Detection of ASK Signal in Presence of White Gaussian Noise

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Abstract – **In this paper, we investigate ASK signal detection in the presence of white Gaussian noise. We determine the lilelihood functions and ratio. Based on obtained ratio, the optimal receiver configuration is formed.**

Keywords – **Detection of ASK Signal, White Gaussian Noise Likelihood Function, Likelihood Ratio, Optimal Receiver**

I. INTRODUCTION

The received signal depends on the signal formed in the transmitter and on the interferences accumulated along the line. Interferences and random fluctuations in the signal components make the received signal a random process, and one can calculate its probability density function (PDF). PDF of the signal at the receiver input, assuming the appropriate hypothesis, is known as the likelihood function. The ratio of the likelihood functions represents the likelihood ratio. By comparing likelihood ratio with carefully chosen decision threshold, one can obtain the appropriate decisions.

In this paper, we determine the likelihood function for chosen hypothesis assuming the presence of white Gaussian noise and then we form the likelihood ratio. Based on the likelihood ratio, we then form the optimal receiver configuration.

II. LIKELIHOOD FUNCTIONS AND RATIO

We examine the ASK signal of the fallowing form

$$H_0 : s_0(t) = 0$$
$$H_1 : s_1(t) = A\cos(\omega_0 t + \varphi) \tag{1}$$

The signal is then impaired by Gaussian noise with phase PDF

$$p(\varphi) = Ce^{-\alpha\cos\varphi} \tag{2}$$

Under this conditions, the signal at the receiver input is

$$H_0 : r_0(t) = n(t)$$
$$H_1 : r_1(t) = A\cos(\omega_0 t + \varphi) + n(t) \tag{3}$$

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Likelihood function for the H_0 hypothesis is

$$p_0(r) = Fe^{-\frac{1}{N_0}\int_0^T r^2(t)dt} \tag{4}$$

For the H_1 hypothesis, the likelihood function is computed as

$$p_1(r) = \int_{-\pi}^{\pi} p_1(r/\varphi)p(\varphi)d\varphi \tag{5}$$

$$p_1(r/\varphi) = Fe^{-\frac{1}{N_0}\int_0^T [r(t) - A\cos(\omega_0 t + \varphi)]^2 dt}$$
$$= Fe^{-\frac{1}{N_0}\int_0^T r^2(t)dt - \frac{A^2T}{2N_0}} e^{\frac{2A}{N_0}\int_0^T r(t)A\cos(\omega_0 t + \varphi)dt}$$

$$p_1(r) = FC\int_{-\pi}^{+\pi} e^{-\frac{1}{N_0}\int_0^T r^2(t)dt - \frac{A^2T}{2N_0}} e^{\frac{2A}{N_0}\int_0^T r(t)A\cos(\omega_0 t + \varphi)dt} e^{-\alpha\cos\varphi}d\varphi \tag{6}$$

By variable interchange, we write

$$D = \int_{-\pi}^{+\pi} e^{-\frac{2A}{N_0}\int_0^T (r(t)\cos\omega_0 t\cos\varphi - r(t)\sin\omega_0 t\sin\varphi)dt} e^{-\alpha\cos\varphi}d\varphi \tag{7}$$

and then

$$X = \int_0^T r(t)\cos\omega_0 t dt$$
$$Y = \int_0^T r(t)\sin\omega_0 t dt$$
$$\beta = \frac{2A}{N_0} \tag{8}$$

further, we get

$$D = \int_{-\pi}^{+\pi} e^{\beta X\cos\varphi - \beta Y\cos\varphi} e^{-\alpha\cos\varphi}d\varphi \tag{9}$$

We expand the exponential expressions to series

$$e^{\beta X\cos\varphi - \beta Y\cos\varphi} = \sum_{k=-\infty}^{+\infty} I_k(q)\cos k(\varphi + \varphi_0) \quad \text{and}$$
$$e^{-\alpha\cos\varphi} = \sum_{l=-\infty}^{+\infty} J_l(-\alpha)\cos l\varphi \tag{10}$$

where

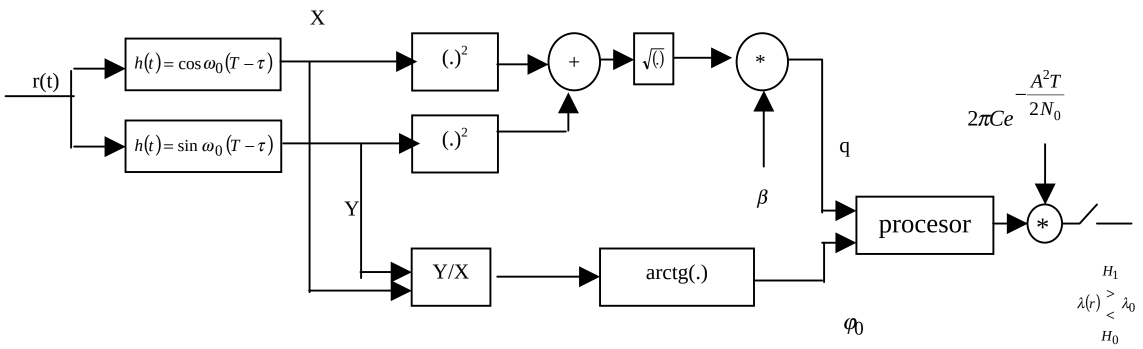

Fig.1. Optimal receiver configuration

$$q = \beta\sqrt{X^2 + Y^2}$$

$$\varphi_0 = arctg\,\frac{Y}{X} \tag{11}$$

Expression for D becomes

$$D = \int_{-\pi}^{+\pi} \sum_{k=-\infty}^{+\infty} I_k(q)\cos k(\varphi + \varphi_0) \sum_{l=-\infty}^{+\infty} J_l(-\alpha)\cos l\varphi d\varphi$$

$$D = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} I_k(q) J_l(-\alpha) \int_{-\pi}^{+\pi} \cos k(\varphi + \varphi_0)\cos l\varphi d\varphi \tag{12}$$

By variable interchange, we write

$$I = \int_{-\pi}^{+\pi} \cos k(\varphi + \varphi_0)\cos l\varphi d\varphi$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \cos[(k+l)\varphi + k\varphi_0]d\varphi + \frac{1}{2}\int_{-\pi}^{\pi} \cos[(k-l)\varphi + k\varphi_0]d\varphi \tag{13}$$

For k=l and k=-l we get

$$I = \frac{1}{2} 2\pi \cos k\varphi_0 = \pi \cos k\varphi_0 \tag{14}$$

and then

$$D = \Big|_{k=\pm l} \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\pi \cos k\varphi_0$$

$$+ \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\pi \cos k\varphi_0$$

$$D = \Big|_{k=\pm l} 2\pi \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\cos k\varphi_0 \tag{15}$$

The likelihood function is

$$p_1(r) = FCe^{-\frac{1}{N_0}\int_0^T r^2(t)dt - \frac{A^2T}{2N_0}} 2\pi \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\cos k\varphi_0 \tag{16}$$

Finally, we get the likelihood ratio as the ratio of the likelihood functions

$$\lambda(r) = \frac{p_1(r)}{p_0(r)} = \frac{FCe^{-\frac{A^2T}{2N_0}} e^{-\frac{1}{N_0}\int_0^T r^2(t)dt} 2\pi \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\cos k\varphi_0}{Fe^{-\frac{1}{N_0}\int_0^T r^2(t)dt}} \tag{17}$$

$$\lambda(r) = 2\pi Ce^{-\frac{A^2T}{2N_0}} 2\pi \sum_{k=-\infty}^{+\infty} I_k(q) J_l(-\alpha)\cos k\varphi_0 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_0 \tag{18}$$

By comparing the likelihood ratio with appropriate decision threshold, a deccision is made whether the transmitted bit is 0 or 1. Based on the likelihood ratio expression, the optimal receiver can be formed, as shown in Fig.1.

III. CONCLUSION

In this paper, we have determined likelihood functions for the set of hypothesis. Then, we have determined the likelihood ratio and formed the optimal receiver configuration. Optimal receiver calculates the likelihood ratio and compares it to the decision threshold. Thus, tke likelihood ratio illustrates the optimal receiver as shown. For the given input signal, receiver computes the random variables X and, then determines q and φ_0 , and finally the likelihood ratio $\lambda(r)$, which is compared to the decision threshold λ_0 to yield the decision on the transmitted data hypothesis.

An optimal receiver constitutes of two parts. First part calculates the likelihood ratio $\lambda(r)$, i.e. it performs the signal processing based on our knowledge of the communication channel characteristics, while the second part of the receiver performs decision, based on our knowledge of the data source characteristics.

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