Systematic and Random Errors in Radiation Therapy

Georgi Y. Gluhchev

Abstract - **A comprehensive approach to the detection of systematic and random errors in radiation therapy is described. Before looking for a systematic error Δ, an attempt to detect a random error δ is made. Δ is evaluated on the base of the measured displacement μ using a statistical classification approach, and a maximum likelihood correction is suggested.**

Keywords: **Systematic Error, Random error, Radiation Therapy.**

I. INTRODUCTION

In time detection and correction of systematic and random errors in field placement parameters is of primary importance in radiation cancer treatment. Wrong parameters leading to insufficient radiation delivery to the target volume or over irradiation of normal tissue will increase patient morbidity. Evaluation of the accuracy of treatment field set-up in radiation therapy is based on the comparison between a portal image and a reference image, which may be a digitized simulation film, a digital reconstructed radiograph, or another portal image. The objective estimation of the differences between two fields requires one-to-one correspondence to be established between them. This is usually achieved by the alignment of two sets of characteristic (fiducial) points A={Ai} and B={Bi} (i=1,2,…,n) from the reference and portal image respectively searching for an orthogonal transformation ***T*** between them. In case that all points are properly placed, the transformation parameters will be properly evaluated, a good registration will be achieved and treatment field parameters will be estimated correctly. However, different errors influence the accuracy of the evaluation of these parameters [3,10], stemming from the following sources: 1) improper data transfer between the simulator and treatment machine, 2) misinterpretation of the anatomical structure, 3) improper machine calibration, 4) operator errors. Some errors are of systematic type and have to be detected and corrected as soon as possible. They usually relate to the first three sources. Two types of random errors concern the operator's work. Operator's error due to improper couch and collimator parameter settings could be easily detected after comparison between the actual image and reference one. But it is too difficult to detect random errors due to improper placement of fiducial points, which happens frequently because of the poor quality of portal images.

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Different approaches have been suggested aimed at the early detection and compensation of systematic errors [2,3,4,5,8,10]. Most of the works concern only the detection problem and there is practically no studies dealing with the processing of random error.

In this paper a comprehensive approach to error processing in radiation therapy is described. It consists of the following three-step procedure:

(i) Evaluation of the operator's random error,

(ii) Evaluation of the systematic error,

(iii) Evaluation of the correction magnitude.

The problem is formulated as follows.

Let μ denotes a measured displacement of a parameter m at a particular treatment session. μ is assumed to be a sum of a systematic error Δ and errors δ_1 and δ_2, i.e.

$$\mu = \Delta + \delta_1 + \delta_2 \ , \qquad (1)$$

where δ_1 concerns the improper point placement by the operator, and δ_2 reflects the accuracy of treatment machine.

While Δ is constant for a particular patient but may vary from patient to patient, δ_1 and δ_2 may vary within a treatment course, i.e. from fraction to fraction. While there is no way to eliminate δ_2, Δ and δ_1 may be detected and eliminated at an early stage of the treatment course.

II. OPERATOR'S ERROR

As mentioned above, the registration between reference and actual image is usually achieved via an orthogonal transformation ***T*** that minimizes the average square distance between corresponding points ***T***Ai and Bi. This technique requires the same number of points in the sets and the correspondence Ai $\leftrightarrow$Bi to be set up.

In this study we assume that images are of same scale, which is the case of portal-to-portal image comparison. This means that only rotation between the images may be present. The case of automatic paring and different scaling between them is described somewhere else [7]. However, since an operator selects the points manually, and since the quality of portal images is far from satisfactory, an error in point placement is possible. As a result, the transformation parameters will not be properly evaluated and wrong conclusion about the accuracy of treatment session will be made. Therefore, it is highly desirable to detect such operator errors and eliminate them before the evaluation of the displacement μ. For this an intuitively sound approach is suggested. The idea is as follows.

Let point P_i is moved away from its correct position and is placed at the position P_i'. Let g_{ij} denotes the difference between the distances $r(P_i, P_j)$ and $r(P_i', P_j)$, i.e. $g_{ij} = | r(P_i, P_j) - r(P_i', P_j) |$. Since $P_i' \neq P_i$, the difference

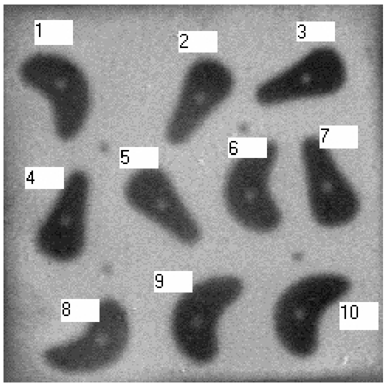
$gij > 0$ for all P_j (j=1,2,…,n; $j \neq i$). Let $g_i = \sum_{j \neq i} g_{ij}$ (i=1,..,n). It is clear that maximum value of g_i will be obtained for the inaccurate point P_i. The average value $G = \frac{1}{n} \sum_i g_i$ of all g_i is evaluated. Since more than one point may be incorrect, an iterative search has to be applied, leading to the detection and deletion of all wrong points. In practice, we assume that points Ai from the reference image are placed correctly and g_{ij} is evaluated as a difference $g_{ij} = | r(A_i, A_j) - r(B_i, B_j) |$. However, point placement by mouse may lead to offsets of 1 or 2 pixels and the evaluated measures g_i will be always different from 0. This requires a threshold t to be used permitting to decide whether point Bi is incorrectly placed. After that same procedure applies to the remaining points.

In practice it is difficult to assign t a particular value *a priori.* To solve this problem following empirical approach can be used.

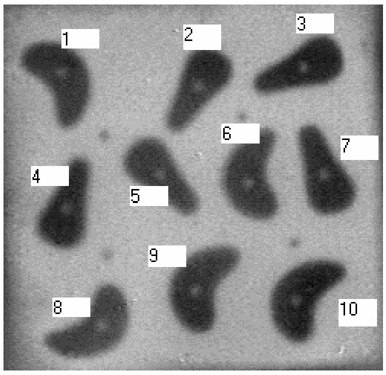
Let Bi be the point of maximum g_i. One can discard Bi from the set of fiducial points and evaluate the sum $G_{-i} = \frac{1}{n-1} \sum_{k \neq i} g_k$ for the remaining points. The experiments have shown that if the ratio $G/G_{-I} > 1.5$, Bi is incorrect for sure, therefore following decision rule could be accepted

Rule: Discard point Bi *if* $G/G_{-i} > 2$, *else stop*.

This procedure was applied to the images a) and b) in Fig. 1. These are phantom images with 10 lead pieces placed in. The position of pieces 5 and 9 in the image b) is deliberately changed which leads to different position of their central points. The results from the implementation of the iterative procedure are shown in Table I. After the first iteration maximum value of 10.46 was obtained at point 5. The elimination of this point has lead to almost 3 times less value of the total sum G. It was diminished from 6.04 to 2.06. At the second iteration point 9 became the best candidate for deletion and following the decision rule it was taken away. At the third iteration the test for the suspicious point 1 failed and the procedure ended.



a)



b)

Fig. 1. Phantom images. Pieces 5 and 9 in image b) are deliberately displaced.

The obtained results have shown that the suggested procedure effectively finds all wrong points even if an observer cannot easily detect their displacements.

III. SYSTEMATIC ERROR DETECTION

A significant systematic error is dangerous since it will affect all radiation delivery sessions and may inflict serious damages destroying either healthy tissues or provoking tumour recurrence. That is why it must be detected and corrected at an early stage of the treatment.

Since the systematic error will move all measured values in one direction, it may be easily evaluated by averaging μ_i for all treatment sessions. But this will lead to an evaluation *post factum*, i.e. it will be of no use for the patient. Also, while being constant for a particular series of measurements, Δ may change from series to series like a random variable. In the latter case previous estimations of Δ cannot be used directly for the evaluation of its current value. Therefore, following question has to be answered: “Does the measured displacement μ_i at the ith session require a correction action?”

TABLE I

Values of g_i, G and G_{-i} for the points in phantom images.

Dark cells correspond to the suspected points.

#point	iter.1	iter.2	iter.3
1	4.56	1.63	0.78
2	4.35	1.86	0.77
3	4.87	1.78	0.70
4	5.98	1.84	0.56
5	10.46		
6	6.30	2.06	0.72
7	5.94	2.18	0.76
8	6.27	2.02	0.51
9	6.45	3.26	
10	5.26	1.95	0.75
G	6.04	2.06	0.69
G_{-i}	2.06	0.69	0.67

During the last years different approaches have been suggested aimed at the evaluation of Δ after one [10] or more measurements [2,3,4,5].

Being derived from practical considerations, suggested rules are reasonable, but it is difficult to claim that they are optimal in whatever sense. In an attempt to give a theoretically founded answer to the above formulated question, an approach was developed [8], where the problem for the detection of a significant systematic displacement was formulated as a classification one. For this following assumptions are made.

(i) Δ is a constant for a particular series $S_\Delta = \{\mu_i : \mu_i = \Delta + \delta_{2i},\ \ i = 1,2,...,n\}$ of measurements but may vary from series to series as a normally distributed random variable with a density function $f(\Delta; 0, \sigma_\Delta{}^2)$ with known σ_Δ.

(ii) The random error δ_1 is eliminated.

(iii) The random error δ_2 is normally distributed with a density function $f(\delta; 0, \sigma_\delta{}^2)$ with known σ_δ.

Two classes about μ_i are defined in the following way:

Class I: $\{\mu_i \in [-\alpha, \alpha]\}$ (no significant displacement)

Class II: $\{\mu_i \notin [-\alpha, \alpha]\}$ (significant displacement is present), where α depends on the machine accuracy.

Class II is the class of large systematic errors.

Now the problem for the detection of a significant displacement could be reduced to the evaluation of a threshold β via minimization of the average loss [6]

$$L(\beta) = C_1 P(E_1) + C_2 P(E_2)\,, \qquad (2)$$

where $P(E_1)$ is the probability of the error $E_1 : \Delta \in \mathit{Class\ I},\ \ \mathit{but}\ |\mu_i| > \beta$ (false alarm), $P(E_2)$ is the probability of the error $E_2 : \Delta \in \mathit{Class\ II},\ \ \mathit{but}\ |\mu_i| \le \beta$ (target's omission), and C_1 and C_2 are losses associated with E_1 and E_2, respectively. The solution of the Eq. (2) is given by the formula

$$\Phi(\beta) = (1 + C_1 / C_2)^{-1}\,, \qquad (3)$$

where

$$\Phi(\beta) = \left(\int_{-\infty}^{a\alpha - b\beta} - \int_{-\infty}^{-a\alpha - b\beta} \right) f(x; 0, 1)\, dx\,, \qquad (4)$$

with $a = \dfrac{\sqrt{\sigma_\Delta{}^2 + \sigma_\delta{}^2}}{\sigma_\Delta \sigma_\delta}$ and $b = \sigma_\Delta / \sigma_\delta$.

The obtained solution for the decision-making threshold β is optimal in the sense of minimal average loss due to classification errors.

IV. CORRECTION EVALUATION

The detection of a significant systematic error is just the first part of the problem. The question: "How to make a correction?" has to be answered as well, in order to improve the quality of radiation treatment. The obvious answer: "Make correction equal to $-\mu_i$" may not be the best one. In that case a residual systematic displacement $\Delta_r = \Delta - \mu_i$ will be obtained equal to $-\delta_{2i}$, and $\sigma_{\Delta r} = \sigma_\delta$. This means that no matter how many corrections based on a single measurement will be applied, $\sigma_{\Delta r}$ will remain unchanged. Better result may be expected if a correction $-k\mu_i$ is used, where $0 < k < 1$. Since the correction goal is to reduce Δ, a proper k may be searched for via minimization of the mean square difference $D(\Delta, k)$ between Δ and $k\mu_i$. In the continuous case $D(\Delta, k)$ is presented as

$$D(\Delta, k) = \int_{-\infty}^{\infty} (\Delta - k\mu)^2 f(\mu; \Delta, \sigma_\delta{}^2)\, d\mu\,. \qquad (5)$$

The minimization of this integral about k leads to the following equation [1]

$$k = \Delta^2 / (\Delta^2 + \delta^2) \qquad (6).$$

In that case

$$\sigma_{\Delta r}{}^2 = \frac{\sigma_\Delta{}^2 \sigma_\delta{}^2}{\sigma_\Delta{}^2 + \sigma_\delta{}^2} \qquad (7)$$

will be obtained. The last formula shows that $\sigma_{\Delta r} < \sigma_\delta$. Also, $\sigma_{\Delta r} < \sigma_\Delta$ and every new correction will make $\sigma_{\Delta r}$ smaller and smaller. For example, if $\sigma_\Delta = \sigma_\delta$, then $\sigma_{\Delta r}{}^2 = \sigma_\delta{}^2 / (i+1)$ will be obtained after the *i*th correction.

These results indicate that a better correction in terms of smaller residual systematic displacement Δ_r and a smaller standard deviation σ_{Δ_r} exists. The experiments [9] have shown that it yields also less number of corrections per patient.

V. CONCLUSION AND DISCUSSION

An approach is suggested aimed at the comprehensive processing of the errors in radiation therapy. Until now different approaches have been suggested for the detection of large systematic displacements. However, less attention has been paid to the question about the correction's magnitude, and no attention was paid at all to the problem of random operator errors. In this paper a solution to the latter problem is suggested, based on intuitively sound assumptions. The experience with synthesized sets of points and phantom images have shown the procedure's efficacy in detection of incorrectly placed points.

For the systematic error an optimal value of the correction magnitude is suggested, leading to smaller residual error after correction. Also, if repeatedly applied it leads to a significant decrease in $\sigma_{\Delta_r}{}^2$.

The practical implementation of the suggested approach requires the standard deviations σ_Δ and σ_δ to be known. They could be taken from the literature or could be evaluated from the measurements carried out at the corresponding department.

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