

Computer Modeling and Simulation of the Brightness of Emitting Surfaces

Todor S. Djamiykov¹, Marin B. Marinov², Elissaveta D. Gadjeva³, Helmut Wurmus⁴

Abstract – Passive optoelectronic systems are often used for solving problems related with orientation, navigation and distant observation of landscapes. The capability verification and adjustment of such systems require in many cases the simulation of real surfaces and composite emission sources. Statistical model is developed for the emission surface, assuming that it represents a random set of emission objects. The composite images are divided in elementary fields with figures of equal shape. The autocorrelation function is calculated for each elementary field and for the composite brightness field.

Keywords – Statistical model, Computer simulation, Emitting surfaces, Brightness field

1. INTRODUCTION

The optoelectronic systems are used for solving problems related to the navigation, orientation and observation of environment record the real space. The observed space can be characterized by a certain distribution of the brightness $L(\vec{r})$, depending on an n -dimensional vector of parameters. The basic parameters in this case are the spatial coordinates, the wavelengths of emissions, the time, etc. Such a representation of the brightness distribution entirely characterizes the observed space for the greater part of the main classes of practical problems.

The passive optoelectronic systems work mainly with non-coherent additive emission that consists partly of its own and partly by the reflected irradiation of the objects. They form the image of the remote spatial sources in the focal plane of the optic system. In this context it is reasonable to consider the brightness of the radiating objects in the two-dimensional space. Most of the irradiating objects do not change their brightness during the period when the analysis is performed or the change is negligible. The changes of the brightness depending on the wavelength represent a separate complicated problem that will not be considered here. We assume the restriction for the considered case that the observation takes place in the $(\lambda_1 - \lambda_2)$

spectral range in which the brightness can be accepted to be constant.

A static two-dimensional brightness field $L(x,y)$ will be considered first. The optoelectronic systems operating on the basis of measuring and evaluating the parameters of the observed space, process mainly random optic signals described by their statistical characteristics. In the most general case the observed space represents an anisotropic non-stationary multi-dimensional process. The most complete characteristic for the description of such a random process is provided by its multi-dimensional probability density that is practically unknown. Another characteristic of the random field giving a quantitative measure for the statistical relationships between the random function values for different arguments or information about the random process dynamics, is the autocorrelation function.

Correlation functions are used in most of the investigations on inhomogeneous radiating surfaces, landscapes of such complex structure, etc. The investigations show that the autocorrelation functions of complexly structured surfaces are very different.

The random brightness field can be described using components reflecting its macrostructure. The component energy is concentrated in the low spatial frequencies. For example, the macrostructure components of a forest massif are the tree crowns (the canopy), while the single leaves are the microstructure elements. The degree of the detailed representing of the brightness field depends mainly on the spatial resolution ability of the receiving part of the optoelectronic system (lens, CCD-, CMOS-matrix). Moreover, the anisotropy and non-stationary features of the brightness field can be described using the macrostructure parameters and their distribution density in space.

II. A MODEL OF THE RADIATING SURFACE

In order to generate the statistical model of the radiating surface it is assumed that it consists of an additive mixture of radiating objects and that their shape, situation and spatial density are described by their compound probability density.

Let the brightness distribution of the single elements of the field $L(x,y)$ be given in the plane $x-y$. The elements are determined by their maximal sizes a and b along two perpendicular axes \vec{x}_α and \vec{y}_α , and by the compound probability density along these axes $w(a,b)$. Each object is modeled by an elementary geometrical figure - a circle, an ellipsis, a square, a rectangle. The basic criterion for selecting the elementary figure of approximation is the minimizing of the deviation from the original figure. The orientation of the single objects in space is determined by the rotation angle α with respect to the x -axis.

¹Todor S. Djamiykov is with the Faculty of Electronic Engineering and Technologies, Technical University of Sofia, kv. Darvenitsa, bl.1, 1756 Sofia, Bulgaria, E-mail: tsd@vmei.acad.bg

²Marin B. Marinov is with the Faculty of Electronic Engineering and Technologies, Technical University of Sofia, kv. Darvenitsa, bl.1, 1756 Sofia, Bulgaria, E-mail: mbm@vmei.acad.bg

³Elissaveta D. Gadjeva is with the Faculty of Electronic Engineering and Technologies, Technical University of Sofia, kv. Darvenitsa, bl.1, 1756 Sofia, Bulgaria, E-mail: egadjeva@vmei.acad.bg

⁴Helmut Wurmus is with the Faculty of Mechanical Engineering, Technical University of Ilmenau, PF 100 565, D-98684 Ilmenau, Germany, E-mail: Helmut.Wurmus@mb.tu-ilmenau.de

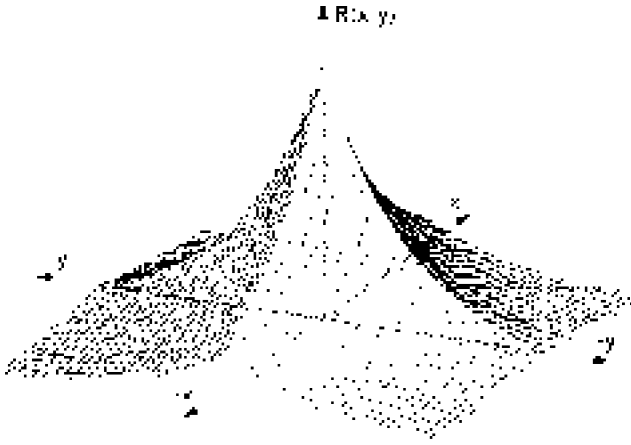


Fig. 1. The autocorrelation function of the brightness field

Taking under consideration the fact that all the parameters (brightness L , maximal sizes a and b , rotation angle α are random magnitudes, their compound probability density $w(L, a, b, \alpha)$ can be determined.

The elementary state with parameters

$$L_i \pm \Delta L, a \pm \Delta a, b \pm \Delta b, \alpha \pm \Delta \alpha$$

in the intervals $\pm \Delta L, \pm \Delta a, \pm \Delta b, \pm \Delta \alpha$ will be considered. N is the total number of elements in a unit area in the x - y -plane. Then the elements in state L_i, a_i, b_i, α_i are given by the following relationship:

$$n_i = N \cdot w(L_i, a_i, b_i, \alpha_i) dL \cdot da \cdot db \cdot d\alpha \quad (1)$$

The autocorrelation function of such an elementary signal in state B_i, a_i, b_i, α_i can be determined by the known relation:

$$K_i(\mu, \nu) = n_i \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} L(x, y) \cdot L(x - \mu, y - \nu) dx \cdot dy \quad (2)$$

Taking under consideration all parameters, the autocorrelation function of the brightness field is:

$$K(\mu, \nu) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} K_i(\mu, \nu) \cdot w(L, a, b, \alpha) dL \cdot da \cdot db \cdot d\alpha \quad (3)$$

A model of a field with random brightness distribution is considered for the case of a finite number of elements with shapes of simple geometrical figures and known statistical parameters for the sizes, orientation and brightness. Three main figures can be examined for the approximation of the spatial formations - the rectangle, ellipsis and triangle of different sizes, brightness and orientation. When the entire field is divided in additive fields formed by figures of equal shape, the autocorrelation function of the j -th elementary brightness field is given by equation (2). Hence the autocorrelation function of the compound field described by the mentioned three basic forms is given by the relationship:

$$K_\Sigma = N \sum_{j=1}^3 p_j \cdot K_j(\mu, \nu) \quad (4)$$

where $N \cdot p_j$ is the density of the equal figures in the j -th field. The distribution according to brightness, size and direction can be assumed as statistically independent for a great class of random optic fields and then the compound probability density is determined by the relationship:

$$w(L, a, b, \alpha) = w(L) \cdot w(a, b) \cdot w(\alpha) \quad (5)$$

On the basis of these considerations the methodology for obtaining of a physical and mathematical model of random optic fields can be summarized as follows:

- The autocorrelation function of the elementary fields is found on the basis of the given probability densities (Eq. (5)) and using the relationship (2) and (3).
- The common autocorrelation function is found using Eq. (4) for the total field composed by N brightness components with rectangular, elliptic and triangular shapes and respective probabilities p_\square, p_\bullet and p_\triangle .

III. EXEMPLARY APPLICATIONS

A. Measuring the Movement Velocity of Rolled Steel

It is necessary to model a random brightness field for the purposes of the construction and adjustment of the optoelectronic equipment of a correlation measuring device for determining the velocity of rolled steel movement. After the preliminary analysis of the surface characteristics, it has been decided that the autocorrelation function of the brightness field should be of the following form (Fig. 1):

$$R(\mu, \nu) = \sigma_L^2 \exp\left(-\frac{\mu}{T_\mu}\right) \cdot \exp\left(-\frac{\nu}{T_\nu}\right) \quad (6)$$

where σ_L^2 is the variance of the random brightness of the field, T_μ - the correlation interval along the x -axis, T_ν - the correlation interval along the y -axis. $T_\mu = 15 \text{ mm}$ and $T_\nu = 10 \text{ mm}$ for the case in particular.

A uniform law of distribution has been chosen for easier generation of the particular sizes:

$$w(L, a, b) = \frac{1}{L_{\max} - L_{\min}} \cdot \frac{1}{a_{\max} - a_{\min}} \cdot \frac{1}{b_{\max} - b_{\min}} \quad (7)$$

The following expression is obtained for an average density of n_0 sources/unit area of the modeled field with ellipsoidal sources of random size and brightness:

$$R_\Sigma(\mu, \nu) = n_0 R_{eq}(\mu, \nu) \quad (8)$$

where $R_{eq}(\mu, \nu)$ is the autocorrelation function of a single source.

After substitution of (6) in (8) the following relationship is obtained for the autocorrelation function:

$$R_\Sigma(\mu, \nu) = \frac{n_0 \left(\bar{L}^2 + \frac{\Delta L^2}{3} \right)}{(a_{\max} - a_{\min}) \cdot (b_{\max} - b_{\min})} A \cdot B \quad (9)$$

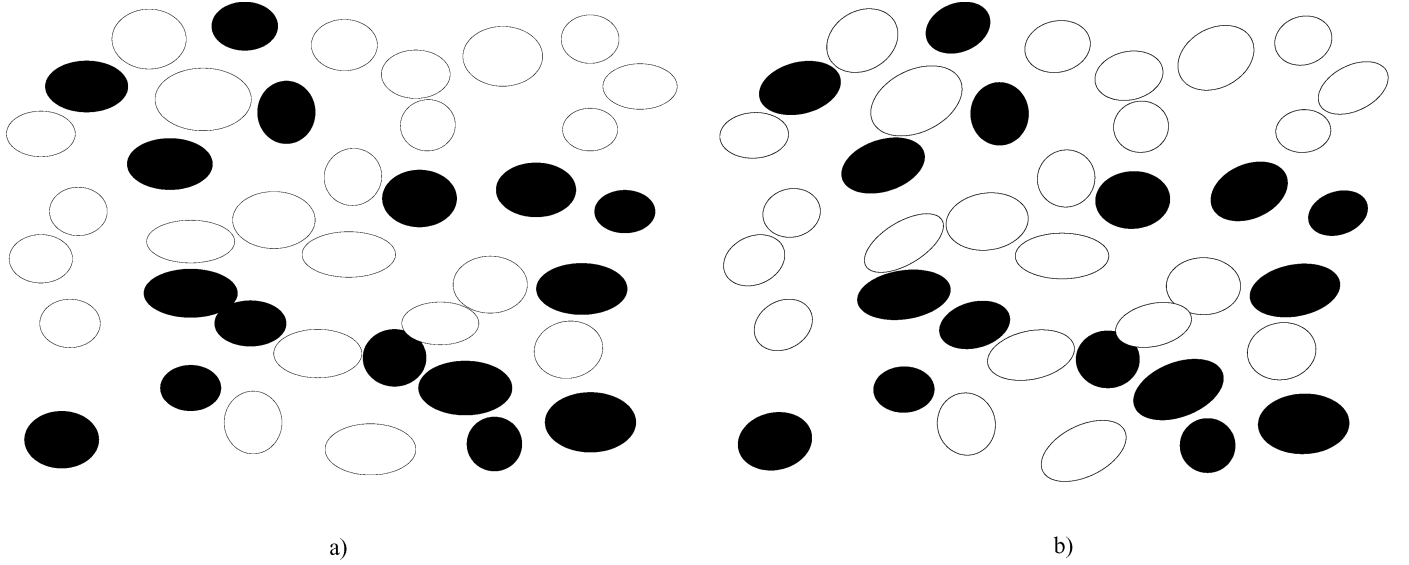


Fig. 2. Fragments of the generated random brightness field: a) ellipsoidal sources of equal orientation; b) ellipsoidal sources rotated at different angles according to the x -axis

where

$$A = \frac{a_{\max}}{2} - a_{\max}|\mu| + \frac{\mu^2}{2}$$

and

$$B = \frac{b_{\max}}{2} - b_{\max}|\nu| + \frac{\nu^2}{2}$$

Additionally, for achieving greater reliability, it can be assumed that the single sources are randomly oriented with respect to the x -axis and the distribution density according to the angle is:

$$w(\alpha) = \frac{1}{\alpha_{\max} - \alpha_{\min}} \quad (10)$$

$$\begin{cases} \bar{L}_{\Sigma} = n_0 \bar{L}_0 \bar{a} \bar{b} \\ \sigma_{\Sigma}^2 = n_0 (\bar{L}_0 + \sigma_{L_0}^2) \bar{a}^2 \bar{b}^2 \end{cases}$$

The modeling of the random brightness field by the autocorrelation function (6) yields for the particular numerical values:

$$\bar{a} = 18 \text{ mm}, a_{\max} = 23 \text{ mm}, a_{\min} = 13 \text{ mm}$$

$$\bar{b} = 12 \text{ mm}, b_{\max} = 15 \text{ mm}, b_{\min} = 9 \text{ mm}$$

$$\bar{L}_0 = 0,5 \text{ (relative units).}$$

It is accepted here that $L_{\min} = 0$ and $L_{\max} = 1$, since the modeled brightness field can be illuminated by an external controlled source. In this case the value is $\sigma_{L_0}^2 = \frac{1}{12}$. The values of n_0 can be consequently within the range from 1 to about 30 sources/dm². Twelve sources/dm² have been accepted for the case in particular. Fragments of the generated random brightness field are presented in Fig. 2.

Emitting surfaces are also modeled using elementary shapes of different sizes. As an example the model of emitting

surface is constructed, by introducing ellipsoidal sources of two different sizes:

$$\bar{a}_1 = 18 \text{ mm}, a_{1\max} = 23 \text{ mm}, a_{1\min} = 13 \text{ mm}$$

$$\bar{b}_1 = 12 \text{ mm}, b_{1\max} = 15 \text{ mm}, b_{1\min} = 9 \text{ mm}$$

$$\bar{a}_2 = 5 \text{ mm}, a_{2\max} = 7 \text{ mm}, a_{2\min} = 3 \text{ mm}$$

$$\bar{b}_2 = 3 \text{ mm}, b_{2\max} = 4 \text{ mm}, b_{2\min} = 2 \text{ mm}$$

Fragments of the generated random brightness field are presented in Fig. 3. Fragments of the tables, containing numerical values X_i , Y_i , \bar{a}_{1i} , \bar{b}_{1i} , \bar{a}_{2i} , \bar{b}_{2i} , α_i and L_i are presented in Table 1 and Table 2.

The computer models are implemented in the MathCAD environment.

V. CONCLUSIONS

The problems related to the modeling and simulation of fields with random brightness distribution consisting of multiple elementary sources with different geometrical shapes and characteristics have been presented. A methodology has been developed for determining the autocorrelation function of fields comprising a finite number of sources with given statistical characteristics in the observed space.

The developed methodology has been illustrated by particular examples, related with the adjustment and testing of passive optoelectronic systems for measuring the velocity of rolled steel movement and for distant observation of moving objects and landscapes. A number of practical applications have proved its efficiency. The computer implementation of the models is presented. The generated models can be also used for the solution of other problems connected with correlation measuring technique.

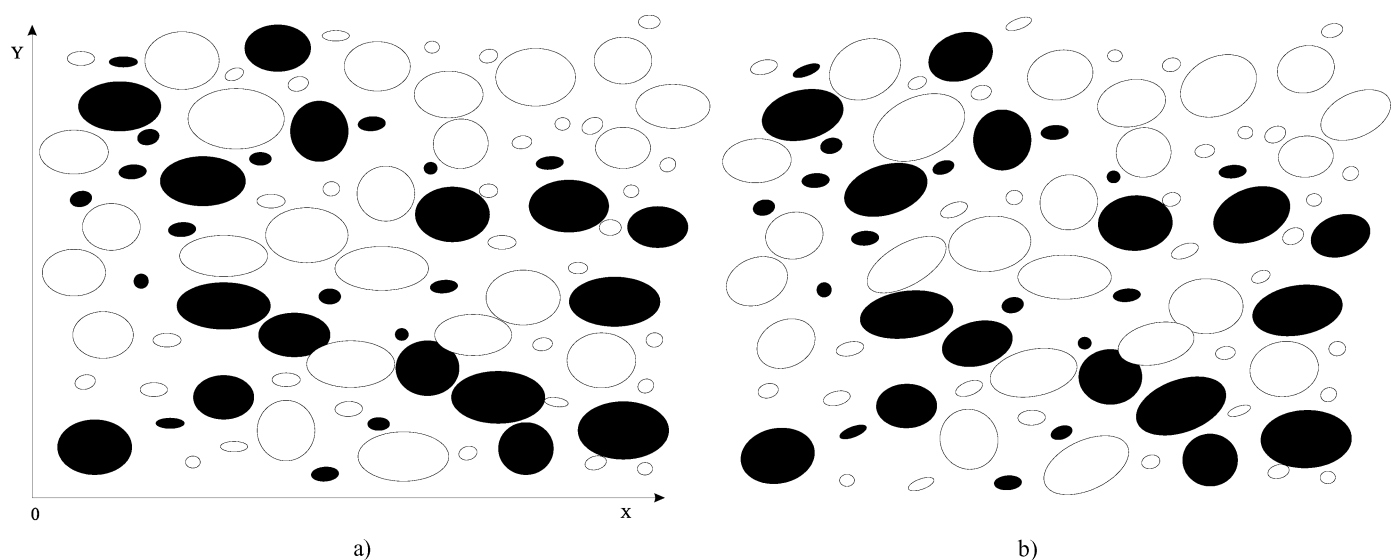


Fig. 3. Fragments of the generated random brightness field: a) ellipsoidal sources of two different sizes and equal orientation; b) ellipsoidal sources of two different sizes and rotated at different angles according to the x -axis

TABLE 1

FRAGMENT OF THE GENERATED RANDOM BRIGHTNESS FIELD $\bar{a}_1 = 18 \text{ mm}$ $\bar{b}_1 = 12 \text{ mm}$

	X_i	Y_i	a_{1i}	b_{1i}	α_i	L_i
1	20.16	19.76	13.14	11.44	16.87	0.96
2	112.79	31.10	16.64	9.37	29.75	0.17
3	79.05	36.72	14.07	10.77	12.65	0.09
4	140.35	79.31	19.41	10.50	7.88	0.69
5	57.09	43.20	14.92	11.95	3.98	0.05
6	12.25	68.57	18.75	11.41	16.28	0.94
7	19.23	89.05	18.50	14.96	12.82	0.33
8	129.39	41.96	17.93	14.50	0.59	0.11

TABLE 2

FRAGMENT OF THE GENERATED RANDOM BRIGHTNESS FIELD $\bar{a}_2 = 5 \text{ mm}$ $\bar{b}_2 = 3 \text{ mm}$

	X_i	Y_i	a_{2i}	b_{2i}	α_i	L_i
1	56.78	37.49	6.89	2.19	22.04	0.37
2	3.64	4.35	4.91	3.10	7.45	0.63
3	18.50	93.74	6.87	2.03	2.75	0.98
4	47.48	27.75	4.00	2.57	16.60	0.59
5	38.08	38.48	6.99	4.00	18.80	0.86
6	63.94	28.20	3.06	2.16	27.02	0.34
7	24.28	40.60	3.77	2.32	22.14	0.51
8	46.70	4.55	3.67	2.02	8.86	0.65

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