Influence of Parameters on Image Compression with Inverse Difference Pyramid

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Abstract – **In the paper is analyzed the influence of some of the parameters used in the process of still image compression with IDP. Special attention is paid to the choice of approximating orthogonal transforms in the pyramid levels. The results got in result of the research are compared with the results obtained with compression with standard JPEG.**

Keywords – **Still image compression, Inverse difference pyramid, Layered image transfer, Compression ratio.**

I. Introduction

In the paper is presented one investigation on the new image compression method – the Inverse Difference Pyramid (IDP) [1,2], aimed at the influence of multiple parameters on the compression ratio and on the quality of the restored image. Specific feature of the method is the pyramidal image decomposition in the frequency or parametric domain, in result of which every successive layer consists of larger number of coefficients, i.e. the pyramid, constituted of these coefficients, is inverse. The higher pyramid levels correspond with higher image quality after the restoration. The special header of the compressed image data contains information about the number of pyramid levels, the used coefficients, etc. The ability for flexible change of these parameters permits easy application of the method for different aims – medical images archiving and filtration, e-commerce, remote learning, etc.

II. Algorithms of Basic IDP Method

The coding process in accordance with the basic IDP method [1], using orthogonal transforms, is performed following the steps below:

Step 1: The matrix $[B(i,j)]$ which represents the original digital halftone image is divided in K sub-images with size $2^n x 2^n$, defined in accordance with pixel's correlation interval.

Step 2. Elements $[S_{k_p}(i,j)]$ of the sub-image matrix with number $k_p=1,2,..,4^pK$ from level $p=0,1,..,P-1$ of IDP pyramid with P levels are defined in accordance with the relation:

$$S_{k_p}(i,j)=\begin{cases} B_{k_0}(i,j) & \text{for} \quad p=0; \\ E_{k_{p-1}}(i,j) & \text{for} \quad p=1,2,...,P-1, \end{cases} \tag{1}$$

where $k_p=1,2,..,4^pK$ and $i,j = 0,1,2,.., 2^{n-p}-1$.

Here $B_{k_0}(i,j)$ is pixel (i,j) from sub-image $k_o=1,2,..,K$ in the zero level ($p = 0$), which is the original image; $E_{k_{p-1}}(i,j)$ - the pixel (i,j) from the difference image k_p in pyramid level p for $p=1,2,..,P-1$.

Step 3: The elements from the sub-image $[S_{k_p}(i,j)]$ are processed with "truncated" orthogonal transform. The coefficients of the corresponding transform are defined with the relation:

$$s_{k_p}(u,v)=\begin{cases} s_{k_p}(u_r,v_r) & \text{for } m_p(u,v)=1 \\ 0 & \text{- for } m_p(u,v)=0 \end{cases} \tag{2}$$

where $u, v = 0,1,2,..,2^{n-p}-1$ and $p = 0,1,..,P-1$;

$$s_{k_p}(u_r,v_r)=\frac{1}{4^{n-p}}\sum_{i=0}^{2^{n-p}-1}\sum_{j=0}^{2^{n-p}-1}S_{k_p}(i,j)\,t_p(i,j,u_r,v_r)$$

for $r=1,2,..,R_p$, when $m_p(u,v)$ are the elements of the binary matrix-mask $[M_p]$ with size $2^{n-p}x2^{n-p}$ which defines the position of the corresponding "meaning" spectral coefficients $s_{k_p}(u_r,v_r)$; $R_p=\sum_{i=0}^{2^{n-p}-1}\sum_{j=0}^{2^{n-p}-1}m_p(i,j)$ - the number of "meaning" coefficients in level p, selected in advance in the interval $1\le R_p<4^{n-p}$; $t_p(i,j,u_r,v_r)$ – element (i,j) from the "basic" image (the kernel of the transform) with spatial frequency (u_r,v_r) in level p, defined by the selected orthogonal transform.

Elements $m_p(u,v)$ from the matrix-mask $[M_p]$ in Eq. (2) are defined in accordance with the algorithm:

3.1. Calculation of mean sub-image pixels:

$$\overline{S}_p(i,j)=\frac{1}{4^pK}\sum_{k_p=1}^{4^pK}S_{k_p}(i,j) \tag{3}$$

for $i, j = 0,1,2,..,2^{n-p}-1$;

3.2: Calculation of mean transform coefficients:

$$\overline{s}_p(u,v)=\frac{1}{4^{n-p}}\sum_{i=0}^{2^{n-p}-1}\sum_{j=0}^{2^{n-p}-1}\overline{S}_p(i,j)\,t_p(i,j,u,v) \tag{4}$$

for $u, v = 0,1,2,..,2^{n-p}-1$;

3.3: The modules of the mean coefficients' values are arranged in monotonously decreasing order, limited by the chosen value of R_p:

$$|\overline{s}(u_1,v_1)|\ge|\overline{s}(u_2,v_2)|\ge\ldots\ge|\overline{s}(u_{R_p},v_{R_p})| \tag{5}$$

3.4: The elements $m_p(u,v)$ are defined using Eq. (5):

$$m_p(u,v)=\begin{cases} 1 & \text{for } u=u_r \text{ and } v=v_r \text{ when } r=1,2,..,R_p; \\ 0 & \text{-} \qquad \text{in other cases.} \end{cases} \tag{6}$$

Step 4: The quantizated value of every "meaning" coefficient is calculated in accordance with:

$$s^q_{k_p}(u_r,v_r)=[s_{k_p}(u_r,v_r)/\Delta_p(u_r,v_r)]_{\text{integer}} \quad \text{for } r = 1,2,.., R_p \tag{7}$$

where $\Delta_p(u_r,v_r)$ is one element of the quantizating matrix $[Q_p]$ for level p, chosen in advance on the basis of

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experimental results showing the influence of the quantization error on the restored image quality, $[*]_{\text{integer}}$ – operator used for the definition of the integer of the number in brackets.

Step 5: Dequantizated value of every quantizated coefficient is calculated in accordance with Eq. (7):

$$s'_{k_p}(u_r, v_r) = s_{k_p}(u_r, v_r).\Delta_p(u_r, v_r) \qquad (8)$$

Step 6: The approximating model $\tilde{S}_{k_p}(i,j)$ for pyramid sub-image k_p is defined using inverse orthogonal transform:

$$\tilde{S}_{k_0}(i,j)=\tilde{B}_{k_0}(i,j)=4^{-n}\sum_{u=0}^{2^n-1}\sum_{v=0}^{2^n-1} s'_{k_0}(u,v)\, t_0(i,j,u,v) \text{ for } p = 0 \quad (9)$$

and $i, j = 0,1,2,..,2^n-1$,

$$\tilde{S}_{k_p}(i,j) = 4^{p-n}\sum_{u=0}^{2^{n-p}-1}\sum_{v=0}^{2^{n-p}-1} s'_{k_p}(u,v)\, t_p(i,j,u,v) \qquad (10)$$

for $p=1,.., P-1$ and $i, j = 0,1,2,..,2^{n-p}-1$

Here $4^{p-n} t_p(i,j,u,v)$ is the kernel of the selected inverse orthogonal transform for level p.

Step 7: The elements of the difference image k_p of level p are defined:

$$E_{k_p}(i,j)=\begin{cases} B_{k_0}(i,j)-\tilde{B}_{k_0}(i,j) & \text{for } \quad p=0; \\ S_{k_p}(i,j)-\tilde{S}_{k_p}(i,j) & \text{for } \; p=1,2,..,P-1. \end{cases} \qquad (11)$$

and $i, j = 0,1,2,..,2^{n-p}-1$.

Step 8: Coefficients $s^q_{k_p}(u_r,v_r)$ from all sub-images in pyramid level p are arranged in R_p two-dimensional (2D) massifs in accordance with their spatial frequency (u_r, v_r) for $r=1,2,..,R_p$.

Step 9: Every 2D massif of spectral coefficients is transformed in one-dimensional, using recursive Hilbert scanning [3]. These massifs for every IDP layer are arranged in one sequence, at the beginning of which is added special header, containing information about the mask [M_p], the number of the selected matrix [Q_p], the values of R_p and P, the kind of orthogonal transform for every layer, initial pyramid level, etc.

Step 10: Lossless coding, performed in 2 stages, is applied on the data in the one-dimensional massif:

10.1: adaptive coding of the lengths of series of equal symbols (ARLE);

10.2: adaptive coding with modified Huffman code. In result is obtained the compressed image data, which could be transferred via communication channel, or saved in memory.

The compressed data decoding is performed as follows:

1. Huffman and ARLE decoding;
2. Dequantization of coefficients $s^q_{k_p}(u_r,v_r)$, as in Eq. (8);
3. Calculation of the model for sub-image $\tilde{S}_{k_p}(i,j)$ using inverse orthogonal transform, Eqs. (9)-(10);
4. Image restoration in correspondence with:

$$B'(i,j) = \tilde{B}_{k_0}(i,j) + \sum_{p=1}^{P-1}\tilde{S}_{k_p}(i,j) \text{ for } i,j = 0,1,2,..,2^n-1 \qquad (12)$$

Here $B'(i,j)$ is the greylevel of element (i,j) in restored image.

Algorithms for implementing IDP method with polynomial functions (resp. "oriented" surfaces) are described in [4].

For color images, represented in standard 4:4:4, the described algorithm is applied for each of the initial color components R,G,B. In order to obtain higher compression ratio these components are transformed in the components Y,Cr,Cb for standard 4:2:0 in accordance with REC. 601-R of ITU [5]. For every component is build single pyramidal decomposition. The decoding is performed in reverse order.

II. Influence of Pyramid Parameters

The IDP method permits the approximating model for every pyramid level to be different. In the research were used following models: Discrete Cosine Transform (DCT), Walsh-Hadamard Transform (WHT), "oriented" planes (3 coefficients for sub-image) and surfaces (polynomials of 2-d order with 4 coefficients for sub-image). In TABLE I is shown the influence of the selected approximation model in the pyramid layers on the compression ratio and image quality.

TABLE I

INFLUENCE OF APPROXIMATION MODEL

NO.	WHT		DCT		PLANE		SURFACE	
	K	PSNR	K	PSNR	K	PSNR	K	PSNR
1	4.89	42.02	4.12	42.06	4.14	41.98	4.03	41.68
2	5.01	41.00	4.15	41.05	4.18	40.98	4.06	40.75
3	5.15	39.52	4.21	39.55	4.24	39.50	4.12	39.33
4	6.47	36.80	4.52	36.85	4.53	36.78	4.37	36.73
5	6.72	34.99	4.56	35.03	4.58	34.98	4.42	34.94
6	7.58	32.65	7.42	32.70	6.92	32.72	6.49	32.72
7	8.14	31.21	7.87	31.26	7.31	31.26	6.84	31.24
8	8.72	29.89	8.42	29.98	7.82	29.94	7.32	29.77
9	9.23	28.75	8.90	28.90	8.27	28.84	7.73	28.60
10	10.28	26.99	9.73	27.41	8.96	27.35	8.35	27.02
11	11.10	26.54	10.83	26.91	9.77	26.86	9.69	26.50
12	12.70	25.82	12.33	26.14	10.99	26.09	10.77	25.78
13	14.47	25.09	13.97	25.37	12.31	25.33	12.01	25.07
14	16.12	24.15	15.50	24.39	13.56	24.35	13.13	24.14
15	24.10	22.59	22.21	22.82	18.72	22.74	17.40	22.59
16	31.67	22.24	29.07	22.46	27.48	22.35	24.27	22.21
17	41.57	21.56	37.35	21.77	38.26	21.63	33.02	21.51
18	67.89	19.96	61.66	20.70	85.38	18.75	79.39	16.90
19	99.64	19.29	87.74	19.85	No change		No change	
20	164.7	18.85	137.9	19.44	No change		No change	
21	189.5	18.60	168.7	19.06	No change		No change	

For the experiments was used the color image "Myanmar", 1024x768 pixels, 24 bpp. In all cases the used number of pyramid layers was 2 (positions 1-17, R_0=4, R_1=16) or 1 (positions 18-21, R_0=4), starting with sub-image of 8x8 pixels for the lower layer and changing the quantization matrices [Q_0] and [Q_1]. Column WHT contains results obtained using only the Walsh-Hadamard transform in the two pyramid levels; column DCT contains results from the case when coefficients in the lower pyramid layer was calculated with DCT, and in the higher – with WHT; the combination for column PLANE is similar - the approximation in the lower layer is with "oriented" plane; and in column SURFACE lower layer was approximated with polynomial model. The last 3 positions in columns PLANE and SURFACE have no change because for these approximations the number of coefficients cannot be less than 3 (correspondingly 4) in the lower layer. Results show that higher compression ratio is obtained using WHT in both layers, but the quality of the restored image is worse. The compromise is to use DCT for the lower layer and WHT – for the higher one. Approximations with "oriented" planes and surfaces offer worse results.

The IDP method permits to use different kinds of color transform as well. The original images were 24 bpp bmp files. In TABLE II are shown results obtained using the following 4 kinds of color transforms: PAL, NTSC, RCT[6] and ITU REC. 601-R. All results in the table are for the image Myanmar (1024x768, 24 bpp). The comparison shows that results obtained with ITU REC. 601-R are the best. There is possibility results to differ for some specific images but the selection of ITU REC. 601-R suits most frequently processed natural images very well. Data in TABLE II shows that very good results are obtained using NTSC too. Transforms with PAL and RCT give close results, but the visual quality of the restored images is slightly worse. Anyway all these color transforms could be successfully used in the pyramidal decomposition.

TABLE II
INFLUENCE OF THE COLOR TRANSFORM

NO.	REC.601		PAL		NTSC		RCT	
	K	PSNR	K	PSNR	K	PSNR	K	PSNR
1	4.89	42.02	4.87	42.08	4.93	42.10	4.87	40.64
2	5.01	41.00	4.98	41.05	5.05	41.06	4.99	39.89
3	5.15	39.52	5.13	39.56	5.20	39.56	5.13	38.74
4	6.47	36.80	6.44	36.88	6.55	36.86	6.32	36.32
5	6.72	34.99	6.69	35.05	6.82	35.04	6.58	34.69
6	7.58	32.65	7.55	32.73	7.70	32.72	7.72	32.43
7	8.14	31.21	8.10	31.27	8.29	31.27	8.29	31.06
8	8.72	29.89	8.69	29.94	8.87	29.97	8.89	29.77
9	9.23	28.75	9.19	28.78	9.38	28.80	9.39	28.66
10	10.28	26.99	10.23	27.00	10.49	27.00	10.46	26.95
11	11.10	26.54	11.06	26.56	11.38	26.56	11.30	26.50
12	12.70	25.82	12.65	26.83	13.06	25.84	12.94	25.78
13	14.47	25.09	14.43	25.11	14.92	25.12	14.76	25.05
14	16.12	24.15	16.06	24.16	16.64	24.18	16.43	24.12
15	24.10	22.59	22.12	22.83	22.81	22.84	22.47	22.80
16	31.67	22.24	29.02	22.47	30.13	22.49	29.61	22.43
17	41.57	21.56	41.44	21.58	43.67	21.62	42.47	21.53
18	67.89	19.96	67.81	20.16	72.34	20.18	69.25	20.12
19	99.64	19.29	99.29	19.30	105.1	19.32	101.2	19.26
20	164.7	18.85	165.3	18.87	175.1	18.90	167.8	18.82
21	189.5	18.60	190.3	18.62	201.8	18.66	192.8	18.56

The IDP method permits for every pyramid layer to be used different color transform, but because of the close results for the compression ratio and image quality, usually for one pyramid is used only one kind of color transform. Anyway the big number of experiments with images of any kind – faces, natural pictures, documents, etc., showed slight but constant advantage for ITU REC. 601-R.

More interesting results are obtained in the case when the color image is processed in accordance with the used color standard: 4:4:4, 4:2:2, 4:1:1 or 4:2:0 (TABLE III).

TABLE III
COMPARISON OF STANDARDS 4:4:4, 4:2:0, 4:2:2 and 4:1:1

No.	4:2:0		4:4:4		4:2:2		4:1:1	
	K	PSNR	K	PSNR	K	PSNR	K	PSNR
1	4.89	42.02	3.44	45.80	4.26	42.46	4.94	38.99
2	5.01	41.00	3.53	43.83	4.36	41.32	5.05	38.35
3	5.15	39.52	3.61	41.46	4.48	39.68	5.19	37.31
4	6.47	36.80	4.96	37.74	5.85	36.86	6.53	35.34
5	6.72	34.99	5.12	35.66	6.06	35.04	6.80	33.87
6	7.58	32.65	5.93	33.10	6.88	32.72	7.63	31.91
7	8.14	31.21	6.30	31.53	7.38	31.23	8,21	30.59
8	8.72	29.89	6.82	30.17	7.92	29.90	8.79	29.35
9	9.23	28.75	7.14	29.01	8.34	28.76	9.28	28.25
10	10.28	26.99	7.80	27.17	9.19	26.99	10.35	26.60
11	11.10	26.54	8.29	26.72	9.84	26.55	11.20	26.19
12	12.70	25.82	9.26	25.99	11.12	25.84	12.77	25.51
13	14.47	25.09	10.25	25.27	12.50	25.12	14.50	24.81
14	16.12	24.15	11.16	24.34	13.83	24.19	16.16	23.90
15	24.10	22.59	15.73	22.74	20.15	22.62	24.09	22.40
16	31.67	22.24	20.54	22.39	26.40	22.27	31.64	22.06
17	41.57	21.56	26.89	21.72	34.59	21.61	41.64	21.40
18	67.89	19.96	41.33	20.31	55.20	20.19	67.95	20.03
19	99.64	19.29	60.38	19.46	81.19	19.34	100.3	19.17
20	164.7	18.85	102.9	19.01	136.3	18.90	164.8	18.47
21	189.5	18.60	115.35	18.77	154.2	18.66	189.1	18.50

When standard 4:4:4 is used all parameters, taking part in the processing (pyramid layers, number of coefficients, approximation model) of color components, must be same. In this case the number of calculations is bigger than in the other cases and respectively the time necessary for the processing is longer. For all other standards parameters could be set individually for each color component. The results show that the certain favorite is standard 4:2:0. The standard 4:4:4 ensures highest quality for the restored image but the compression ratio is worse. The standard 4:1:1 gives high compression ratios (anyway not much higher than in case of 4:2:0), but the quality of the restored image is worse not only in absolute values but visually as well.

Another interesting relation shows the influence of the transform coefficients on the compression ratio and image quality. Some investigations show that in case when coefficients with spatial frequencies (u,v) = (0,0), (1,0), (0,1) (1,1) are used in the lower layer, there is no need coefficient

(0,0) to be calculated and transferred for the following higher pyramid layer. This is illustrated with the experimental results again for image "Myanmar" (1024x768, bpp). The results show the changes due to removal of coefficient (0,0) are negligible concerning the restored image quality, and the compression ratio is higher. The removal of coefficient (1,1) results in worse image quality and obviously lower compression ratio. The meaning of curves in Fig. 1 is as follows: the curve, called "data 1" corresponds with the default pyramid parameters configuration; the curve, called "data 2" shows the results for the case when coefficient (0,0) was removed, and "data 3" – the case with coefficient (1,1) removed. All other parameters were the same.

Specific for the inverse pyramidal decomposition is that it ensures very high compression ratios for which image quality is better than when using standard JPEG. This is illustrated with the examples shown in Fig. 2. The example images are: Myanmar and Birds (both 1024x768 pixels, 24 bpp). Maximum compression for JPEG was obtained with Microsoft Photo Editor 3.01, which does not permit higher compression.

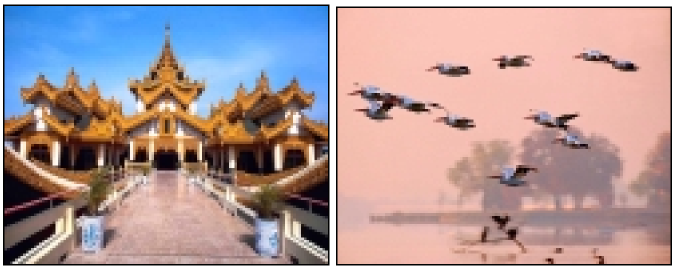

Fig.2a. Image "Myanmar" 24 bpp   Image "Birds" 24 bpp

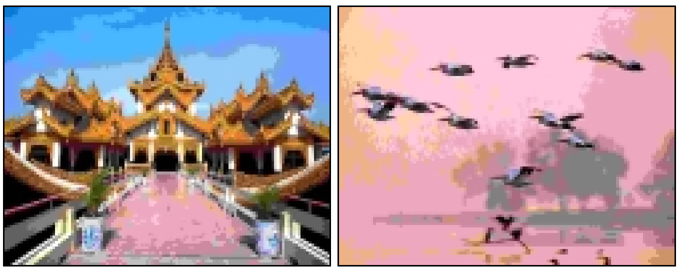

Fig.2b. 0.14 bpp (JPEG)   0.22 bpp (JPEG)

Fig.2c. 0.14 bpp (IDP)   0.22 bpp (IDP)

III. Summary

The investigation on the influence of participating parameters on the compression ratio and image quality confirms the efficiency of the presented algorithms. On the basis of the obtained results were made the following conclusions.

1. The definition of matrix-mask [M_p] in accordance with Eqs. (2)-(6) improves method efficiency and permits its adaptation in accordance with image contents.

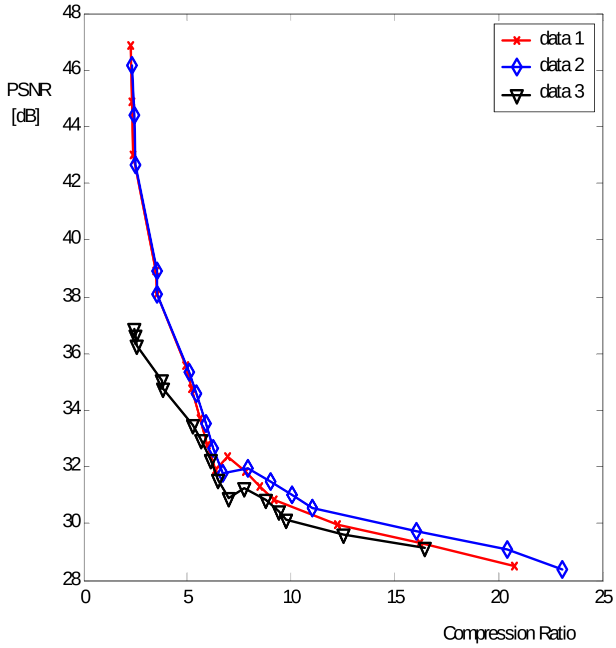

Fig.1. Image "Myanmar": influence of coefficients (0,0) and (1,1): data1 – combination of coefficients set with matrix-mask; data2 - missing coefficient (0,0); data3 - missing coefficient (1,1).

2. The experiments show that best results for compression of natural images are obtained in case when the approximation model in the lowest pyramid level (zero level) is DCT, and in the upper levels (levels one and two) – WHT (illustrated with Table I).

3.The comparison of color space models shows that best results concerning compression are obtained with ITU Rec. 601-R and standard 4:2:0.

4. The results for big values of the compression ratio show that algorithm IDP ensures much better image quality than JPEG, and comparable with that of JPEG 2000 [6].

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