Algorithms for Local Adaptive Image Processing

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Abstract **- Two algorithms for local adaptive image processing are developed using generalized two-dimensional LMS approach. The first algorithm presents an error diffusion method for image quantisation and the second - adaptive linear predictive coding of images. Experiments suggest that the effective use of local information contribute to minimize the processing error.**

Keywords **- image halftonning, linear prediction, 2D LMS algorithm, local adaptation, image processing.**

I. Introduction

The linear filtration is related to the common used methods for image processing and is separated into the two basic groups - non-adaptive and adaptive [1], [2]. In the first group the filter parameters are obtained by the principles of the optimal (Winner) filtration, which minimizes the mean square error of signal transform and assumes the presence of the apriory information for image statistical model. The model inaccuracy and the calculation complexity required for their description might by avoided by adaptive estimation of image parameters, by iteration minimization of mean-square error of the transform.

Depending on the processing method, the adaptation is divided into global and local. The global adaptation algorithms refer principally to the basic characteristics of the images, while the local ones are connected to adaptation in each pixel of the processed image based on the selected pixel neighborhood.

In the present paper two local adaptive image processing algorithms for image halftoning and linear prediction are developed. The coefficients of filters are adapted with the help of generalized two-dimensional LMS algorithm [3], [4].

II. Mathematical Description

Adaptive error-diffusion quantisation algorithm.

The input m-level halftone image and the output n-level ($2 \le n \le m/2$) image of dimensions *M x N* can be represented by the matrices:

$$\boldsymbol{C} = \{\mathbf{c}(k,l) \;/\; k = \overline{0, M-1};\; l = \overline{0, N-1}\},$$
$$\boldsymbol{D} = \{\mathbf{d}(k,l) \;/\; k = \overline{0, M-1};\; l = \overline{0, N-1}\}. \qquad (1)$$

Transformation of the image elements **c**(k,l) into **d**(k,l) is accomplished by the adaptive error diffusion quantiser (AEDQ) shown on Fig. 1. The quantiser operation is described by the following equation:

$$\mathbf{d}(k,l) = Q[\mathbf{c}_f(k,l)] =$$

$$= \begin{cases} q_{0,} \text{ if } \mathbf{c}_f(k,l) < T_0 \\ q_{p,} \text{ if } T_{p-1} < \mathbf{c}_f(k,l) < T_p \;\; (p = \overline{1, n-2}) \\ q_{n-1,} \text{ if } \mathbf{c}_f(k,l) < T_0 \end{cases} \qquad (2)$$

where $q_p \le q_{p+1} \le m$ $(p = \overline{0, n-2})$ are the values of the function Q[.]. Thresholds for comparison are calculated by $T_p = (C_p + C_{p+1})/2$, where C_p represents the gray values dividing the normalized histogram of the input halftone image $\boldsymbol{C}$ into n equal parts. The value of the filtered element $\mathbf{c}_f(k,l)$ in Eq. (2) is:

$$\mathbf{c}_f(k,l) = \mathbf{c}(k,l) + \mathbf{e}_0(k,l). \qquad (3)$$

The summarized error can be expressed as:

$$\mathbf{e}_0(k,l) = \sum_{(r,t)\in \boldsymbol{W}}\sum \mathbf{w}_{k,l}(r,t)\mathbf{e}(k-r, l-t) = \mathbf{W}_{k,l}\mathbf{E}_{k,l}, \qquad (4)$$

where $\mathbf{e}(k,l) = \mathbf{c}_f(k,l) - \mathbf{d}(k,l)$ is the error of the current filtered element when its value is substituted by q_p; $\mathbf{w}_{k,l}(r,t)$ are the filter weights defined in the certain causal two -dimensional window $\boldsymbol{W}$; $\mathbf{W}_{k,l}$ and $\mathbf{E}_{k,l}$ are the vectors of the weights and their summarized errors, respectively.

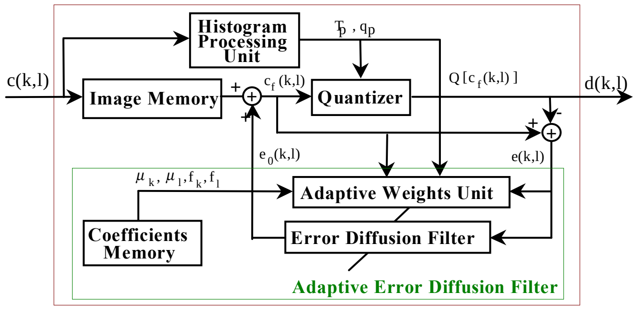

Fig.1 Adaptive error-diffusion quantiser

According to 2D-LMS algorithm [5] the AEDF weights can be determined recursively:

$$\mathbf{W}_{k,l} = f_k \mathbf{W}_{k,l-1} - \mu_k \nabla_{k,l-1} + f_l \mathbf{W}_{k-1,l} - \mu_l \nabla_{k-1,l}, \qquad (5)$$

where: $\nabla_{k,l-1}$ and $\nabla_{k-1,l}$ are the gradients of the squared errors by the quantisation in horizontal and vertical directions; f_k, f_l - coefficients, considering the direction of the adaptation, where : $f_k + f_l = 1$; μ_k, μ_l - adaptation steps in the respective direction.

According to [3] the convergence and the stability of the AEDF adaptation process is given by the following condition:

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$$\left|f_k - \mu_k \lambda_i\right| + \left|f_l - \mu_l \lambda_i\right| < 1 \ , \tag{6}$$

where λ_i are the eigenvalues of the gray-tone image covariance matrix .

Sequence (5) is 2D-LMS algorithm of Widrow summary from which the following two particular cases should hold:

First. If $f_k = 1, \mu_k = \mu, f_l = \mu_l = 0$ then the adaptive calculation of the weights is proceeded only in the horizontal direction:

$$\mathbf{W}_{k,l} = \mathbf{W}_{k,l-1} + \mu_k (-\nabla_{k,l-1}) = \mathbf{W}_{k,l-1} - \mu \frac{\partial \mathbf{e}^2(\mathbf{k}, \mathbf{l}-\mathbf{1})}{\partial \mathbf{W}_{k,l-1}} . \tag{7}$$

Second. If $f_l = 1, \mu_l = \mu, f_k = \mu_k = 0$ then the adaptive calculation is proceeded only in the vertical direction:

$$\mathbf{W}_{k,l} = \mathbf{W}_{k-1,l} + \mu_l (-\nabla_{k-1,l}) = \mathbf{W}_{k-1,l} - \mu \frac{\partial \mathbf{e}^2(k-1,l)}{\partial \mathbf{W}_{k-1,l}} . \tag{8}$$

The derivatives by the quantization error in the respective directions are determined by the Eqs. (1), (2), (3), (4) and (5). For the derivative in horizontal direction is obtained:

$$\begin{aligned}
\frac{\partial \mathbf{e}^2(k,l-1)}{\partial \mathbf{W}_{k,l-1}} &= 2\mathbf{e}(k,l-1)\frac{\partial}{\partial \mathbf{W}_{k,l-1}}\left[\mathbf{c}_f(k,l-1) - Q'_{\mathbf{c}_f}(k,l-1)\right] \\
&= 2\mathbf{e}(k,l-1)\frac{\partial \mathbf{c}_f(k,l-1)}{\partial \mathbf{W}_{k,l-1}}\left[1 - Q'_{\mathbf{c}_f}(k,l-1)\right] \\
&= 2\mathbf{e}(k,l-1)\frac{\partial \mathbf{e}_0(k,l-1)}{\partial \mathbf{W}_{k,l-1}}\left[1 - Q'_{\mathbf{c}_f}(k,l-1)\right] \\
&= 2\mathbf{e}(k,l-1)\,\mathbf{E}_{k,l-1}\left[1 - Q'_{\mathbf{c}_f}(k,l-1)\right]
\end{aligned} \tag{9}$$

where:

$$\begin{aligned}
Q'_{\mathbf{c}_f}(k,l-1) &= \frac{\partial Q_{\mathbf{c}_f}(k,l-1)}{\partial \mathbf{c}_f(k,l-1)} = \\
&\lim_{\Delta \mathbf{c}_f \to 0} \frac{Q\left[\mathbf{c}_f(k,l-1) + \Delta \mathbf{c}_f\right] - Q\left[\mathbf{c}_f(k,l-1)\right]}{\Delta \mathbf{c}_f} \\
&\approx \Delta Q_{\mathbf{c}_f}\left[\mathbf{c}_f(k,l-1)\right] \\
&= \Delta \quad Q_{\mathbf{c}_f}(k,l-1) = \begin{cases} 0, \text{if}: \mathbf{c}_f(k,l-1) \neq T \\ q_1 - q_0, \text{if}: \mathbf{c}_f(k,l-1) = T. \end{cases}
\end{aligned}$$

In the same way for the derivative in the vertical direction is obtained:

$$\frac{\partial \mathbf{e}^2(k-1,l)}{\partial \mathbf{W}_{k-1,l}} = 2\mathbf{e}(k-1,l)\,\mathbf{E}_{k-1,l}\left[1 - Q'_{\mathbf{c}_f}(k-1,l)\right] \tag{10}$$

For the AIHF weights the condition must be hold:

$$\sum\sum_{(r,t)\in \boldsymbol{W}} \mathbf{w}_{k,l}(r,t) = 1 \ , \tag{11}$$

which guarantees that $\mathbf{e}(k,l)$ is not increased or decreased by its passing through the error filter.

On the basis of analysis, made in Eqs. (7) to (11) the sequence for the components of $W_{k,l}$ is

$$\begin{aligned}
w_{k,l}(r,t) = f_k\, w_{k,l-1}&(r,t) \\
&- 2\mu_k\, e(k,l-1)e(k-r,l-t-1)\left[1 - Q'_{\mathbf{c}_f}(k,l-1)\right] \\
&+ f_l\, w_{k-1,l}(r,t) \\
&- 2\mu_l\, e(k-1,l)e(k-r-1,l-t)\left[1 - Q'_{\mathbf{c}_f}(k-1,l)\right]
\end{aligned} , \tag{12}$$

where $Q'_{\mathbf{c}_f}(k,l) = \begin{cases} 0, \text{if}\, \mathbf{c}_f(k,l) \neq T_p, \\ q_{p+1} - q_p, \text{if}\, \mathbf{c}_f(k,l) = T_p. \end{cases}$

Adaptive linear predictive coding algorithm.

The basic equation of the liner prediction is presented by the following way:

$$\hat{\mathbf{x}}(k,l) = \sum\sum_{(r,t)\in \boldsymbol{A}} \mathbf{a}_{k,l}(r,t)\mathbf{x}(k-r,l-t) = \mathbf{A}_{k,l}\mathbf{X}_{k,l} \ , \tag{13}$$

where $\hat{\mathbf{x}}(k,l)$ is the value of the predicted element from the input image $\mathbf{x}$(k,l). The prediction error is described by the equation:

$$\mathbf{e}(k,l) = \mathbf{x}(k,l) - \hat{\mathbf{x}}(k,l) \ , \tag{14}$$

and the quantisation error by:

$$\mathbf{e}_q(k,l) = \boldsymbol{Q}\left[\mathbf{e}(k,l)\right]. \tag{15}$$

Related to above calculations the adaptive prediction coefficients are described by the equation:

$$\mathbf{A}_{k,l} = f_k\, \mathbf{A}_{k,l-1} - \mu_k \nabla_{k,l-1} + f_l\, \mathbf{A}_{k-1,l} - \mu_l \nabla_{k-1,l} \ , \tag{16}$$

where: $\nabla_{k,l-1}$ and $\nabla_{k-1,l}$ are the gradients of the squared errors by the prediction in horizontal and vertical directions; f_k, f_l - coefficients, considering the direction of the adaptation, where : $f_k + f_l = 1$; μ_k, μ_l - adaptation steps in the respective direction.

The convergence and the stability of the adaptation process is given by the following condition:

$$\left|f_k - \mu_k \lambda_i\right| + \left|f_l - \mu_l \lambda_i\right| < 1 \ , \tag{17}$$

where λ_i are the eigenvalues of the gray-tone image covariance matrix .

The sequence for the components of $\mathbf{A}_{k,l}$ is given by:

$$\begin{aligned}
\mathbf{a}_{k,l}(r,t) = f_k\, \mathbf{a}_{k,l-1}(r,t) + 2\mu_k e(k,l-1)\mathbf{x}(r,t-1) \\
+ f_l\, \mathbf{a}_{k-1,l}(r,t) + 2\mu_l e(k-1,l)\mathbf{x}(r-1,t)
\end{aligned} . \tag{18}$$

The following equations are used for decoding of images:

$$\mathbf{x}'(k,l) = \mathbf{e}'(k,l) + \hat{\mathbf{x}}'(k,l) \ , \tag{19}$$

$$\mathbf{e}'(k,l) = \boldsymbol{Q}^{-1}\left[\mathbf{e}_q(k,l)\right]. \tag{20}$$

The adaptive line prediction codec, synthesized by the Eqs. (13) to (20) is shown on the Fig. 2.

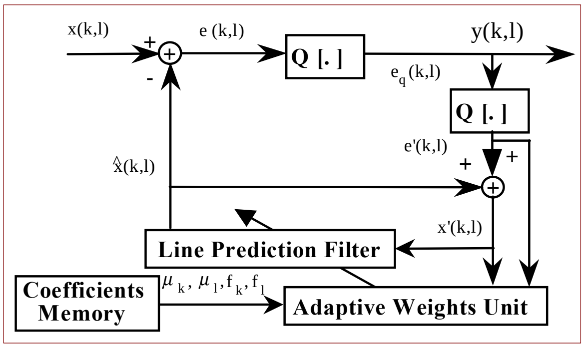

Fig.2 Adaptive line prediction codec

III. Experimental Results

An error diffusion filter with 4 coefficients has been used for the evaluation of the efficiency of the developed algorithm. The spatial disposition, shown on Fig.3, and the initial values of weights correspond to these in the Floyd-Steinberg filter [6].

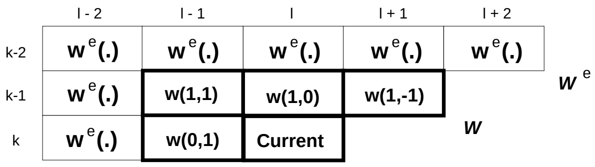

Fig.3. Spatial disposition of the weights $w_{k,l}(r,t)$ in $\boldsymbol{W}$.

For the calculation of each one $w_{k,l}(r,t)$ are used the weights $w^e_{k,l}(.)$ from the extended window $\boldsymbol{W}^e$.

The variation of peak signal to noise ratio (PSNR in dB) from the simulation of AEDQ for the test image "LENNA" with M=N=m=256, n=2,...16, $q_p = C_p$, $f_k = 0.7, f_l = 0.3, \mu_k = \mu_l = 1.67x10^{-6}$ is presented in Table 1.

The PSNR in dB is determined by the equation

$$PSNR=10lg(m^2/E\{[e(k,l)]^2\}\ ,$$

where E{.} is the mathematical expectation.

The use of AEDQ leads to increasing of PSNR with about 0.6 dB in comparison with the 4 weights non-adaptive filter.

A second image transform with scanning the pixels in reverse direction is performed for further decrease of mean square error. The initial values of the weights are equal to the last ones obtained from the direct transform. In this case a better convergence of the adaptation is obtained, the phase distortions are decreased and PSNR increased in addition with about 0.2 dB.

Table 1.

Levels n	Non-adaptive EDQ	AEDQ	AEDQ Reverse direction
2	9.5995	9.6019	9.6192
3	15.8332	16.4755	16.6350
4	19.5003	20.2235	20.4351
5	21.7430	22.3138	22.5224
6	23.0939	23.6004	23.8177
7	24.6805	25.1299	25.3104
8	26.0728	26.4997	26.5336
9	27.3401	27.7489	27.8631
10	28.6571	29.1020	29.2995
11	29.6944	30.1560	30.3015
12	30.5843	31.0828	31.2609
13	31.2506	31.7249	31.9008
14	31.8294	32.3176	32.4957
15	32.5293	33.0186	33.2112
16	33.0700	33.5310	33.7420

An adaptive line prediction coder (ALPC) with 3 coefficients has been used for the evaluation of the efficiency of the developed algorithm. The spatial disposition, shown on Fig.4, and the initial values of weights correspond to the basic adaptive line prediction coder [1].

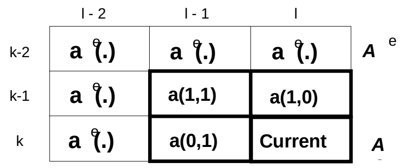

Fig.4. Spatial disposition of the weights $a_{k,l}(r,t)$ in $\boldsymbol{A}$.

For the calculation of each one $a_{k,l}(r,t)$ are used the weights $a^e_{k,l}(.)$ from the extended window $\boldsymbol{A}^e$.

The variation of peak signal to noise ratio (PSNR in dB) from the simulation of ALPC for the test image "LENNA" with M=N=m=256, n=2,3,4 bits, $f_k = 0.7, f_l = 0.3$, and $\mu_k = \mu_l = 1.67x10^{-6}$ is presented in Table 2.

Table 2.

Quntization bits	Non-adaptive LPC	2D ALPC
2	21.083227	21.227181
3	24.163650	24.370046
4	32.898031	33.149066

IV. Conclusion

The developed generalized AEDQ results in the following particular cases: the wide-spread non-adaptive error diffusion filter of Floyd and Steinberg (for n=2,

$f_k = 1, \mu_k = \mu_l = f_l = 0$); adaptive error diffusion using the weights only in the horizontal (from the same image row - $f_k = 1, f_l = 0$) or only in the vertical direction (from the previous image row - $f_l = 1, f_k = 0$). The quantiser provides minimum reconstruction error, uniform distribution of the arranged structures in the homogeneous areas and precise reproduction of edges in the output multilevel images. The coefficients f_k, f_l, μ_k, μ_l must be selected on the basis of PSNR analysis and keeping of Eq. (7) as is done in [5]. The developed AEDQ is appropriate for realization on special VLSI circuit to accelerate calculation of image transform.

The presented error diffusion method can be used for transformation of colour palettes or brightness of pixels in multimedia systems, for printing colour and halftone images and transmition by facsimile devices.

The developed 2D-ALPC provides minimum processing error and lieds to increase of PSNR with about 0.3 dB in comparison with 3 coefficients non-adaptive prediction coder.

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