Reliability Allocation and Optimisation of Microsystems in Design Phase

Anna V. Andonova[1], Natasha G. Atanasova[2], Dimitar G. Dimitrov[3]

Abstract – **A general model estimates the minimum reliability requirement for multiple components within a microsystem (MS) that will yield the goal reliability value for the MS. A general behavior of the cost as a function of a component' reliability is assumed for this matter. Once the reliability requirement for each component is estimated, one can then decide whether to achieve this reliability by fault tolerance or fault avoidance. The model can be applied to any type of MS, simple or complex, and for a variety of distributions. It is very flexible and requires very little processing time. Using the software package BlockSim these advantages make the proposed reliability allocation solution a great MS design tool.**

Keywords – **Reliability optimization, Microsystems design,**

I. Introduction

The problem of reliability allocation and optimization has been widely treated by many authors. In this paper is taken a different approach to the problem. Instead of concentrating exclusively on redundancy allocation, the minimum required reliability for each component/block of an electronic system would be estimated in order to achieve a system reliability goal with minimum cost. Thereafter, the engineer can decide whether this minimum required component/block reliability would be achieved via fault avoidance or redundancy. Several methods for addressing this type of allocation problem are available. The majority of them however are limited in their application to simple systems of exponential failure distribution of units. In this paper the allocation problem is formulated as a constrained nonlinear optimization problem [1]. With this approach reliability can now be allocated to the components of any type of system, complex or not, and for a mixture of failure distributions for the components of the system. Although the nonlinear programming formulation to the problem has been proposed in the past [1, 2] a very little attention has been given to its implementation. First major factor of this situation is that the model requires the system's analytical reliability equation as input. Second, the model also requires cost as a function of the component's reliability as input and this is not always available to engineers. This problem has been addressed [1, 2] through the introduction of general mathematical formulations for the required cost functions. These mathematical formulations depend on certain parameters that must be supplied by the engineers. Quantifying these parameters has no been an easy task in many instances, however, since a number of them are constants with no close relation to reliability principles. These shortcomings in attempts to formulate the allocation problem as a nonlinear programming problem have been resolved in the paper.

A breakthrough in the implementation of the model was achieved through the using software package BlockSim. This software provides the system's (simple or complex) analytical reliability equation. This equation can now be improve directly as an input to the optimization algorithm. Secondly, the cost function problem is addressed through the proposal of a general cost function, which is a function of parameters that can easily be quantified by engineers and is simple in its use.

II. Reliability Problem Allocation

The allocation problem addressed in this paper is of great practical importance. Reliability engineers are often called upon to make decisions as to whether to improve a certain component or components in order to achieve a minimum required MS reliability. For example, consider an MS consisting of three components connected reliability-wise in series. The reliability of each of the component is 0,7; 0,8; 0,9 respectively. Under the independence assumption, the reliability of the system will be 0,504. Assuming that a MS reliability performance of 0,8 where sought, the current design is clearly inadequate. The question now becomes one of how this goal can be reached. The reliability goal can not be reached by increasing the reliability of just one component and how feasible is it to improve a component's reliability? Fig. 1 is an illustration of this typical example of a decision-making dilemma. In order for these question to be answered another quantity is consider – cost. The challenge then becomes to model the cost as a function of reliability. The preferred approach would be to formulate the cost function from actual cost data. In many cases however this data is not available and is hard to obtain. This problem is addressed with the introduction of a general mathematical formulation for the cost function, which is assumed to have an exponential behavior. This function will act as the penalty of increasing a component's reliability. The overall MS cost, which is the

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objective function to be minimised, is assumed to be the summation of each component's cost.

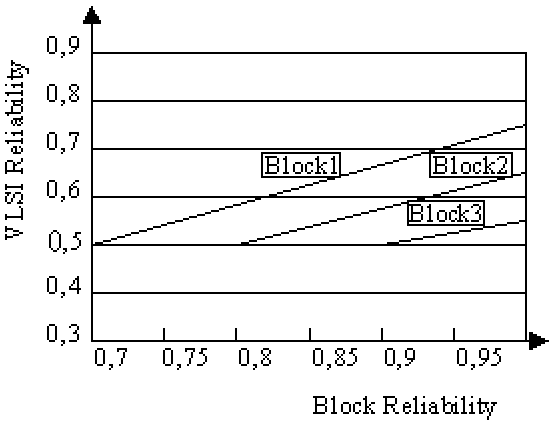

Fig.1. Meeting a reliability goal by changing the reliability of one component at a time

Consider a MS consisting of n components. Goal reliability is sought for this MS. The objective is to allocate reliability to all or some of the components of that MS, in order to meet that goal with a minimum cost. The problem is formulated as:

$$P : min\,C = \sum_{i=1}^{n} c_i(R_i)$$
$$s.t. R_S \geq R_G \qquad (1)$$
$$s.t. R_{i,min} \leq R_i \leq R_{i,max}, i = 1,2,\ldots,n$$

This formulation is designed to achieve a minimum total MS cost, subject to R_G a lower limit on the MS reliability. The first step will be to obtain the MS's analytical reliability function in terms of its component's reliability. Several methods exist for obtaining the MS's reliability equation, a review of them can be found in [3]. In this paper the BlockSim software [4] will be used which is designed to solve for the system's analytical reliability function.

II. Cost Function Allocation

The next step is to obtain a relationship for the cost of each component as a function of its reliability. An empirical relationship can be derived based on past experiences and/or data for similar components. In many cases however, such data is not available. In order to overcome this problem, a general behavior for the cost function is proposed in this paper, as follows:

$$c_i(R_i; f_i; R_{i,min}, R_{i,max}) = exp\left[(1 - f_i)\frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right] \qquad (2)$$

This is an exponential behavior, as shown in Fig. 2, and it contains three parameters namely f_i, $R_{i,min}$ and $R_{i,.max}$. The first parameter is the feasibility of increasing a component's reliability and it assumes values between 0 and 1. The second parameter is the maximum achievable reliability of the i^{th} component. For example in Fig. 2 the initial reliability for that particular component is 70%, the maximum achievable reliability is 99%, and the cost function is plotted for 0,1 feasibility.

The important assumptions in the model are that the system and its independent components/blocks can only assume two states – failed and operational.

It can be seen that the cost function of Eq. (2), which is less of dimension, is easy to implement, with only two required inputs (in addition to the failure distribution of the components), namely the feasibility and the maximum achievable reliability. It essentially acts as a weighting factor that describes the difficulty in increasing the component reliability from its current value.

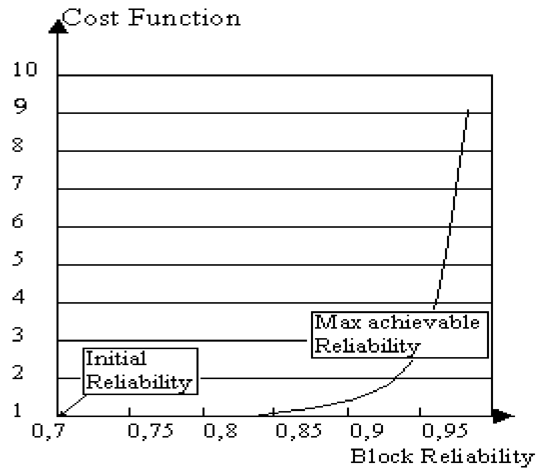

Fig. 2. Behavior of the assumed cost function (0,99 max achievable reliability and $f=0,1$)

Examining the cost function given by Eq. (2) yields the following observations:

- ✓ The cost increases as the allocated reliability departs from the minimum (current) reliability;
- ✓ The cost is a function of the range of improvement;
- ✓ The exponential Eq. (2) approaches infinity as the component/block's reliability approaches its maximum achievable value.

A. Feasibility

The feasible parameter is a constant which represents the difficulty in increasing a component/block's reliability relative to the rest of the components in the MS. Depending on the design complexity, technological limitations etc. certain components can be very hard to improve relative to other components in the system. Clearly, the more difficult it is to improve the reliability of the component/block, the greater the cost. Examining the effect of the feasible on the cost function of Eq. (2), it can be seen that the lower the feasibility value, the more rapidly the cost function approaches infinity (see Fig. 3).

Several methods can be used to obtain a feasibility value. Weighting factors for allocating reliability have been proposed by many authors and can be used to quantify feasibility. These weights depend on certain factors of influence [3] such as the complexity of the component, the state of the art, the operational profile, the criticality, etc. Engineering judgement based on past experience, supplier quality availability, etc., can also be used in determining a feasibility value.

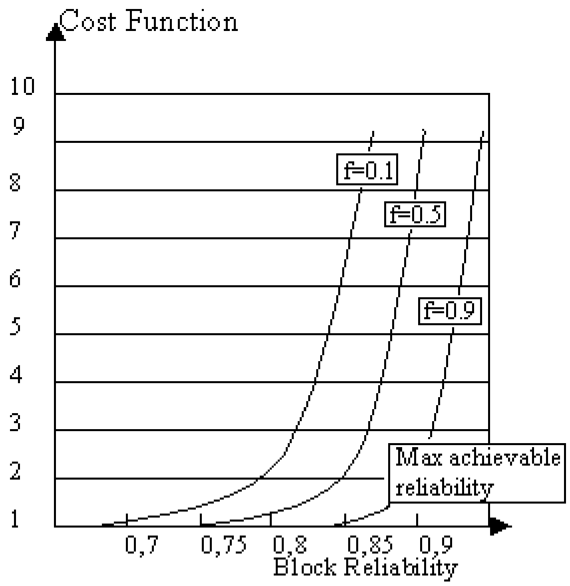

Fig. 3. Effect of feasibility on the cost function for different feasibility values

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B. Maximum Achievable Reliability

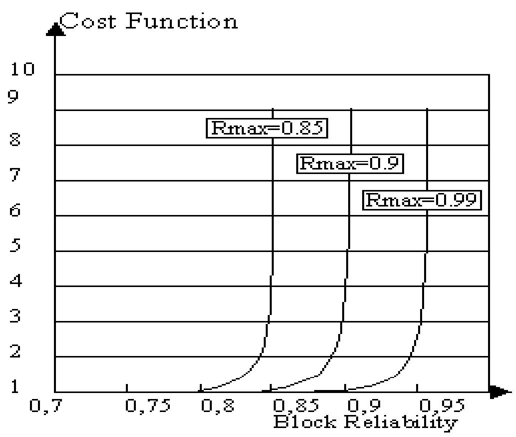

Fig. 4. Effect of the maximum achievable reliability on the cost function for different Maximum Achievable Values

The maximum achievable reliability is a limiting reliability value. For example a reliability of 100% is a limiting reliability. However, technological or financial constrains might dictate a maximum achievable reliability for certain components/blocks other than 100%. For this reason the maximum achievable reliability is incorporated in Eq. (2) as one of the parameters. The maximum achievable reliability acts as a scale parameter for the cost function. By decreasing $R_{i,max}$ the cost function is compressed between $R_{i,min}$ and $R_{i,max}$ as shown in Fig. 4.

C. Other forms of the cost function

The proposed cost function in this paper, given in Eq. (2) represents a general behaviour of the cost as function of reliability, to be used in the case where an actual function is not available. In [3] is given other general form as well. It is suggested however that these general functions are used individually within a system and do not get mixed with other cost functions. For example it is not recommended to use a cost equation such as the one given by Eq. (2) for some of the components in the MS and a different cost equation for the rest of the components particularly when these functions do not represent actual costs. Empirical forms for the cost function can also be derived based on past data or models can be fitted on cost data obtained from the development phase of the product. In BlockSim the engineer has the flexibility of using the cost function given by Eq. (2) or any other user-defined cost function.

III. Example

Consider a MS consisting of three components connected reliability-wise in series. Assume the objective reliability for the MS is 90% for mission time of 100 hours.

The first step is to obtain the MS's reliability equation:

$$R_s = R_1 \cdot R_2 \cdot R_3 \tag{3}$$

Eqs. (1) and (2) can now be written respectively as:

$$\begin{aligned} &P: \min C = \sum_{i=1}^{3} c_i\left(R_i\right) \\ &s.t. R_1 \cdot R_2 \cdot R_3 \geq R_G \\ &s.t. R_{i,\min} \leq R_i \leq R_{i,\max}, i = 1,2,3 \end{aligned} \tag{4}$$

$$\begin{aligned} &P: \min C = \sum_{i=1}^{3} \exp\left[\left(1 - f_i\right)\frac{R_i - R_{i,\min}}{R_{i,\max} - R_i}\right] \\ &s.t. R_{1,\min} \leq R_1 \leq R_G \\ &s.t. R_{2,\min} \leq R_2 \leq R_{2,\max} \\ &s.t. R_{3,\min} \leq R_3 \leq R_{3,\max} \end{aligned} \tag{5}$$

Five cases will be considered for allocation problem:

Case 1: All three components are identical whose times-to-failure are described with a Weibull distribution with β=1,318 and η=312 hours. All components have the same feasibility value.

Case 2: Same as in Case 1, but component 1 has a feasibility of 0,9, component 2 a feasibility of 0,5 and component a feasibility of 0,1.

Case 3: Component 1 has 70% reliability, component 2 – 80% and component 3 – 90%, all for mission duration of 100 hours. All components have the same feasibility value of 0,9.

Case 4: Component 1 has 70% reliability and a 0,9 feasibility, component 2 – 80% reliability and a 0,5 feasibility and component 3 – 90% reliability and a 0,1 feasibility, all for mission duration of 100 hours.

Case 5: Component 1 has 70% reliability and 0,1 feasibility, component 2 – 80% reliability and a 0,9 feasibility and component 3 – 90% reliability and a 0,5 feasibility, all for mission duration of 100 hours.

In all cases the maximum achievable reliability $R_{i,\ ,max}$ for each component is 99,9% for mission duration of 100 hours.

Let consider the Case 1. The components are identical with

$$R_i(100) = \exp\left[-\left(\frac{100}{312}\right)^{1,318}\right] = 0.8$$

This reliability value (the initial reliability) corresponds to the minimum reliability $R_{i,\ min}$. Using Eq. (2), the optimisation algorithm in BlockSim and the specified parameters of this case, the resulting optimal reliability allocation for each component is:

$$R_1(100) = R_2(100) = R_3(100) = 96{,}55\%$$

In other words, each component's reliability should be at least 96,55% at 100 hours in order for the system's reliability to be 90% at 100 hours. This result was expected since the components are identical, thus all components will be assigned the same reliability. The results for Cases 1 through 5 are summarised in Table I.

The reliability importance of component i is given by [3]:

$$I_R(i) = \frac{\partial R_S}{\partial R_i} \qquad (6)$$

TABLE I

SUMMARY TABLE FOR THE FIVE CASESEFERENCES

№ of comp	***Case 1***	***Case 2***	***Case 3***	***Case 4***	***Case 5***
1	0,9655	0,9874	0,9552	0,9790	0,9295
2	0,9655	0,9633	0,9649	0,9553	0,9884
3	0,9655	0,9463	0,9765	0,9624	0,9797

The partial derivations of Eq. (3) with respect to each component/block's reliability were calculated and the results are plotted in Figure 5, where it can be seen that component 1 has the greatest reliability importance and component 3 the smallest (this reliability importance also applies in Cases 4 and 5). This indicates that the reliability of component 1 should be significantly increased since it has the biggest impact on the overall MS reliability.

From Table I it can be seen that in Case 3 there was a 25,52% improvement for component 1, a 16,49% for component 2 and a 7,65% for component 3. On the other hand in Case 4 component 1 was assigned an even greater increase of 27,9% with components 2 and 3 receiving a lesser increase than in Case 3, of 15,53% and 6,24% respectively. This is due to the fact that component 1 has the largest feasibility value of 0,9 and component 3 the lower of 0,1 which means that is more difficult to increase component 3 than component 1.

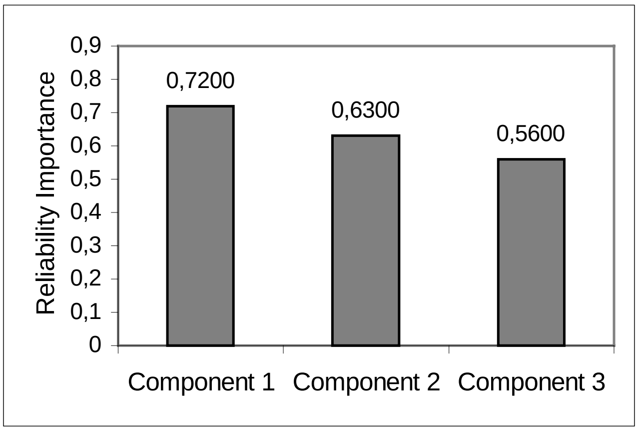

Fig. 5. Reliability importance for cases 3, 4 and 5.

Since all three components have the same maximum achievable reliability, component 1 is the most cost efficient component to improve. It has the largest range for improvement.

III. CONCLUSION

In the paper the MS reliability optimisation problem through reliability allocation at the component/block level was examined. Further research should be concentrated in obtaining such functions based on actual cost data. The advantage of the model is that it can be applied to any system [5] including VLSI. As long as the system's reliability equation can be derived analytically, the model can be used to solve the reliability allocation problem. Different components/blocks of the system can be selected for optimisation. In other words, reliability can be allocated to some or all of the components/blocks of the MS. The parameters of the proposed cost function can be altered, allowing the engineers to investigate different allocation cases. Design engineers can decide and plan on how to achieve the assigned minimum required reliability for each of the components/blocks.

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