QPSK System Performance in the Presence of Nakagami Fading and Interference

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Abstract - **The aim of this paper is the determining the system performance in detecting the QPSK signal. The error probability is determined when the signal, Gaussian noise, interference, Nakagami fading and imperfect carrier phase recovery are taken into consideration. Phase locked loop, as the constituent part of the receiver, is used in providing the synchronization reference signal extraction, which is assumed to be imperfect. The reference carrier is extracted by the first order loop. The system performance is determined when the signal affected by Nakagami fading, interference and Gaussian noise are applied at the input of the receiver.**

Key words – **QPSK, PLL, Nakagami fading**

I. INTRODUCTION

The performance evaluation of binary and *M*-ary (*M*>2) phase-shift-keying communication systems have been analysed in a great variety of papers, which have appeared in the literature [1-7]. Quaternary phase-shift-keying (QPSK or 4-PSK) systems have the greatest practical interest of all nonbinary (multiposition) systems of digital transmission of messages by phase modulated signals. Currently, QPSK is one of the prevalent modulations in use for digital communication systems (since bandwidth efficiency) [1,2]. The only significant penalty factor is an increased sensitivity to carrier phase synchronisation error.

Any successful transmission of information through a digital phase-coherent communication system requires a receiver capable of determining or estimating the phase and the frequency of the received signal with as few errors as possible; any noise associated with carrier leads to degradation of the detection performance of the system. In practice, quite often the phase locked loop (PLL) is used in providing the desired reference signal [3,4,5,6]. Frequently, a PLL system must operate in such conditions where the external fluctuations due to the additive noise are so intense that classical linear PLL theory neither characterises adequately the loop performance, nor explain the loop behaviour [7]. Numerical results for QPSK system is presented so that this results combined with the characteristic of the phase recovery circuit will enable the best practical design of a QPSK system.

The error probability, as a measure of systems quality, is an important issue and has received much attention in the literature. Noise influence and interference are often fundamental limiting factors in digital transmission systems. An expression for the bit error probability was calculated when the signal and Gaussian noise are applied at the input of the QPSK system [8]. QPSK system performance when the signal, Gaussian noise, interference, Nakagami fading and imperfect carrier phase recovery are considered as source of degradation, are determined in this paper.

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II. SYSTEM MODEL

Let the input signal at QPSK receiver consists of the signal, interference and Gaussian noise:

$$r(t) = A\cos\omega_0 t + A_1\cos(\omega_0 t + \theta) + n(t), \qquad (1)$$

where A is a signal amplitude, ω_o is a constant carrier frequency, A_1 is a interference amplitude, $n(t)$ is a Gaussian noise and θ is the uniformly distributed phase with the probability density function:

$$p_1(\theta) = \frac{1}{2\pi}, \quad \{-\pi \le \theta \le \pi\}. \qquad (2)$$

Input signal can be also written with the form:

$$r(t) = AR\cos(\omega_0 t + \psi) + n(t),$$
$$\eta = \frac{A_1}{A}, \quad R = \sqrt{1+\eta^2 + 2\eta\cos\theta}, \quad \psi = \mathrm{arctg}\frac{\eta\sin\theta}{1+\eta\cos\theta}, \qquad (3)$$

where η is a interference to signal ratio. It is assumed that the signal is affected by the Nakagami fading:

$$p(A) = \frac{2m^m A^{2m-1}}{\Gamma(m)\Omega_o} e^{-\frac{mA^2}{\Omega_o}}, \quad A \ge 0. \qquad (4)$$

where $\Omega = \overline{A^2}$. The constant m is the fading parameter. Hence the one side Gaussian pdf is obtained for m)0.5 and m)1 gives the Rayleigh pdf. The nonfading case correspodents to $m \to \infty$.

From now on, interference, additive Gaussian noise, fading and imperfect phase carrier recovery, are taken into account in our detection analysis. All other functions are considered ideal. The block diagram of a QPSK receiver would be adopted is shown in Figure 1. The recovered carrier signal is assumed to be in the form of the sin wave. Also, it would be adopted that a original message is in binary form.

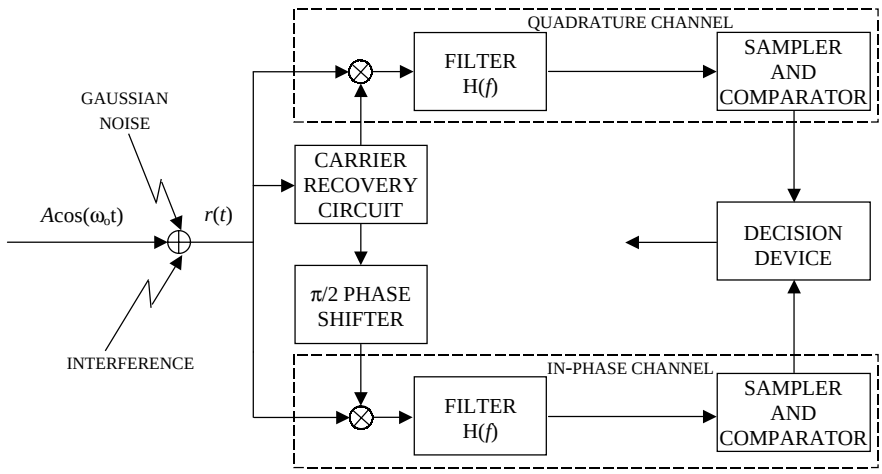


Figure 1. Block diagram of a QPSK receiver

Under the assumption of a constant phase in the symbol interval, the conditional error probability for the given phase error ϕ (the phase error ϕ is the difference between the receiver incoming signal phase and the voltage controlled oscillator output signal phase) can be written as [8],

$$P_{e/\phi}(\phi)=\frac{1}{4}\{\mathrm{erfc}\left[\sqrt{2R_b}\cos[(\pi/4)+\phi]\right] + \mathrm{erfc}\left[\sqrt{2R_b}\cos[(\pi/4)-\phi]\right]\} \tag{5}$$

where the function erfc(x) is the well known complementary error function defined as:

$$\mathrm{erfc}(x)=\frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-z^2/2}dz. \tag{6}$$

The received signal to noise spectral density ratio in the data channel (demodulator) denoted by R_b, is given by $R_b=E/N_o$, where E is a signal energy per bit duration T. N_o represents the normalised noise power spectral density in W/Hz, referenced to the input stage of the demodulator, since the signal to noise ratio is established at that point. The signal detection in receiver is accomplished by cross-correlation-and-sampling operation. The effect of filtering due to $H(f)$ in Figure 1. is not considered here.

The conditional steady state probability density function, for the non-linear PLL model with a known signal and noise at the PLL input, of modulo 2π reduced phase error is given by the following approximation [7]:

$$p(\phi)=\frac{e^{\beta\phi+\alpha\cos\phi}}{4\pi^2 e^{-\pi\beta}\,|I_{j\beta}(\alpha)|^2}\int_{\phi}^{\phi+2\pi} e^{-\beta x+\alpha\cos x}dx, \tag{7}$$

$I_{j\beta}(\alpha)$ is the modified Bessel function of complex order $j\beta$ and real argument α. The range of definition for ϕ in the previous equation is any interval of width 2π centered about any lock point $2n\pi$, with n an arbitrary integer. The parameters α and β, that characterise Eq.(7), for the first order non-linear PLL model in this case are:

$$\alpha=\alpha_0 R, \qquad \beta=\beta_0\Omega, \tag{8}$$

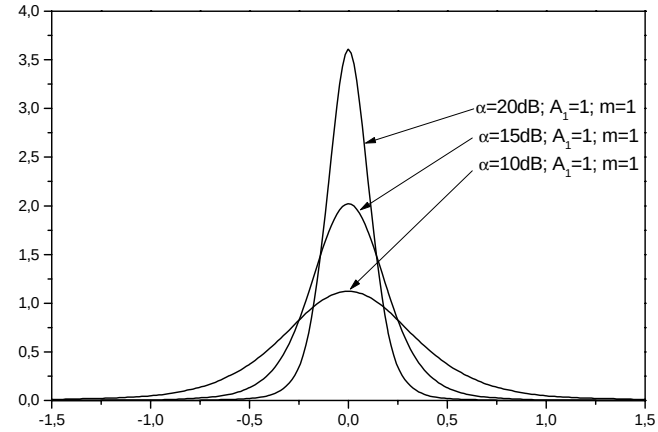
where α_0 and β_0 are constants [7,9]. The parameter α is a measure of the loop signal to noise ratio in the sense that the larger the value of α, the smaller are the deleterious effects due to noise reference signal. The parameter β is a measure of the loop stress. Ω is the loop detuning, i.e. the frequency offset of the first term in Eq.(3) defined by:

$$\Omega=\frac{d}{dt}(\omega_0 t+\psi)-\omega_0=\frac{\eta(\eta+\cos\theta)}{R^2}\frac{d\theta}{dt}. \tag{9}$$

Since $(d\theta/dt) = 0$, it follows $\Omega = 0$, i.e. $\beta = 0$. Therefore, the average steady-state probability density function of the phase error is:

$$p(\phi)=\int_A\int_\theta p(\phi/A,\theta)p(A)p(\theta)dAd\theta = \int_0^\infty\int_{-\pi}^{\pi}\frac{e^{\alpha_0 R\cos\phi}}{2\pi I_0(\alpha_0 R)}\frac{2m^m A^{2m-1}}{\Gamma(m)\Omega_o}e^{-\frac{mA^2}{\Omega_o}}\frac{1}{2\pi}dAd\theta. \tag{10}$$

Substituting Eq.(3) into Eq.(10) yields the probability density function of the phase error that is shown in Figure 2. The values of parameters α_o, σ_f and A_1 are given in figure.



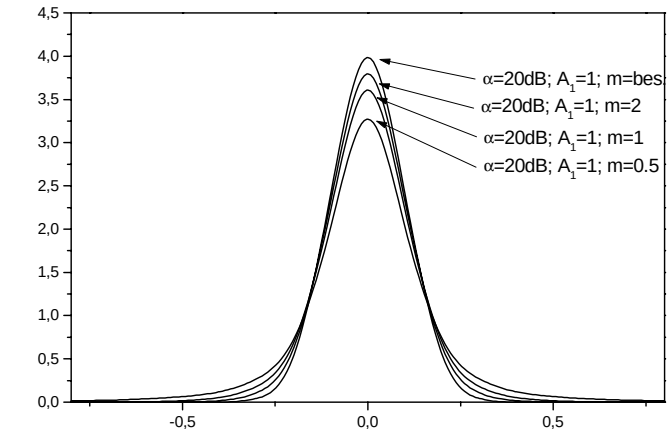


Figure 2. Probability density function of the phase for the non-linear first order PLL model

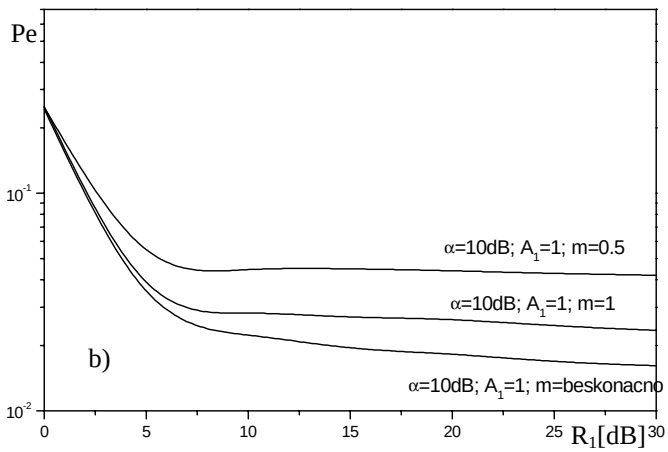


Figure 3. Average error probability performance of a QPSK coherent detector with a noisy carrier synchronization reference signal when αo is a parameter, while A1 and m are constants a), when m is a parameter, while αo and A1 are constants b)

III. SYSTEM PERFORMANCE

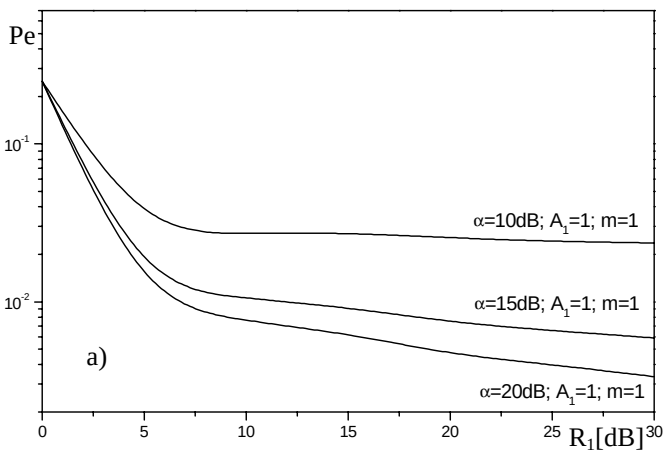
Substituting $R_b = R_1R^2$ in Eq. (5), where R_1 corresponds to the case when there is no interference, the conditional bit error probability, given both ϕ and θ is determined. The total error probability is determined by averaging the conditional error probability over random variables ϕ, θ and A :

$$P_e = \int_{\phi}\int_{\theta}\int_{A} P_{e/\phi}\, p(\phi / A,\theta)\, p(A)\, p(\theta)\, d\phi\, d\theta\, dA\,. \qquad (11)$$

On the basis on the Eq.(11) the bit error probability in the presence of the Nakagami fading becomes:

$$P_e = \frac{1}{16\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{0}^{\infty}\left\{\mathrm{erfc}\left[R\sqrt{R_1}\left(\cos\phi-\sin\phi\right)\right]+\mathrm{erfc}\left[R\sqrt{R_1}\left(\cos\phi+\sin\phi\right)\right]\right\} \frac{e^{\alpha_o R\cos\phi}}{I_0(\alpha_o R)}\frac{2m^m A^{2m-1}}{\Gamma(m)\Omega_o} e^{-\frac{mA^2}{\Omega_o}}\, d\phi\, d\theta\, dA \qquad (12)$$

The total error probability is computed on the basis of the Eq. (12) and is plotted versus signal to noise ratio (R_1 [dB]) at the demodulator input in Figure 3 a), and b). The values of α_o, m and A_1 are given in figures.



The total error probability, when the signal, Gaussian noise and interference are applied at the input of the receiver, as a function of the signal to noise ratio for Nakagami fading is shown in Figure 3. From the figures follows that the system error probability decreases with the increase of the signal to noise ratio (R_1). One can see that the system error probability increases with decrease of both, PLL parameter α_o, Figure 3 a), and fading parameter m, Figure 3 b).

IV. CONCLUSION

The quaternary PSK system is analysed by means of the system error probability, in this paper. Noise influence, interference, fading and imperfect carrier phase recovery are the limiting factors in the observed system performance. The interference is represented by cosinusoidal signal with the uniform distributed phase. The influence of the imperfect reference signal extraction is expressed by the probability density function of the PLL phase error.

The detailed analysis of the obtained numerical results is performed in this paper. Case when the signal affected by the Nakagami fading, Gaussian noise and interference are applied at the input of the receiver have been considered in this paper. The influence of the parameters α_o as well as the influence of the fading and the signal to noise ratio R_1 on the system error probability are especially considered. On the basis of the shown analysis one can conclude that the system has better performances if both, PLL parameter, α_o, and fading parameter, m, have a greater values.

However, from all figures, the large signal to noise ratio system error tends to a constant value (BER floor). In the BER floor area, the signal to noise ratio is relatively large with respect to parameter α_o and has therefore a small influence on the system error probability. It is seen from Figure 3 a) that this BER floor can be reduced by increasing the parameter α_o which depends on the applied PLL loop. On the basis of the shown analysis it is possible to determine the

QPSK system parameter α_o and useful signal power necessary to compensate the imperfect carrier extraction. This means that the presented conclusions can be useful in practice for the QPSK system design.

REFERENCES

[1] Stefanovi} M., Djordjevi} G., Djordjevi} I., Stojanovi} N., *Influence of Imperfect Carrier Recovery on Satellite QPSK Communication System Performance*, International Journal of Satellite Communications, Vol. 17, pp. 37-49, 1999.

[2] Stefanovi} M., Zlatkovi} N., Nikoli} Z., Stojanovi} N., *Performance of M-QAM System in the Presence of Phase Jitter and Cochannel Interference*, International Journal of Electronics, pp. 863-867, 1994.

[3] Stefanovi} M., Djordjevi} I., Djordjevi} G., Basta J., *Coherent Optical Heterodyne PSK Receiver Performance in Multi Channel Environment*, Journal of Optical Communications, Vol. 20, pp. 12-15, April 1999.

[4] Stefanovi} M., Dra~a D., Vidovi} A., Milovi} D., *Coherent Detection of FSK signal in the presence of Cochannel interference and noisy carrier reference signal*, International Journal of Electronics and Communications, No. 2, pp. 77-82, 1999.

[5] Stefanovi} M., Djordjevi} G., Djordjevi} I., *Performance of Binary CPSK Satellite Communication System in the Presence of Noises and Noisy Carrier Reference Signal*, International Journal of Electronics and Communications, No. 2, pp. 70-76, December 1999.

[6] Dra~a, D., Stefanovi} M., *Bit error probability of phase coherent communication systems in presence of noise and interference*, Electronics Letters 26 , No. 16, pp. 1234-1235, 1990.

[7] Viterby, J., *Principles of coherent communication*, McGraw-Hill, New York, 1966.

[8] Prahbu. K. V., *PSK Performance with Imperfect Carrier Phase Recovery*, IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-12, No.2, pp. 275-285, March 1976.

[9] Lindsey, C., *Synhroization systems in communication and control*, Prentice-Hall, Englewood Cliffs, 1972.

[10] Lindsey, C., *Nonlinear analysis and synthesis of generalized tracking system*, Proc. IEEE, 57, pp. 1705-1722, 1969.

[11] Abramowitz, M., Stegun, A., *Handbook of mathematical functions with formulas, graphs and mathematical tables*, New York, Dover Publications, Inc., 1970.

[12] Papulis A., *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, NewYork, 1965.