of the each control method - the implementation of the self-tuning algorithms and the introduction of an additional integrator in order to eliminate the disturbances in the case with MV control, or, the difficulties arisen in the implementation of VSC with sliding mode on the minimum phase plants. However, the main advantage of such combination is the chattering alleviation, which is achieved by filtering the relay sliding mode control component through the proportional-plus-integral term, included in the control law



(a)



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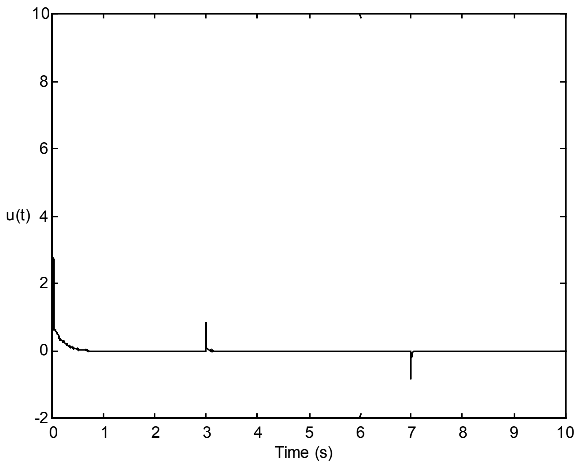
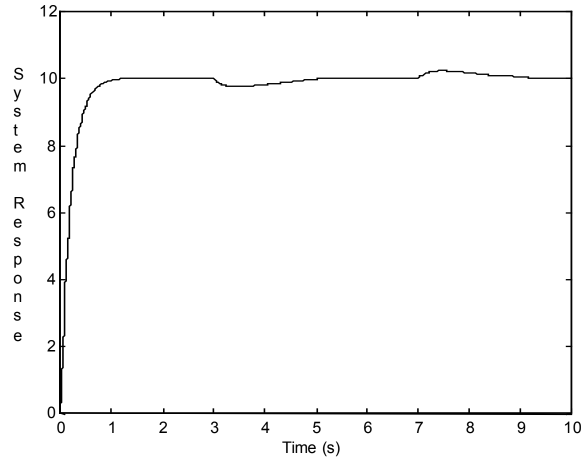
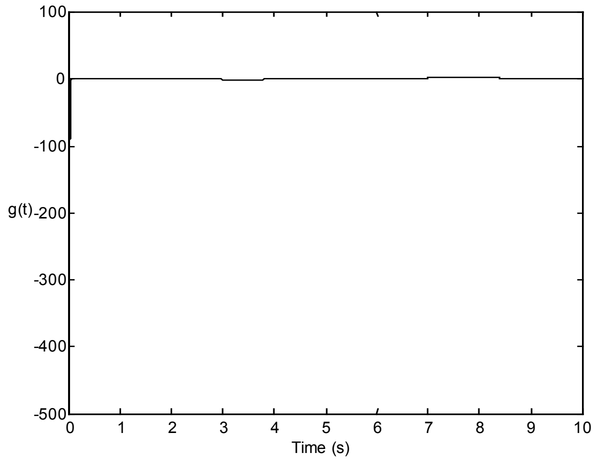


(c)

Fig. 1 The system response and the disturbance elimination (a), the switching function response g(t) (b) and the control u(t) (c). (the case with the disturbance acting on the plant output)



(a)



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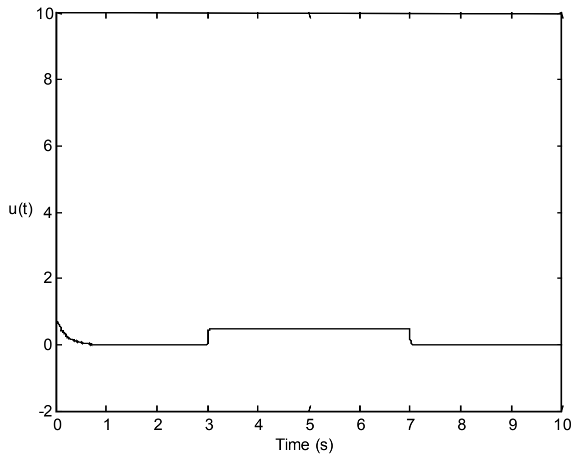

(c)

Fig. 2 The system response and the disturbance elimination (a), the switching function response g(t) (b) and the control u(t) (c). (the case with the disturbance acting on the plant input)

By choosing the proportional component of proportional-plus-integral term to be as small as possible, the proportional-plus-integral term tends to the pure integrator, resulting in significantly reduced chattering.

Nevertheless, all good characteristics of the sliding mode control are preserved, including the predefined system dynamics in the sliding mode which does not depend on the plant parameters, as well as the system robustness to the external disturbances.

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