# C-GM Tuning for Constant Loop-Gain of Voltage-controlled Oscillators

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*Abstract* - **Due to technology limitations as well as stringent operating conditions that are imposed, the design of fully integrated analog RF front-end circuits is aimed at the edge of the required performances. Such an approach implies that each dB of power or noise is of high importance, as it directly determines the range of operation of a system. Accordingly, a design trajectory that offers certain power savings, while still keeping the system within the required range of operation, appears to be a promising candidate for nowadays highly stringent low-power RF front-end circuit design. The concept of frequency-transconductance tuning, introduced in this paper, offers the possibility for voltage-controlled oscillators to trade power consumption for loop-gain and phase-noise. Detailed analysis shows how this concept is employed to achieve full control over the operation of the oscillator that is being changed as a result of a frequency tuning.**

## I. INTRODUCTION

Wireless telecommunication transceivers of both today and future have to be broadband, low power and adaptive; broadband, so as to support high data rate demanded by the applications; adaptive, so as to accommodate the varying channel conditions as well as the varying user and application requirements; and low power, i.e., to consume as little energy as possible for a given performance and to ensure long battery life.

So far, only the low-power aspect of the design of RF front-end circuits has been addressed, while a concept like adaptivity is rather referred to a high-level modeling of the telecommunication systems – adaptive channel-coding for example. Current analog front-end circuits are often designed to perform only one specific task while the parameters such as dynamic range, bandwidth, selectivity and the like are set by the design in a fixed way, and not by the communication system in an adaptive way. As a result, many of today's receiver topologies, which are designed to be functional under highly stringent conditions, suffer from a lot of overhead, both in circuit complexity and power consumption. Therefore, the concept of adaptivity must be considered at the circuit level as well.

Accordingly, the possibility to trade power, phase-noise and loop-gain of oscillators among each other in an adaptive way, will allow the oscillators to respond actively to varying channel conditions as well as the current requirements of the system as a whole.

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The concept of a frequency-transconductance ($C$-$g_m$) tuning, presented in this paper, stands for compensating the changes of the oscillation condition imposed by a frequency tuning [1]. It is well known that as a result of a change in frequency by tuning the varactor ($C$) of the LC-tank, the loop-gain, the voltage swing and the phase-noise of the oscillator are changed as well.

If the oscillator is designed at the very edge of the required specifications, it can turn out that the corresponding oscillation condition, i.e., loop-gain, or the amount of produced noise, i.e., phase-noise, put the oscillator out of operation. Therefore, it is necessary to apply a controlling mechanism, i.e., control of the biasing condition ($g_m$), being able to keep the oscillator functioning. The presented concept of $C$-$g_m$ tuning shows what is the relation between the tuning voltage of the varactor of the LC-tank and the tuning biasing current of the transistors of the oscillator's active part. Also, it is shown to what extent the power consumption and the phase-noise of oscillator are affected by the tuning mechanism.

This paper is divided into five sections. The concept of frequency-transconductance tuning is presented in Section 2. On the example of a quasi-tapped VCO, in Section 3, it is shown what is the sensitivity of the tuning voltage to a tuning current, in case of a constant loop-gain and a constant phase-noise, respectively. Section 4 is left for discussion, while Section 5 summarizes the conclusions resulting from the presented analysis.

## II. FREQUENCY-TRANSCONDUCTANCE TUNING

The ever-present philosophy of low-voltage low-power design has directly resulted in a design at the very edge of the required performance. Namely, to meet stringent requirements posed by the standardization comities, on one hand, and to stay at the lowest power levels, on the other hand, fully integrated analog RF front-end circuits are targeting the limits of the circuits' functionality in all its aspects. For the oscillators, this means that design is usually aimed at a loop-gain slightly larger than the necessary minimum of one, e.g., two. In such a case, the increase in capacitance of the LC-tank of the oscillator, in order to lower the oscillating frequency, will result in an increase of the effective tank conductance. If the design is rather "fixed" than adaptive, the oscillation condition will be deteriorated as the loop-gain is lowered. Accordingly, this might bring full front-end system to a halt, as there might be no oscillations if the loop-gain becomes critically small.

In another situation where power consumption is not of such a concern, but rather the phase-noise of the oscillator, the repercussions are different but not less detrimental. This

means that the oscillating condition is rather relaxed, as the loop-gain might be substantially larger than two, but not the phase-noise requirements. If the previous example is reconsidered, then as a result of the increase in the effective tank conductance, the voltage swing over the tank will be reduced and therefore the phase-noise.

In both the above mentioned examples, the oscillator would have still been fulfilling the required specs, i.e., oscillation condition and phase-noise, respectively, if the control mechanism had been applied over the tank, by means of adapting the biasing conditions of the corresponding transistors in the cross-coupled differential pair, playing the role of a negative conductance. Named "frequency-transconductance ($C$-$g_m$) tuning", this concept of a control will be both qualitatively and quantitatively exposed in the remainder of the paper.

First, for the purpose of the analysis to come, we will refer to a bipolar VCO, shown in Fig. 1, as to a quasi-tapped bipolar VCO [2]. The main parameters of the oscillator are defined as:

$$G_{TK} = \frac{1}{R_P} + \frac{R_L}{(\omega_0 L)^2} + R_C(\omega_0 C)^2 \quad (1)$$

$$n = 1 + \frac{C_A}{C_B}, \qquad G_M = g_m/2, \qquad G_{M,TK} = G_M/n \quad (2)$$

$$L_{TOT} = L, \; C_{TOT} = C + \frac{C_A C_B}{C_A + C_B}, \; \omega_0 = \frac{1}{\sqrt{L_{TOT} C_{TOT}}} \quad (3)$$

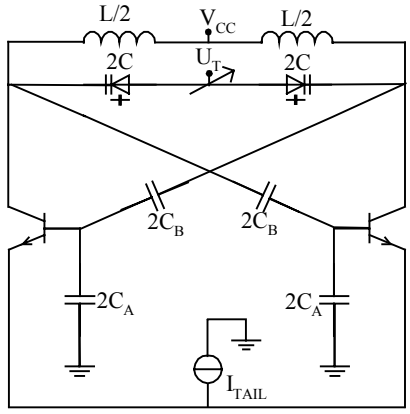


Fig. 1 Quasi-tapped LC-oscillator.

where $L$ is the inductance, $C$ the capacitance, $R_L$, $R_C$ and $R_P$ their parasitic resistances, $G_{TK}$ the effective tank conductance, $C_A$ and $C_B$ the quasi-tapping capacitances, $n$ is the quasi-tapping factor, $G_M$ the transconductance of the active part of the oscillator, $G_{M,TK}$ the transconductance seen by the LC-tank, $g_m$ the transconductance of the bipolar transistors, $U_T$ the tuning voltage of the varactor $C$, and $I_{TAIL}$ the tail current of the differential pair.

The objective is to find the relation between the tuning voltage $U_T$, and the effective conductance $G_{TK}$ of the LC-tank, on one hand, as well as the relation between the tank conductance and the biasing current $I_{TAIL}$, on the other hand. The resulting sensitivity of a tail current to a tuning voltage will show to what extent the biasing conditions should be changed, due to a change in frequency, so as to keep the oscillator operating under the specified conditions.

Starting from Eqs. (1) and (3), the sensitivity of the LC-tank to a change in angular frequency $\omega_0$ ($2\pi f_0 = \omega_0$) defined as:

$$S_{\omega_0}^{G_{TK}} = \partial G_{TK} / \partial \omega_0 \quad (4)$$

has a final form:

$$S_{\omega_0}^{G_{TK}} = -4\omega_0 C_{TOT}\left[C R_C\left(1 - C/2C_{TOT}\right) + C_{TOT} R_L/2\right] \quad (5)$$

Further, let as define the sensitivity of the resonant frequency to the tuning voltage as:

$$S_{U_T}^{\omega_0} = \frac{\partial \omega_0}{\partial U_T}\Big|_{U_T = U_{T0}} = \frac{\partial \omega_0}{\partial C_{TOT}} \frac{\partial C_{TOT}}{\partial C} \frac{\partial C}{\partial U_T}\Big|_{U_T = U_{T0}} \quad (6)$$

where tuning voltage $U_{T0}$ corresponds to the frequency $f_0$.
If the capacitance of the varactor is related to a tuning voltage:

$$C(U_T) = \frac{C_0}{(1 + U_T/\varphi)^{1/a}} \quad (7)$$

where $C_0$, $\varphi$ and $a$ are the corresponding parameters of the varactor, the resulting sensitivity has a form:

$$S_{U_T}^{\omega_0} = \frac{\omega_0 C}{2aC_{TOT}(U_{T0} + \varphi)} \quad (8)$$

Linearising the addressed sensitivity characteristics around resonant frequency, the following relations hold as well:

$$\Delta G_{TK} = S_{\omega_0}^{G_{TK}} \Delta\omega_0, \qquad \Delta\omega_0 = S_{U_T}^{\omega_0} \Delta U_T \quad (9)$$

Now, the change of the effective tank conductance depends on the change in the tuning voltage as:

$$\Delta G_{TK} = S_{\omega_0}^{G_{TK}} S_{U_T}^{\omega_0} \Delta U_T \quad (10)$$

To compensate for such a change in the tank, the conductance seen by the tank, $G_{M,TK}$, should be changed proportionally. From Eq. (2), it can be written:

$$\Delta g_m = 2k \cdot n \cdot \Delta G_{M,TK} \qquad \Delta I_{TAIL} = 2V_T \Delta g_m \quad (11)$$

where $k$ is the loop-gain of the oscillator and $V_T$ a thermal voltage. Accordingly, a change of the tail current is related to a change of the conductance seen by the LC-tank as:

$$\Delta I_{TAIL} = S_{G_{M,TK}}^{I_{TAIL}} \Delta G_{M,TK}, \qquad S_{G_{M,TK}}^{I_{TAIL}} = 4k \cdot n \cdot V_T \quad (12)$$

Finally, from Eqs. (10) and (12), the sensitivity of the tail current to the tuning voltage, referred to the increase or the reduction in the tail current or power so as to sustain oscillations and keep system operating under required conditions, has a final form:

$$\Delta I_{TAIL} = S_{U_T}^{I_{TAIL}} \Delta U_T, \qquad S_{U_T}^{I_{TAIL}} = S_{G_{M,TK}}^{I_{TAIL}} S_{\omega_0}^{G_{M,TK}} S_{U_T}^{\omega_0} \quad (13)$$

$$S_{U_T}^{I_{TAIL}} = -\frac{8k \cdot n \cdot V_T}{a(U_{TO} + \varphi)}\left[R_C\left(1 - \frac{C}{2C_{TOT}}\right) + \frac{C_{TOT} R_L}{2C}\right](\omega_0 C)^2 \quad (14)$$

For the oscillator under consideration, this expression allows us to estimate to what extent the tail current should be changed, as a result of a change in frequency, so as to keep the oscillator operating under the required conditions.

A unique representation of the *C-gm* phenomenon is given in Fig. 3, in the form of, for the first time introduced, *k-loop diagram*.

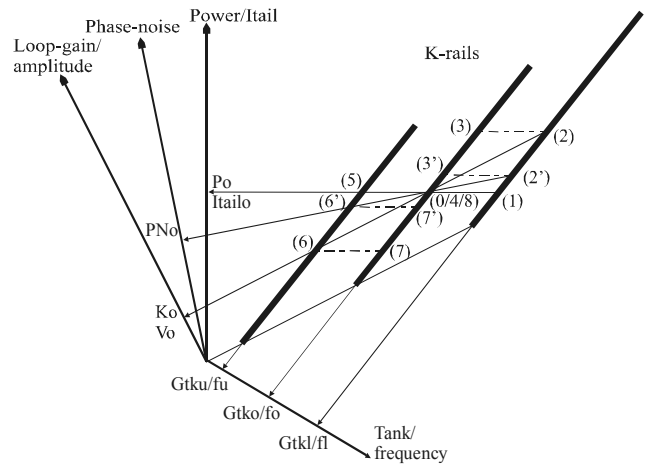


Fig. 2 K-loop diagram.

Index $_0$ refer to the resonant frequency, i.e., loop-gain $k_0$, amplitude $V_0$, phase-noise $PN_0$, and power consumption $P_0$, at frequency $f_0$. Apart from the axes, corresponding to the above mentioned parameters of the oscillator, three, so-called, *k-rails* are shown as well. They correspond to the LC-tank at lower, central and upper tuning frequency, respectively, as indicated at the tank/frequency axis.

To explain its meaning, let us now make one loop, for example, from point (0) up to point (4) in the *k-loop* diagram. As a result of tuning to a lower frequency, the total capacitance as well as the effective tank conductance become larger, which is equivalent to a moving from a position (0) to a position (1), as the inserted power (tail current) is at the same level. It can be noticed that at point (1), the voltage swing over the resonator, the loop-gain and the phase-noise are decreased, accordingly. To compensate for such a deterioration of the performances, the power level (tail current) must be increased for an amount indicated by Eqs. (13) and (14). This corresponds to the next position in a diagram – point (2). As seen, the loop-gain and the amplitude are brought again to the level as being before the tuning action. What is more, the phase-noise is even improved compared to the starting point. Next, the reduction of the capacitance of the varactor and according increase in oscillating frequency correspond to point (3). As the phase-noise, loop-gain, amplitude, and power consumption are at unnecessary higher levels, the tail current can be reduced, so that all the specifications are met again – point (4). Namely, as shown in the diagram, point (4) corresponds to the starting point (0). Similar reasoning holds for the left loop, being the one consisting of points (4) to (8).

On a journey throughout the *k-loop*, not only can all the previously addressed situations in this section be recognized, but also all the possible trade-offs between the power consumption, phase-noise and loop-gain can be qualitatively interpreted. Ending this journey, let us just name this particular phenomenon "frequency-transconductance tuning for a *constant loop-gain*".

Note that the counterpart of the preceding concept in the circuitry is a simple amplitude control mechanism, as a constant loop-gain means a constant amplitude of the signal across the LC-tank of the oscillator.

## III. AN ALL-ROUND EXAMPLE

To prove the validity of the introduced concept of frequency-transconductance tuning as well as to show how powerful tool the *k-loop* diagram is for both qualitative and quantitative representation of the addressed phenomena, a complete and fully realistic example is presented. As in the previous section, we will refer to the quasi-tapped bipolar VCO, shown in Fig. 1, in the forthcoming analysis. Also, we will introduce two forms of a k-loop diagram, i.e., a *constant loop-gain loop* and a *constant phase-noise loop*, respectively.

The values of the parameters of the oscillator are as follows:

$f_0$=900MHz, $2C$=2pF, $Q_C$=15, $L/2$=12.5nH, $Q_L$=4, $2C_A$=1pF, $2C_B$=1pF, $V_{CC}$=2V, $U_{T0}$=1V, φ=0.5V, $a$=2, and $k$=2, where $Q_C$ and $Q_L$ are the quality factors of the corresponding varactor and the inductor. To the case when the loop-gain $k$ equals 2, we will refer as to the safety start-up condition, and denote it with the index $_{S\_S\text{-}UP}$.

### A. *Constant loop-gain loop*

First, from Eq. (14), the sensitivity of the tail current to the tuning voltage is found to be $S_{U_T}^{I_{TAIL}} = -0.25mA/V$ .

To see what are the effects of both voltage tuning and the corresponding $C$-$g_m$ tuning, we will use the *k-loop* diagram, shown in Fig. 3, as the cornerstone of the analysis. Let as therefore show how the loop-gain, the tail current, the effective tank conductance and the phase-noise can be found for any point in the diagram.

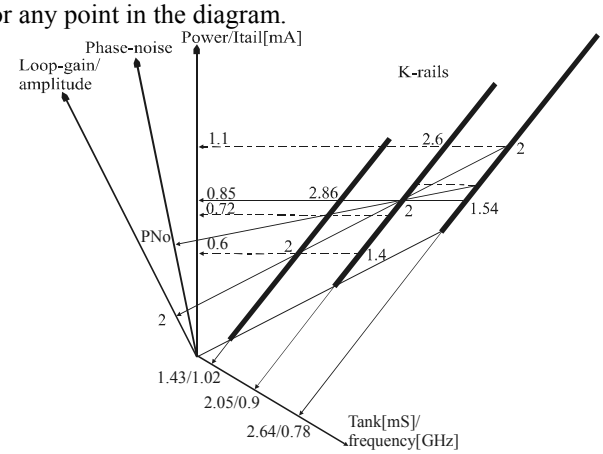


Fig. 3 K-loop diagram for the given parameters of the VCO.

For that purpose, we should find the values of the tank conductance for the lower, central and upper resonant frequency of the oscillator. From Eq. (1), it is found that the tank conductance $G_{TK}$ equals 2.05mS at a frequency $f_0$=900MHz, while from Eq. (2) the tail current is found to be $I_{TAIL,S\_S\text{-}UP}$ =0.85mA. For a maximum voltage tuning range of 1V, the maximum change of a tail current is, from Eq. (14), expected to be 0.25mA. This means that in order to sustain oscillations under the same condition, being a loop-gain of two, the tail current should be either increased or reduced by this amount of current. As the order of the points is the same as in Fig. 2, points (2) and (3) of the diagram correspond to a tail current of 1.1mA, while points (6) and (7) correspond to

a current of 0.6mA. Now, knowing the loop-gain at a position (2) and referring to a tail current of 1.1mA as to the safety start-up current for a new tank, it is rather straightforward to calculate, from Eq. (2), that a new tank conductance $G_{TK,L}$, corresponding to a lower resonant frequency $f_L$=780MHz, has a value 2.64mS. The tuning range of ±120MHz is found from the Eq. (9). Finally, the loop-gain for the point (3) is simply found to be $k$=2.6. The parameters of the left-loop, (4) to (8), are calculated following the same procedure.

B. C*onstant phase-noise loop*

So far, we have been concerned with how to keep the oscillator under the conditions that enable its proper functioning, i.e., not to have the oscillations faded away. However, apart from keeping the loop-gain at a satisfactory level, there are the applications where signal amplitude or phase-noise is of greater concern. If for example, the provided loop-gain is large enough, e.g. $k$>3, then the deterioration of the phase-noise as a result of the tuning can be the main problem.

To track the influence of $C$-$g_m$ tuning on the phase-noise, let us first, for the already defined parameters of the oscillator, find the sensitivity of the tail current to the tuning voltage. Unlike the previous example when the condition was a constant loop-gain, here, the condition is a constant phase-noise.

Starting from the expression for the phase-noise of a quasi-tapped bipolar VCO [2]

$$\mathcal{L} = KT\frac{\pi^2 G_{TK}}{4(\omega_0 C_{TOT})^2}\frac{1+2n^2 r_B G_{TK} + I_{TAIL}/8V_T G_{TK}}{(I_{TAIL}/G_{TK})^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2 \quad (15)$$

where $K$ is Boltzmann's constant and $T$ the absolute temperature, the change in phase-noise as a result of change in both a tuning voltage and a compensating tuning tail current can be written:

$$d\mathcal{L} = S^{\mathcal{L}}_{U_T} dU_T + S^{\mathcal{L}}_{I_{TAIL}} dI_{TAIL}\,,\; S^{\mathcal{L}}_{U_T} = \frac{\partial\mathcal{L}}{\partial U_T},\; S^{\mathcal{L}}_{I_{TAIL}} = \frac{\partial\mathcal{L}}{\partial I_{TAIL}} \quad (16)$$

Now, with the condition that the phase-noise is constant, the sensitivity of the tail current to the tuning voltage is found to be:

$$S^{I_{TAIL}}_{U_T} = -\frac{S^{\mathcal{L}}_{U_T}}{S^{\mathcal{L}}_{I_T}} = -\frac{\left(\frac{\partial\mathcal{L}}{\partial G_{TK}}\frac{\partial G_{TK}}{\partial C_{TOT}} + \frac{\mathcal{L}}{\partial C_{TOT}}\right)\frac{\partial C_{TOT}}{\partial C}\frac{\partial C}{\partial U_T}}{\frac{\partial\mathcal{L}}{\partial I_{TAIL}}} \quad (17)$$

For the same parameters of the oscillator as in the above example, the sensitivity is calculated to be $S^{I_{TAIL}}_{U_T} = -0.13mA/V$, which is lower than the sensitivity for a constant loop-gain. For the purpose of better understanding of the phenomenon of $C$-$g_m$ tuning for a *constant phase-noise*, let us switch again to the *k-loop* diagram, shown in Fig. 2.

As a direct result of the lower sensitivity of the tuning current to the tuning voltage, in the case of a constant phase-noise, than in a case of a constant loop-gain, the corresponding *phase-noise loop* (0-1-2'-3'-4) is encompassed by a *loop-gain loop*. This also implies that a constant phase-noise loop is less "power expensive", as already expected from the obtained numerical values.

The questions like which loop to follow, or what criterion to apply, when combating the deteriorating effects of frequency tuning by $C$-$g_m$ tuning, are addressed in the next section.

Finally, note that both the exposed concept and the obtained results are fully confirmed by the CADENCE simulation tool SpectreRF.

## IV. DISCUSSION

Depending on the application and accordingly the required specifications, depends which trajectory the oscillator should follow, the constant loop-gain loop or the constant phase-noise loop.

For the power critical applications, the design should be aimed at the lowest possible power level, i.e., the lowest allowable loop-gain, let's say two. In such a case, and for a tuning to a lower resonant frequency, tuning for a constant phase-noise would not mantain the system in its safety region, being the one with $k$≥2. Therefore, the choice should be $C$-$g_m$ tuning for a constant loop-gain.

For phase-noise critical applications, a problem might arise in a case of a tuning to a higher resonant frequency, thus when the corresponding effective tank conductance gets reduced – points (4) to (8) in the *k-loop* diagram. Here, the loop-gain loop puts the system out of operation, as the phase-noise is below the level required by the application. Therefore, either the constant phase-noise loop must be followed (4-5-6'-7'-8), or the oscillator must be left in a position (5) of the k-loop diagram. This situation corresponds to frequency tuning only.

Finally, as each *dB* of phase-noise and each *mW* of power are of concern in nowadays analog RF front-end circuits, the concept of $C$-$g_m$ tuning can be beneficially employed, offering a possibility to trade power, phase-noise and loop-gain in an adaptive way. Just note that in the previous example, as a result of the applied concept, the trade-off of ±30% in power for ±2dB [2] in phase-noise is possible.

## V. CONCLUSIONS

As a result of stringent operating conditions, very often required by the applications, the design of fully integrated analog RF front-end circuits is aimed at the very edge of the required performance. In case of oscillators, this implies that each *dB* of phase-noise and each *mW* of power are of high importance, as they directly determine their range of operation.

The concept of frequency-transconductance ($C$-$g_m$) tuning, introduced in this paper, offers a possibility for voltage-controlled oscillators to trade power consumption for loop-gain and phase-noise. The presented analytical expressions and diagrams show how this concept is employed so as to achieve full control over the operation of the oscillator that is being changed as a result of frequency tuning.

Furthermore, the introduced concepts of tuning for constant loop-gain and constant phase-noise give the opportunity to a designer of voltage-controlled oscillators to have broader insight into all the possible trade-offs existing among the parameters of the oscillator, and accordingly to be capable of responding to all the requirements imposed by the versatile applications and the communication systems of today.

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