The Performance of ASK System in the Presence of Fading

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Abstract - **In this paper we will consider noncoherent ASK digital communication system in the presence of gaussian noise and fading. For this case we will calculate the error probability.**
Keywords - **ASK system, fading**

I. INTRODUCTION

In this paper we will consider the digital communication system in the presence of gaussian noise and fading.

The problem of the detection of signals with the signal to noise ratio is constant at the output of the receive filter is considered in the paper [1]. But, it is not the case always. Sometimes, the signal can disappear in the receiver partially or even more completely. This phenomenon is fading [2]. Because of that, in our paper we consider the signal in the presence of white gaussian noise and fading.

II. THE MODEL OF THE RECEIVER

The model of the receiver, which is discussed in this paper, is at Fig.1.

The receiver consists of the narrow bandpass filter and the envelope detector. The signal is amplitude shift keying modulated (ASK).

We have two hypothesis; first, when we send "0", the hypothesis is H_0, and when we send "1", the hypothesis is H_1. The signals are $s_0(t)$ and $s_1(t)$:

$$H_0 : s_0(t) = 0$$
$$H_1 : s(t) = A\cos\omega_0 t \tag{1}$$

A is the amplitude of the useful signal, ω_0 is the frequency. The amplitude A is not constant. Because of the fading the probability density function of A is:

$$p(A) = \frac{A}{\sigma_F^2} e^{-\frac{A^2}{2\sigma_F^2}} \tag{2}$$

The white gaussian noise has the influence to the useful signal. The signal at the input of the receiver is [3]:

$$H_0 : r(t) = s_0(t) + n(t) = n(t)$$
$$H_1 : r(t) = s_1(t) + n(t) = A\cos\omega_0 t + n(t) \tag{3}$$

The noise after the narrow bandpass filter is:

$$n(t) = x(t)\cos\omega_0 t - y(t)\sin\omega_0 t \tag{4}$$

At the input of the envelope detector the signal is:

$$H_0 : z_0(t) = x(t)\cos\omega_0 t - y(t)\sin\omega_0 t$$
$$H_1 : z_1(t) = A\cos\omega_0 t +$$
$$+ x(t)\cos\omega_0 t - y(t)\sin\omega_0 t \tag{5}$$

After the envelope detector the signal is:

$$H_0 : r_0(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$
$$H_1 : r_1(t) = \sqrt{[A + x(t)]^2 + [y(t)]^2} \tag{6}$$

III. THE PROBABILITY DENSITY FUNCTIONS

Now, we will find the probability density function for these signals:

$$p_0(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \tag{7}$$

That is Reilly's distribution.

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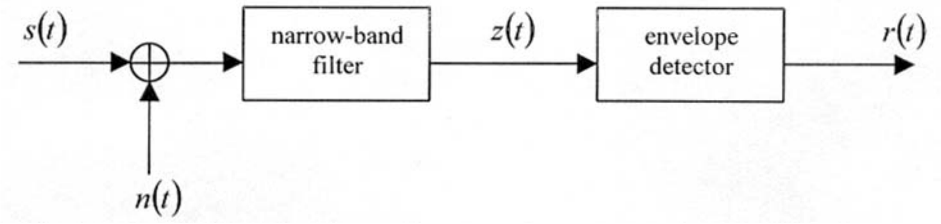

Fig.1. The model of the system

$$p_1(r/A)=\frac{r}{\sigma^2}e^{-\frac{A^2+r^2}{2\sigma^2}}I_0\left(\frac{Ar}{\sigma^2}\right) \tag{8}$$

known as Rice's distribution.
The probability density function for the hypothesis H_1 is conditional with respect to A, and we have to integrate:

$$p_1(r)=\int_0^\infty p_1(r/A)p(A)dA \tag{9}$$

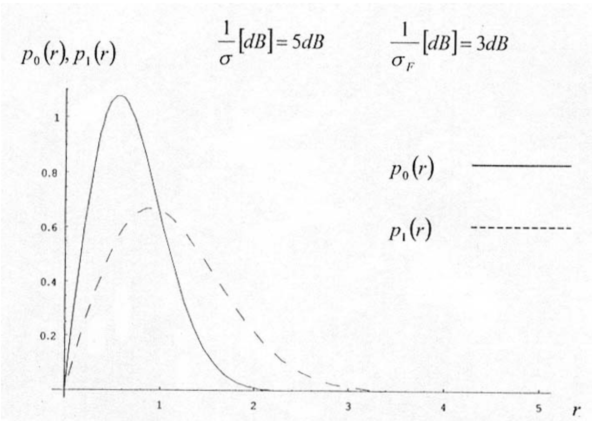

Fig. 2. The probability density functions

Finally, the probability density function for the hypothesis H_1 is:

$$p_1(r)=\int_0^\infty \frac{Ar}{\sigma^2\sigma_F^2}e^{-\frac{A^2}{2\sigma_F^2}}e^{-\frac{A^2+r^2}{2\sigma^2}}I_0\left(\frac{Ar}{\sigma^2}\right)dA \tag{10}$$

The graphics of the probability density function are given at Fig. 2. and Fig. 3.

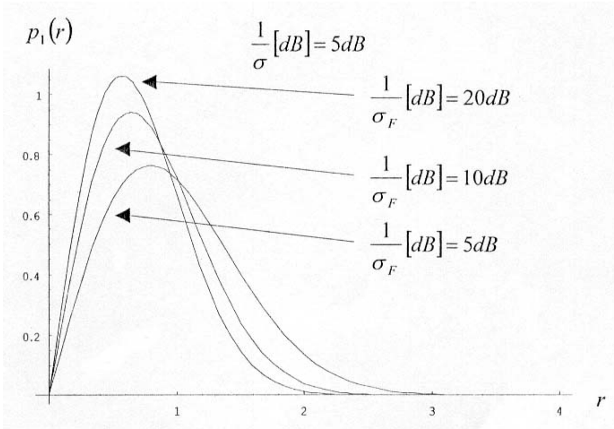

Fig. 3. The probability density function for the hypothesis H_1

IV. THE BIT ERROR PROBABILITY

The next step is to calculate the bit error probability. The bit error probability is:

$$\begin{aligned}P_e &= P(D_1,H_0)+P(D_0,H_1)= \\ &= P(H_0)P(D_1/H_0)+P(H_1)P(D_0/H_1)\end{aligned} \tag{11}$$

Because of $P(H_0)=P(H_1)=1/2$, P_e is:

$$\begin{aligned}P_e &= \frac{1}{2}P(D_1/H_0)+\frac{1}{2}P(D_0/H_1)= \\ &= \frac{1}{2}\int_{r_T}^{\infty}p_o(r)dr+\frac{1}{2}\int_0^{r_T}p_1(r)dr\end{aligned} \tag{12}$$

At the end:

$$\begin{aligned}P_e &= \frac{1}{2}\int_{r_T}^{\infty}\frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}}dr+ \\ &+\frac{1}{2}\int_0^{r_T}\int_0^{\infty}\frac{Ar}{\sigma^2\sigma_F^2}e^{-\frac{A^2}{2\sigma_F^2}}e^{-\frac{A^2+r^2}{2\sigma^2}}I_0\left(\frac{Ar}{\sigma^2}\right)dAdr\end{aligned} \tag{13}$$

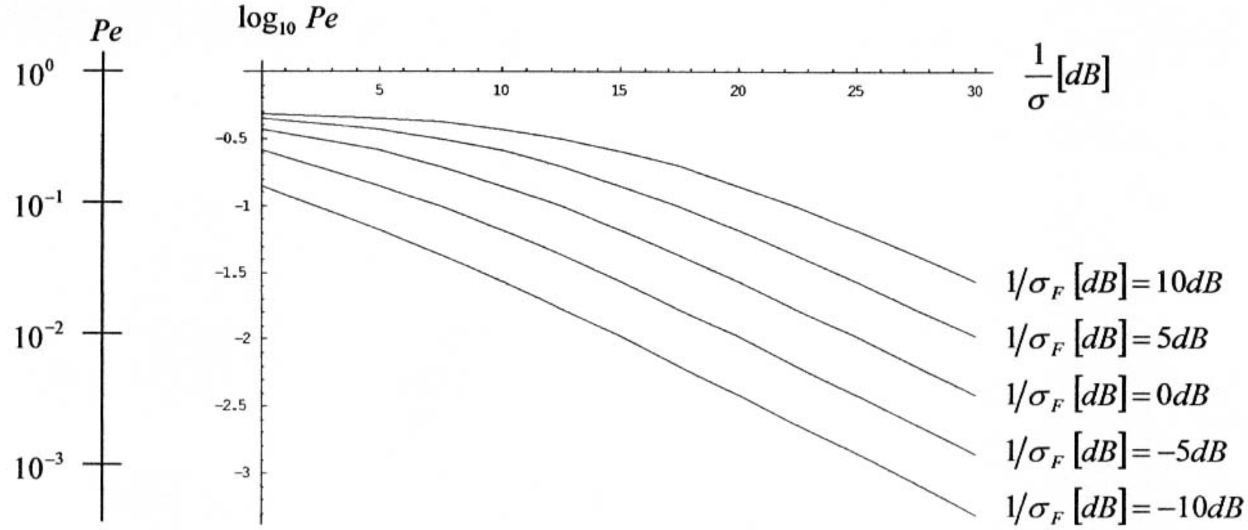

Fig. 4. The probability of error

The graphic of the bit error probability is at Fig. 4. We can see that when the fading increases the bit error probability increases too. The variance of the gaussian noise has the same influence to the increasing of the bit error probability.

V. CONCLUSION

In this paper we analyzed the digital communication system in the presence of gaussian noise and fading. The signal to noise ratio is not constant at the output of the receive filter Sometimes, the signal can disappear in the receiver partially or even more completely. This phenomenon is fading. That is one of the limiting factors to the system performance. Because of that, in our paper we considered the signal in the presence of white gaussian noise and fading

Also, we calculated for this case the probability density functions for both of hypothesis, H_0 and H_1 and then, the bit error probability for different values of the noise variance. This graphic is at Fig. 4.

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