The Performance of the OFDM System in Presence of Frequency and Timing Offset

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Abstract - **The influence of the number of subcarriers on the error probability in presence of frequency and timing offset in the system with orthogonal frequency division multiplexing (OFDM) is investigated. In paper we use Gaussian approximation of the inter-carrier interference (ICI). The results show that, in the presence of frequency offset, the number of subcarriers does not affect the error probability, and the frequency offset significantly increases the error probability (in considered case for two orders of magnitude). The presence of timing offset also causes the error probability increase. In this case the number of subcarriers has influence on the value of error probability. At higher timing offsets the error probability depends only on the subcarriers number.**

Keywords - **Orthogonal Frequency Division Multiplexing, frequency offset, timing offset.**

I. Introduction

The effect of the timing offset on the error probability in OFDM system was investigated in paper [1]. The effect of the frequency offset was investigated in paper [2]. In this paper the effect of the frequency and timing offset as well as the number of subcarriers on the error probability of the system with orthogonal frequency division multiplexing (OFDM) will be investigated.

In OFDMA system variable data rate can be achieved using different number of subcarriers. By increasing the number of subcarriers, the spacing between the samples is reduced and sensitivity to the timing errors is increased. In this paper we will focus only on the inter-carrier interference (ICI) introduced by the frequency offset. The power of ICI introduced by the frequency offset also depends on the number of the subcarriers. If we assume that the OFDM symbol is cyclically extended, and that the delay spread of the channel does not exceed the guard time, then the timing offset introduces only a phase rotation that linearly changes with the order of subcarrier.

It will be shown that as the number of subcarriers *N*, increase, when the subcarrier spacing is constant, the error probability changes more rapidly in presence of the timing offset than in the case when only frequency offset is present.

This paper is organized as follows. In Section II we will derive the equations for symbol error probability in case when the receiver makes a timing error τ in the sampling process and frequency downconversion error Δf. Numerical results are presented in Section III, while conclusions are presented in Section IV.

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II. Error Probability

Input binary data stream is portitioned into blocks of *2N* bits. Each block is used to modulate *N* orthogonal subcarriers. Each subcarrier is QAM modulated. We assume that at the reception we use digital phase locked loop. Frequency offset Δf is normalized to the subcerrier spacing f_0, $\varepsilon = \Delta f / f_0$. Timing offset is modeled as zero mean random Gaussian variable. Standard deviation of timing offset is normalized to the interval $T_0 = 1/f_0$ OFDM symbol duration is equal to $T = T_0 + T_{cp}$, where T_{cp} denotes the duration of the cyclic prefix.

If we assume that the modulation is performed using inverse discrete Fourier transform (IDFT), complex envelope of the OFDM transmission symbol is given by the equation:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{T_0} nt}$$

for $-T_{cp} \le t \le T_0$, where N is the number of subcarriers, $X_n = a_n + jb_n$ are complex QAM symbols and a_n and b_n are two information bits in the *I* - and *Q* - channel, respectively. We will assume that $a_n, b_n \in \{-1, 1\}$. Complex envelope of the signal at the input of the receiver is equal to:

$$r(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{T_0}(n+\varepsilon)t} \tag{1}$$

where *n(t)* is the complex envelope of white additive Gaussian noise (AWGN) at the input of the receiver, with zero mean and variance σ_n^2. Received signal is sampled at the rate $N f_0$. We assume that the receiver makes a timing error τ in the sampling process. If OFDM symbol is cyclicly extended, then the sample of the receved signal is equal to:

$$r_k \equiv r(t)\Big|_{t = k\frac{T_0}{N} + \tau} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{T_0}(n+\varepsilon)\cdot\left(k\frac{T_0}{N} + \tau\right)} + n_k \tag{2}$$

where n_k represents AWGN noise sample. We use this samples to compute the discrete Fourier transform (DFT):

$$\tilde{X}_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_k e^{-j\frac{2\pi}{N} mk} \tag{3}$$

By substituting (2) in (3) we get the estimation of the complex data symbol transmitted on the mth subcarrier:

$$\tilde{X}_m = X_m \frac{\sin \varepsilon\pi}{N \sin \frac{\varepsilon\pi}{N}} \exp\left[j \left(\frac{2\pi}{T_0}(m+\varepsilon)\tau + \varepsilon\pi \frac{N-1}{N} \right)\right] + \sum_{\substack{n=0 \\ n\neq m}}^{N-1} X_n \frac{\sin(n-m+\varepsilon)\pi}{N \sin \frac{(n-m+\varepsilon)\pi}{N}} e^{j\left(\frac{2\pi}{T_0}(n+\varepsilon)\tau + (n-m+\varepsilon)\pi\frac{N-1}{N}\right)} + n_m \tag{4}$$

where the first term represents user signal, and the second term represents the inter-carrier interference. The third term, n_m , is equal to:

$$n_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} n_k e^{-j\frac{2\pi}{N}mk}$$

This term represents AWGN at the output of DFT block, with zero mean and variance σ_n^2. The second term, representing ICI, will be modelled as a Gaussian process with zero mean and variance:

$$\sigma^2_{ICI/\varepsilon} = \sum_{\substack{n=0 \\ n\neq m}}^{N-1} \left(\frac{\sin(n-m+\varepsilon)\pi}{N \sin(n-m+\varepsilon)\frac{\pi}{N}} \right)^2 \tag{5}$$

Variance of ICI is conditioned on relative frequency offset. Equation (4) then can be written as:

$$\tilde{X}_m = X_m \frac{\sin \varepsilon\pi}{N \sin \frac{\varepsilon\pi}{N}} \exp\left[j \left(\frac{2\pi}{T_0}(m+\varepsilon)\tau + \varepsilon\pi \frac{N-1}{N} \right)\right] + g_m \tag{6}$$

where g_m denotes Gaussian random variable with zero mean and variance conditioned on relative frequency offset ε:

$$\sigma^2_{g/\varepsilon} = \sigma^2_{ICI/\varepsilon} + \sigma^2_n \tag{7}$$

This random variable represents the overall noise that includes thermal noise and intecarrier interference.

If $\tilde{X}_m = \tilde{a}_n + j\tilde{b}_m$ then from (6) we can write for $\tilde{a}_m$ and $\tilde{b}_m$:

$$\tilde{a}_m = \frac{\sin \varepsilon\pi}{N \sin \frac{\varepsilon\pi}{N}} \left(a_m \cos \frac{2\pi}{T_0} m\tau - b_m \sin \frac{2\pi}{T_0} m\tau \right) + g_{mI} \tag{8}$$

$$\tilde{b}_m = \frac{\sin \varepsilon\pi}{N \sin \frac{\varepsilon\pi}{N}} \left(b_m \cos \frac{2\pi}{T_0} m\tau + a_m \sin \frac{2\pi}{T_0} m\tau \right) + g_{mQ} \tag{9}$$

where g_{mI} and g_{mQ} represent quadrature components of the overall noise.

The probability of correct decision on the mth subcarrier is conditioned on the value of timing offset and signals in I and Q channels of the QAM signal. This probability is given by the equation (10):

$$\begin{aligned} P_{c/\tau,m,a_m,b_m} = &\left[1 - \frac{1}{2} Q\left(\sqrt{2SNR}\left(\cos\frac{2\pi}{T_0}m\tau + b_m \sin\frac{2\pi}{T_0}m\tau \right)\right) \right. \\ &\left. - \frac{1}{2} Q\left(\sqrt{2SNR}\left(\cos\frac{2\pi}{T_0}m\tau - b_m \sin\frac{2\pi}{T_0}m\tau \right)\right) \right] \times \\ &\times \left[1 - \frac{1}{2} Q\left(\sqrt{2SNR}\left(\cos\frac{2\pi}{T_0}m\tau + a_m \sin\frac{2\pi}{T_0}m\tau \right)\right) \right. \\ &\left. - \frac{1}{2} Q\left(\sqrt{2SNR}\left(\cos\frac{2\pi}{T_0}m\tau - a_m \sin\frac{2\pi}{T_0}m\tau \right)\right) \right] \end{aligned} \tag{10}$$

where SNR represents the signal-to-overall noise ratio per information bit at the input of the decision device:

$$SNR = \frac{\left(\frac{\sin \varepsilon\pi}{N \sin \frac{\varepsilon\pi}{N}} \right)^2}{2\sigma^2_{g/\varepsilon}} \tag{11}$$

Error probability on the mth subcarrier conditioned on the timing offset is equal to:

$$P_{e/\tau,m} = 1 - \frac{1}{4} \sum_{a_m} \sum_{b_m} P_{c/\tau,a_m,b_m} \tag{12}$$

Because of the ICI, the value of the error probability depends on the index of the subcarrier that we observe. Because of this, all of the results for the error probability that are presented here, are obtained by averaging the probability given by the eqation (12) over the index m:

$$P_{e/\tau} = \frac{1}{N} \sum_{m=0}^{N-1} P_{e/\tau,m} \tag{13}$$

The error probability in presence of the frequency offset and Gaussian distributed timing offset is calculated according to the equation:

$$P_e = \int_{-\infty}^{\infty} P_{e/\tau}\, p(\tau)\, d\tau \tag{14}$$

where $p(\tau)$ denotes the pdf function of the timing offset that is modelled as a Gaussian random variable with zero mean and variance σ_τ.

III. NUMERICAL RESULTS

Error probability as a function of relative frequency offset ε, normalized standard deviation of timing offset and ratio $E_b/N_0 = 10\log 1/2\sigma_n^2$ is shown in Fig. 1 and Fig 2. E_b denotes the user signal energy per one bit. These results were obtained using equation (14). It is assumed that the subcarrier spacing is constant f_0.

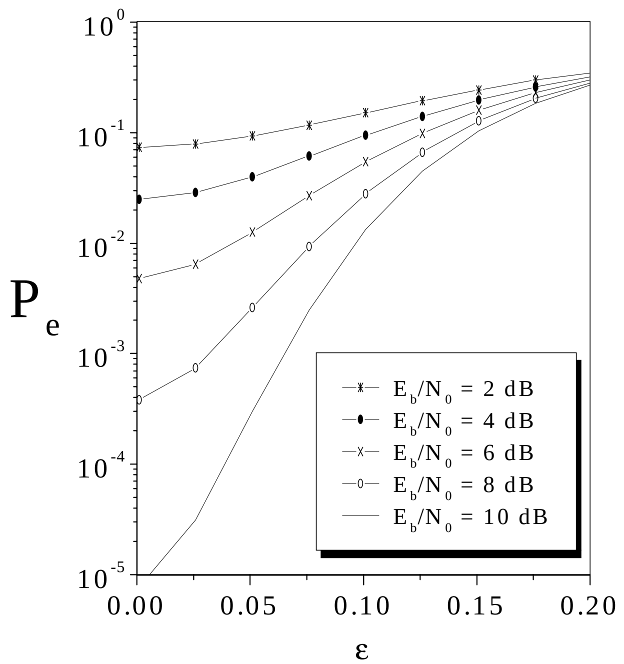

Fig. 1. Error probability as a function of relative frequency offset ε, and E_b/N_0 ratio. *N = 32*, and $\sigma_\tau/T_0 = 0$.

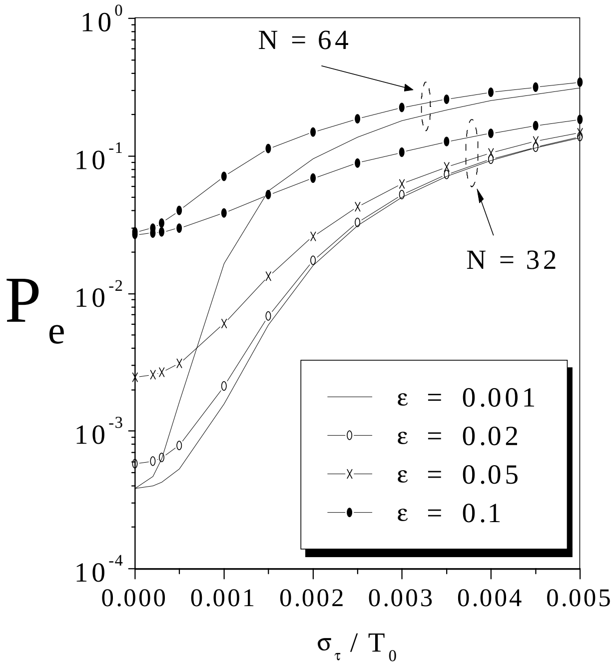

Fig. 2. Error probability as a function of normalized timing offset σ_τ/T_0 and relative frequency offset ε. Ratio E_b/N_0 is equal 8 dB.

Figure 1. represents error probability as a function of the relative frequency offset and signal-to-noise ratio per one bit at the input of the decision device, E_b/N_0.

The error probability curves were computed using N = 32, 64, 96 and 128. With the increase of the number of the subcarriers N, when the subcarrier spacing is constant, the error probability does not change significantly. Because the curves for the various values of N do not differ significantly, only the values of the error probability computed for $N = 32$ are plotted in Fig. 1.

Figure 2. represents error probability per one QAM symbol, obtained using equation (14), versus the normalized timing offset and the relative frequency offset. Signal-to-noise ratio is equal $E_b/N_0 = 8\,dB$.

From Fig. 2. it can be seen that in the case when the timing offset is not present ($\sigma_\tau / T_0 = 0$), the number of the subcarriers does not affect the value of the error probability. The value of the error probability, in that case, depends only on the value of the frequency offset that degrades the system performance by two orders of magnitude.

The increase of the timing offset value results in the increase of the subcarriers number influence on the error probability.

From the figure, one can see that there is the value of timing offset ($\sigma_\tau/T_0 = 0.0025$), above which the value of the error probability depends only on the subcarriers number.

IV. Conclusion

It was shown that, in the presence of frequency offset, the number of subcarriers does not affect the error probability, and the frequency offset significantly increases the error probability (in considered case for two orders of magnitude).

The presence of timing offset also causes the error probability increase. In this case the number of subcarriers has influence on the value of error probability. At higher timing offsets the error probability depends only on the subcarriers number.

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