"Natural" Iterative Learning Control for Uncertain LTV Dynamic System

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Abstract – **The paper presents a new algorithm for iterative learning control (ILC) called "natural" ILC. NILC is developed on the basis of biological analog - principle of self-adaptability in feedback configuration. Sufficient conditions for the convergence of a new type of learning control algorithm for a class of uncertain linear time-varying system- are presented.**

Keywords – **learning control, iterative, principle of self-adaptability,**

I. INTRODUCTION

In recent years, there has been a great deal of study to overcome limitations of conventional controllers against uncertainty due to inaccurate modeling and/or parameter variations. As one of alternatives, the iterative learning control (ILC) method has been developed [1],[2]. The common observation that human beings can learn perfect skills trough repeated trials motivations the idea of iterative learning control for systems performing repetitive tasks. The learning control concept differs from conventional control methodologies in that the control input can be appropriately adjusted to improve its future performance by learning from the past experimental information as the operation is repeated. Therefore, iterative learning control requires less *a priori* knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control. ILC is a technique to control systems operating in a repetitive mode with the additional requirement that a specified output trajectory $y_d(t)$ in an interval $[0,T]$ be followed to a high precision and in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. Examples for such systems are more generally the class of repetitive tracking systems, such as process plants , robotic systems and etc. Iterative learning control is found to be a good alternative, especially when detailed knowledge about the plant is not available.

II. PRELIMINARIES

Here, it is considered a class of repetitive uncertain linear time-varying system described in the form of state space and output equations.

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$$\begin{aligned} \dot{x}_i(t) &= A(t)x_i(t) + B(t)u_i(t) + w_i(t) \\ y_i(t) &= C(t)x_i(t) + v_i(t) \end{aligned} \tag{1}$$

In these equations t denotes time, $t \in [0,T]$, $t \in \Re$, where T presents terminal time which is known; x_i the state vector, $x_i \in \Re^n$, u_i the control vector, $u_i \in \Re^m$, y_i the output vector of the system, $y_i(t) \in R^r$ and i denotes the i-th repetitive operation of the system, $w_i(t), v_i(t)$ are uncertainties or disturbances to the system. $A(t), B(t)$ and $C(t)$ are matrices with appropriate dimensions. Let $x_d(t)$ be the desired state trajectory and $y_d(t)$ be the corresponding output trajectory. Assume that $y_d(t), x_d(t)$ are continuosly differentiable on $[0,T]$.

For later using in proving the convergence of proposed learning control, the following norms are introduced [3] for n-dimensional Euclidean space R^n:

- the Euclidean norm as

$$\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2} \quad x = [x_1, x_2, \ldots x_n]^T \tag{2a}$$

-the sub-norm

$$\|x\|_\infty = \sup_{1 \le i \le n} |x_i| \quad x = [x_1, x_2, \ldots x_n]^T \tag{2b}$$

- the matrix norm as

$$\|A\|_\infty = \max_{1 \le i \le m} \left(\sum_{j=1}^{n} |g_{i,j}| \right), A = [a_{i,j}]_{mxn} \tag{2c}$$

- the λ-norm for a real function:

$$h(t), (t \in [0,T]), h : [0,T] \to \Re^n \quad \text{as} \tag{2d}$$

$$\|h(t)\|_\lambda = \sup_{t \in [0,T]} e^{-\lambda t} \|h(t)\|_\infty, \lambda > 0$$

Motivated by human learning, the basic idea of iterative learning control is to use information from previous executions of the task in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. The learning controller for generating the present control input is based on the previous control history and a learning mechanism. Task is synthesis control $u(t)$ applying learning concept. For given output trajectory $y_d(t)$, the control objective is to find a control input $u_i(t)$ such that when $i \to \infty$, the system output $y_i(t)$ will track the desired output trajectory as close as possible. Learning control law based on iterative learning can be found in literature in following manner [3],[4]:

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) \quad (3)$$

and Γ is gain matrices appropriate dimensions. Also tracking error and their first derivative are defined as:

$$\begin{aligned} e_i(t) &= y_d(t) - y_i(t) \\ \dot{e}_i(t) &= de_i(t)/dt = \dot{y}_d(t) - \dot{y}_i(t) \end{aligned} \quad (4)$$

III. MAIN RESULT

In this paper, it is suggested a new algorithm for iterative learning control which differs from existing learning algorithms. For improving the properties of tracking as well as speed of convergence and especially control a process plant with disturbances, uncertainties and initialization errors it is proposed applying biological analog - principle of self-adaptability [5] which introduce local positive feedback on control with great amplifying. In the simplest case learning control law can be shown such as (Fig.1):

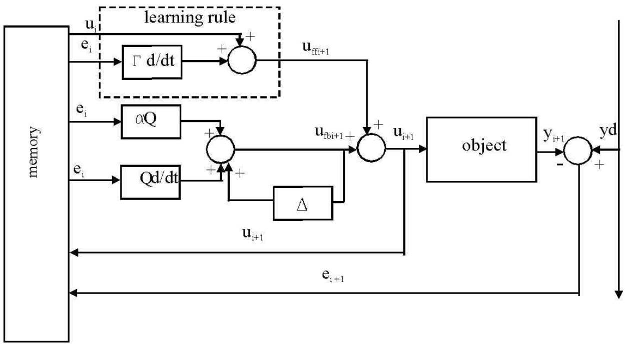

Figure 1. Block diagram of natural iterative learning control for a LTV system

Before presenting a learning control algorithm, the following assumptions on the system are imposed.

(A1) The system is causal. Furthermore, for a given bounded desired output $y_d(t)$, there exists a unique input $u_d(t), t \in [0,T]$ such that when $u(t) = u_d(t)$ the system has a unique bounded state $x_d(t)$ and $y_d(t),\ t \in [0,T]$.

(A2) The matrices $A(t), B(t), C(t), \dot{C}(t), \Gamma_{(.)}(t)$ and functions $v_i(t), \dot{v}_i(t), w_i(t)$ are bounded, with upper bounds which defined as

$$\begin{aligned} &h_v = \sup_{t\in[0,T]} \|v_i(t)\|,\ \forall i \quad h_{v^\bullet} = \sup_{t\in[0,T]} \|\dot{v}_i(t)\|,\ \forall i \\ &h_w = \sup_{t\in[0,T]} \|w_i(t)\|,\ \forall i \quad h_A = \sup_{t\in[0,T]} \|A(t)\|, \end{aligned} \quad (5a)$$

$$\begin{aligned} &h_B = \sup_{t\in[0,T]} \|B(t)\|,\quad h_C = \sup_{t\in[0,T]} \|C(t)\|,\quad h_{C^\bullet} = \sup_{t\in[0,T]} \|\dot{C}(t)\|, \\ &h_{\Gamma_k} = \sup_{t\in[0,T]} \|\Gamma_k(t)\|, \end{aligned} \quad (5b)$$

(A3) Also, following assumption is imposed: as the initial state at each operation may not be the same, i.e random inital state error at each iteration is an a neighborhood of $x_d(0)$ such that

$$\|x_d(0) - x_i(0)\| \le h_0 \quad (6)$$

Here, it is proposed:

$$u_{fbi+1}(t) = \Delta u_{fbi+1}(t^-) + Q(\dot{e}_{i+1}(t) + \alpha e_{i+1}(t)) \quad (7)$$

$$u_{ffi+1}(t) = u_i(t) + \sum_{k=1}^{N} \Gamma_k(t)\dot{e}_l(t),\ l = i - k + 1 \quad (8)$$

$$u_{i+1}(t) = u_{ffi+1}(t) + u_{fbi+1}(t) \quad (9)$$

where $\Delta \in (0,1]$ and $\alpha > 0$ are real constants; N the order of ILC updating law, Q denotes gain matrix appropriate dimensions; $\Gamma_k(t)$ are bounded time-varying learning parameter matrices of proper dimensions, $u_{fb}(t)$ the feedback control input, $u_{ff}(t)$ the feedforward input; $u(t)$ the value of the function at time t and $u(t^-) = u(t - \varepsilon), \varepsilon \to 0^+$ denotes a control vector of the just realised control at time t. If the feedback delay can be neglected then:

$$u_{fbi+1}(t^-) = u_{fbi+1}(t) \quad (10)$$

and

$$u_{fbi+1}(t) = \frac{Q}{1-\Delta}(\dot{e}_{i+1}(t) + \alpha e_{i+1}(t)) \quad (11)$$

or taking $Q^* = Q/(1-\Delta)$ yields:

$$u_{fbi+1}(t) = Q^*(\dot{e}_{i+1}(t) + \alpha e_{i+1}(t)) \quad (12)$$

THEOREM 1: Suppose that the update law Eqs. (7),(8)and (9) is applied to the system Eq. (1) and the initial state at each iteration satisfies Eq.(6). If matrices $\Gamma_k(t)$,Q exist and real numbers Δ and ρ_k satisfying

$$\|[I - \Gamma_k(t)C(t)B(t)][I - D(t)]\| \le \rho_k \quad (13)$$

and

$$\sum_{k=1}^{N} \rho_k = \rho < 1 \quad (14)$$

where $D(t) = [I + Q^*C(t)B(t)]^{-1} Q^*[C(t)B(t)]$ and $Q^* = Q/(1-\Delta)$ then, when $i \to \infty$ the bounds of the tracking errors $\|x_d(t) - x_i(t)\|, \|y_d(t) - y_i(t)\|, \|u_d(t) - u_i(t)\|$, converge asymptotically to a residual ball centered at the origin.

The following lemma is needed in the proof of Theorem 1.

Lemma1 [4] Suppose a real positive series $\{a_n\}_1^\infty$ satisfies

$$\begin{aligned} &a_n \le \rho_1 a_{n-1} + \rho_2 a_{n-2} + ... + \rho_n a_{n-N} + \varepsilon \\ &(n = N+1, N+2, ...), \end{aligned} \quad (15)$$

where $\rho_i \geq 0, (i = 1,2,.., N)$ $\varepsilon \geq 0$ and $\rho = \sum_{i=1}^{n} \rho_i < 1$ then the following holds:

$$\lim_{n\to\infty} a_n \leq \frac{\varepsilon}{1-\rho}. \quad (16)$$

The proof of *Lemma1* is given in the Appendix.

Proof:

For brevity, the time t is dropped from the equations. Let

$$e_i(t) = y_d(t) - y_i(t) \;\; \delta x_i = x_d(t) - x_i(t), \quad (17)$$

$$\delta \dot{x}_i = \dot{x}_d(t) - \dot{x}_i(t), \;\; \delta u_{ffi} = u_d(t) - u_{ffi}(t), \quad (18)$$

Also, the tracking error can be presented as:

$$e_l = C\delta x_l - v_l, \quad l = i-k+1, \; k = 1,2,...,N \quad (19)$$

and

$$\dot{e}_l = \dot{C}\delta x_l + C\delta\dot{x}_l - \dot{v}_l \quad (20)$$

where

$$\delta\dot{x}_l = A\delta x_l + B\delta u_l - w_l \quad (21)$$

$$\dot{e}_l = \left(\dot{C} + CA\right)\delta x_l + CB\delta u_l - Cw_l - \dot{v}_l \quad (22)$$

Taking the proposed control law and Eqs. (20),(21),and (22) gives:

$$\delta u_{ffi+1} = \delta u_{ffi} - u_{fbi} - \sum_{k=1}^{N} \Gamma_k \dot{e}_l, \quad (23)$$

$$\delta u_{ffi+1} = \delta u_{ffi} - u_{fbi} - \sum_{k=1}^{N} \Gamma_k \left(\left(\dot{C} + CA\right)\delta x_l - CB\delta u_l - Cw_l - \dot{v}_l\right), \quad (24)$$

$$\delta u_{ffi+1} = \sum_{k=1}^{N} \left[I - \Gamma_k CB\right]\delta u_l - \sum_{k=1}^{N} \Gamma_k \left(\left(\dot{C} + CA\right)\delta x_l - Cw_l - \dot{v}_l\right) \quad (25)$$

Also one obtains from Eq. (11):

$$u_{fbi} = Q^*\left(C(A + \alpha I)\delta x_i + CB\delta u_i\right) \quad (26)$$

taking in account:

$$\delta u_i = \delta u_{fi} - u_{fbi} \quad (27)$$

it yuields:

$$u_{fbi} = \left[I + Q^* CB\right]^{-1} Q^* \left[C(A + \alpha I)\delta x_i + CB\delta u_{fi}\right] \quad (28)$$

or , taking in account $l = i - k + 1$

$$u_{fbl} = \left[E\delta x_l + D\delta u_{fl}\right] \quad (29)$$

where

$$E = \left[I + Q^* CB\right]^{-1} Q^* \left[C(A + \alpha I)\right]$$
$$D = \left[I + Q^* CB\right]^{-1} Q^* \left[CB\right] \quad (30)$$

$$h_E = \sup_{t\in[0,T]} \left[I + Q^* CB\right] Q^* \left[C(A + \alpha I)\right] \quad (31)$$

$$h_D = \sup_{t\in[0,T]} \left[I + Q^* CB\right]^{-1} Q^* \left[CB\right] \quad (32)$$

By applying Eq. (29) to (25) it results:

$$\delta u_{ffi+1} = \sum_{k=1}^{N} \left[I - \Gamma_k CB\right]\left[I - D\right]\delta u_{ffl} -$$
$$-\sum_{k=1}^{N} \left\{\left[I - \Gamma_k CB\right]E + \left[\Gamma_k\left(\dot{C} + CA\right)\right]\right\}\delta x_l \quad (33)$$
$$+\sum_{k=1}^{N} \Gamma_k\left(Cw_l + \dot{v}_l\right)$$

Estimating the norms of Eq. (33) with $\|(.)\|$ and using the condition of theorem 1 gives

$$\left\|\delta u_{ffi+1}\right\| \leq \sum_{k=1}^{N} \rho_k \left\|\delta u_{fl}\right\| + \sum_{k=1}^{N} \beta_k \left\|\delta x_l\right\| + \varepsilon_1 \quad (34)$$

where

$$\beta_k = h_E + h_E h_{\Gamma_k} h_C h_B + h_{\Gamma_k}\left(h_{C\bullet} + h_C h_A\right) \quad (35)$$

and

$$\varepsilon_1 = +\sum_{k=1}^{N} h_{\Gamma_k}\left(h_C h_w + h_{v\bullet}\right) \quad (36)$$

Also,

$$\delta x_l(t) = \delta x_l(0) + \int_0^t \delta\dot{x}_l(\tau)d\tau$$
$$= \delta x_l(0) + \int_0^t \begin{bmatrix} A(\tau)\delta x_l(\tau) + B(\tau)\delta u_{fl}(\tau) \\ -B(\tau)u_{bl}(\tau) + w(\tau) \end{bmatrix} d\tau \quad (37)$$

From equations (29) and (37) one can obtain:

$$\left\|\delta x_l\right\| \leq h_{x0} + h_w T + \int_0^t \left\|(A - BE)\right\|_\infty \left\|\delta x_l(\tau)\right\| d\tau +$$
$$+\int_0^t \left\|B(I - D)\right\|_\infty \left\|\delta u_{fl}(\tau)\right\| d\tau \quad (38)$$

where

$$\left\|A - BE\right\|_\infty \leq h_A + h_B h_E = h_1$$
$$\left\|B - BD\right\|_\infty \leq h_B + h_B h_D = h_2 \quad (39)$$

For any function $x_l(t) \in R^n$, $t \in [0,T]$ the λ-norm form $\int_0^t \left\|x_i(s)\right\| ds$ is [6]

$$\sup_{t\in[0,T]} e^{-\lambda t} \int_0^t \left\|x_l(s)\right\| ds \leq \left\|\delta x_l(t)\right\|_\lambda \sup_{t\in[0,T]} e^{-\lambda t} \int_0^t e^{\lambda s} ds \quad (40)$$

$$\leq \left\|\delta x_l(t)\right\|_\lambda \sup_{t\in[0,T]} \frac{1 - e^{-\lambda t}}{\lambda} \leq \left\|\delta x_l(t)\right\|_\lambda O(\lambda^{-1}) \quad (41)$$

where $O(\lambda^{-1}) = (1 - e^{-\lambda T})/\lambda \leq 1/\lambda$. (42)

New performing the λ-norm operation for Eq.(38) and using Eq.(41) one obtains

$$\left\|\delta x_l\right\|_\lambda \leq h_o + h_1 \left\|\delta x_l\right\|_\lambda O(\lambda^{-1}) + h_2 \left\|\delta u_{fl}\right\|_\lambda O(\lambda^{-1}) \quad (43)$$

and

$$\|\delta x_l\|_\lambda \le h_o/(1-h_1O(\lambda^{-1})) + h_2\|\delta u_{fl}\|_\lambda O(\lambda^{-1})/(1-h_1O(\lambda^{-1})) \quad (44)$$

or

$$\|\delta x_l\|_\lambda \le \varepsilon' + O'(\lambda^{-1})\|\delta u_{fl}\|_\lambda \quad (45)$$

where

$$\varepsilon' = h_o/(1-h_1O(\lambda^{-1})), \quad O'(\lambda^{-1}) = h_2O(\lambda^{-1})/(1-h_1O(\lambda^{-1})) \quad (46)$$

if a sufficiently large λ is used such that $h_1O(\lambda^{-1}) < 1$. Taking the λ-norm of Eq.(34) with the substitution of Eq.(45) simply yields

$$\|\delta u_{ffi+1}\|_\lambda \le \sum_{k=1}^{N} \rho'_k \|\delta u_{fl}\|_\lambda + \varepsilon_o \quad (47)$$

where

$$\rho'_k = \rho_k + \beta_k O'(\lambda^{-1}),\ \varepsilon_o = \sum_{k=1}^{N} \beta_k \varepsilon' + \varepsilon_1 . \quad (48)$$

Based on condition of theorem 1, one can make

$$\rho' = \sum_{k=1}^{N} \rho_k + \beta_k O'(\lambda^{-1}) < 1 \quad (49)$$

by using a sufficienty large λ .According to Lemma1 it can be concluded that

$$\lim_{i\to\infty} \|\delta u_{ffi+1}\|_\lambda \le \varepsilon_o/(1-\rho') \quad (50)$$

Now, it can be easily shown that

$$\lim_{i\to\infty} \|\delta x_i\|_\lambda \le \varepsilon' + \varepsilon_o O'(\lambda^{-1})/(1-\rho') \quad (51)$$

or

$$\lim_{i\to\infty} \|\delta e_i\|_\lambda \le h_c(\varepsilon' + \varepsilon_o O'(\lambda^{-1})/(1-\rho')) + h_v , \quad (52)$$

If the bounds of the uncertainty, disturbance, and initialization error tend to zero, the final tracking error bound will also tend to zero. This completes the proof of Theorem 1.

IV. Conclusion

A new iterative learning control algorithm utilizing principle of self-adaptability in feedback configuration. A new algorithm has benefits which include control a object with unknown initial state, improving the properties of tracking as well as speed of convergence ILC. Sufficient conditions for the convergence of a new type of learning control algorithm for a class of LTV-system are presented.

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APPENDIX A - Proof of *Lemma 1*

Proof. Let $n_1 \in \{n-1, n-2, ..., n-N\}$ be an index number such that $a_{n_1} = max\{a_{n-1}, a_{n-2}, ..., a_{n-N}\}$.Then, by the assumption in the lemma,

$$a_n \le \rho_1 a_{n-1} + \rho_2 a_{n-2} + ... + \rho_N a_{n-N} + \varepsilon \le \rho a_{n1} + \varepsilon$$

Similarly, let $n_2 \in \{n_1-1, n_1-2, ..., n_1-N\}$ such that

$$a_{n_2} = max\{a_{n_1-1}, a_{n_1-2}, ..., a_{n_1-N}\}$$

then,

$$a_{n_1} \le \rho a_{n_2} + \varepsilon$$

Therefore,

$$a_n \le \rho^2 a_{n_2} + \rho\varepsilon + \varepsilon$$

In general,

$$a_n \le \rho^m a_{n_m} + \rho^{m-1}\varepsilon + \rho^{m-2}\varepsilon + ... + \rho\varepsilon + \varepsilon \le \rho^m a_{n_m} + \frac{1-\rho^m}{1-\rho}\varepsilon$$ w

here m and n_m are positive integers. If m is chosen such that $n_m \le N$, then $[n/N]-1 \le m \le n-N$, and therefore $m \to \infty$ when $n \to \infty$. Let $M = max\{a_1, a_2, ..., a_N\}$ then

$$a_n \le \rho^m M + \frac{1-\rho^m}{1-\rho}\varepsilon$$

which implies

$$\lim_{n\to\infty} a_n \le \frac{\varepsilon}{1-\rho}$$

This completes the proof of *Lemma 1*.