Instantaneous and Average Output Dissipated by an Output Stage Operating in Class AB

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Abstract **Theoretical analysis of the output dissipation of an audio amplifier operating in class AB. Computer simulation of the operation of a real-world output stage.**

Keywords **Instantaneous and average output dissipation. Class AB amplifier.**

I. Introduction

Output is assessed for output stages made on transformer-free circuits, where the end transistors operate in serial and push-pull mode.

The analysis is done at a constant sinusoidal signal or at smooth change of frequency and amplitude.

The theoretical analysis and the computer simulation are performed for an active load (rated impedance) and for a reactive load (real-world complex impedance of an loudspeaker) and the results so obtained are compared.

II. Analysis of the Output Dissipated by the End Transistors at an Active Load

The instantaneous output dissipated by an output stage operating in class AB at a sinusoidal signal $V_O.\sin(\varpi.t)$, at a load of R_L and a bipolar supply of $\pm V_{SS}$ is:

$$P_{d(inst)} = V_{CE}.i_E\,, \quad V_{CE} = V_{ss} - V_O.\sin(\varpi.t) \tag{1}$$

As shown in [3] and [4], the non-linear response of each of the arms of the output stage can be presented by:

$$i_E = \left[V_O.\sin(\varpi.t) + V_{Rt} + \left|V_O.\sin(\varpi.t) + V_{Rt}\right|\right] / \left(2.R_{sp}\right) \tag{2}$$

In case $V_{Rt} = 0$ (class B amplifier) and for $\alpha = \varpi.t$ varying from 0 to 180°, for the half of the sinusoid:

$$i_E = \left[V_O.\sin(\varpi.t)\right] / R_{sp} \quad [1] \tag{3}$$

The peak swing over the transistor will appear for those values of $m = V_0 / V_{ss}$, for which the derivative of the expression with respect to m is nullified:

$$\frac{d}{dm} P_{d(inst)_{norm}} = 0 \tag{4}$$

$$\frac{d}{dm}\left\{0{,}5.\left[1 - m.\sin(x)\right].\left[m.\sin(x) + v + \left|m.\sin(x) + v\right|\right]\right\}$$

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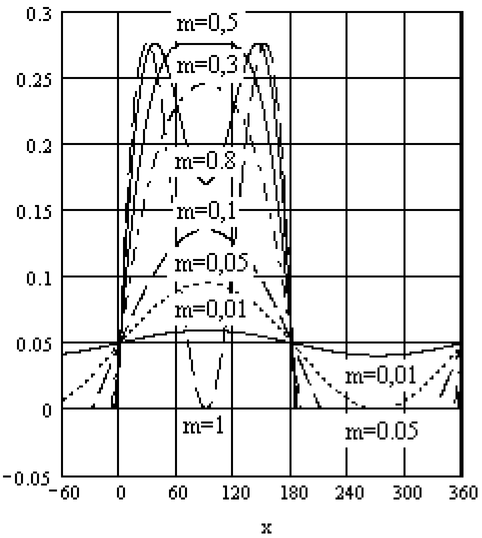

Fig.1. Instantaneous output dissipation $P_{d(inst)}$ in class AB.

The result is: $m = (1 - v)/\left[2.\sin(x)\right]$

$$P_{d(inst)_{norm}}\max(v) = 0{,}25.(v+1)^2 \tag{5}$$

For class AB, the mathematical relations Eqs. (2) and (3) are functions of two variables: the input signal V_O and the bias voltage V_{Rt} (determining the class of operation), normalized to the supply voltage V_{SS} .

The random distribution of the signal output level m is within the range of $0 \le m \le 1$.

Generally, for the class AB amplifiers, the bias voltage V_{Rt} is a fixed value and varies within a narrow range depending on the temperature of the end transistors.

Fig. 1 shows the instantaneous output dissipation $P_{d(inst)}$, normalized to the peak output dissipation $P_{0(\max)} = V_{SS}{}^2 / 2.R_{sp}$ during the whole cycle $\alpha = \varpi.t = 0 \div 360^0$ as a function of the output level $m = V_0 / V_{ss} = 0 \div 1$ and at $v = V_{Rt} / V_{SS} = 0{,}05$ – class AB. The extremums (and their kind) are modeled under a mathematical relation Eqs. (1) and (2), and by a computer simulation (See Fig. 2).

The analysis is performed for the real value of $v = V_{Rt} / V_{ss}$ =0,05, class AB:

- without an output level $m = V_O / V_{SS} = 0$, $P_{d(inst)_{norm}} = 0{,}05$,

- when the output level varies within $0 < m < 0{,}05$, the contour of $P_{d(inst)_{norm}}$ repeats V_O at the maximum for $x = \varpi.t = 90^0$ and minimum for $x = 270^0$;

- when $m \to 0{,}05$, $P_{d(inst)_{norm}\max} \to 0{,}1$, for $x = 90^0$,

  $P_{d(inst)_{norm}\min} \to 0$ for $x = 270^0$,

- when $0 < m < 0{,}5$, the maximum for, $x = \varpi.t = 90^0$,

- when $0{,}5 < m < 1$, there are two peaks with a limiting value of $P_{d(inst)_{norm\max}} = 0{,}276$ at $v = V_{Rt}/V_{ss} = 0{,}05$, which are symmetrical against to a local minimum of $x = 90^0$ (sin(x)=1),

- when $m = 1$, there are two peaks with a limiting value of $P_{d(inst)_{norm\max}} = 0{,}276$ at $v = V_{Rt}/V_{ss} = 0{,}05$ and x being close to $x = 30^0$ and $x = 150^0$, $P_{d(inst)_{norm\min}} = 0$ for $x = 90^0$ and $180^0 < x < 360^0$.

TABLE 1
THE EXTREMUMS IN CLASS AB

$m = V_0/V_{ss} = 0 \div 1$	0	0,05	1
$P_{d(inst)_{norm}\max}$ for x	0,05 -	0,1 90^0	0,276 30^0 and 150^0
$P_{d(inst)_{norm}\min}$ for x	0,05 -	0 270^0	0 $0^0, 180^0 < x < 360^0$

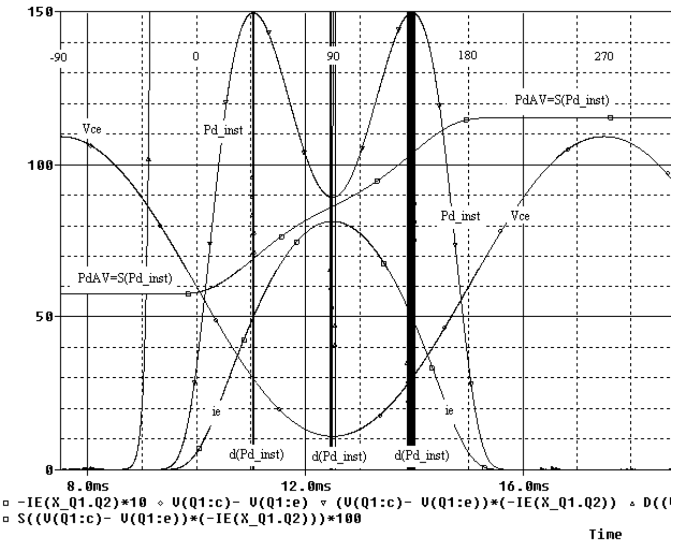

Fig.2. Computer simulation of an output stage operating in class AB at a resistive load.

Fig 2 shows the computer simulated voltage over the transistor $V_{CE} \Rightarrow$ V(Q1:c-V(Q1:e), the current passing through the transistor $i_E \Rightarrow$ IE(X_Q1.Q2), the instantaneous dissipation output $P_{d(inst)} = V_{CE}.i_E \Rightarrow$ -IE(X_Q1.Q2)*(Q1:c-V(Q1:e), and its first derivative (extremums) $d/dm(P_{d(inst)}) \Rightarrow$ D(-IE(X_Q1.Q2)*(Q1:c-V(Q1:e)), as well as the average dissipation output $\int P_{d(inst)} \Rightarrow$ S(IE(X_Q1.Q2)*(Q1:c-V(Q1:e)).

The average dissipation output can be obtained by integrating the instantaneous dissipation output:

$$P_{d(AV)} = \overline{P_{d(inst)}} = \frac{1}{T}.\int_{t_0}^{t_1} P_{d(inst)} dt = \frac{\varpi}{2.\pi}.\int_0^{\frac{2.\pi}{\varpi}} P_{d(inst)} dt \tag{6}$$

When $T = 2.\pi/\varpi = 1/f$ the integration is from $t_0 = 0$ to $t_1 = 2.\pi/\varpi$.

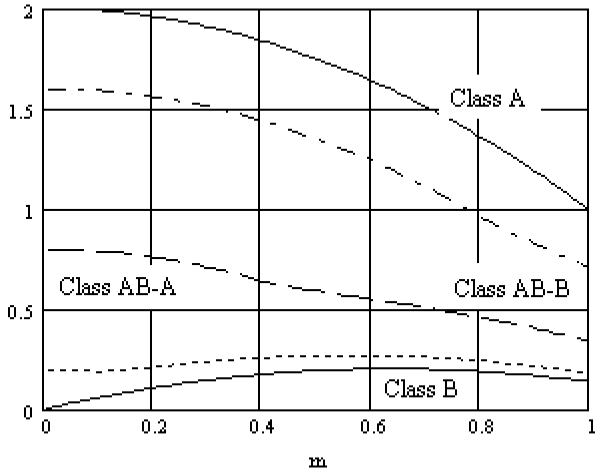

Fig.3. Average dissipation output $P_{d(AV)}$ $P_{d(AV)}$, normalized for maximum output in class A, AB and B and a resistive load, as a function of output level.

In class AB and output level of $m = V_0/V_{ss} = 0 \div 1$ and $v = V_{Rt}/V_{ss}$, but $m > v$, when integrating

from: $-\arcsin(v/m)$ ($t_0 = -\phi_{V_{Rt}} = -\arcsin(v/m)$

to $\pi + \arcsin(v/m)$ ($t_1 = \pi + \phi_{V_{Rt}} = \pi + \arcsin(v/m)$):

$$P_{d(AV)} = \frac{1}{2.\pi}.\int_{-\phi_{V_{Rt}}}^{\pi+\phi_{V_{Rt}}} P_{d(inst)} d\varpi.t = n_0 + (n_{1a} - n_{1b}).m - n_2.m^2 \tag{7}$$

$$n_0 = v.\left[1 + \frac{2}{\pi}.a\sin\left(\frac{v}{m}\right)\right]$$

$$n_{1a} = \frac{2}{\pi}.\sqrt{1 - \left(\frac{v}{m}\right)^2},\quad n_{1b} = \frac{v}{\pi}.\sqrt{1 - \left(\frac{v}{m}\right)^2} = n_{1a}.\frac{v}{2} \tag{7a}$$

$$n_2 = \frac{1}{2} + \frac{1}{\pi}.a\sin\left(\frac{v}{m}\right) = n_0.\frac{1}{2.v}$$

In class B ($v = V_{Rt}/V_{SS} = 0$), $n_0 = 0$, $n_{1a} - n_{1b} = \frac{2}{\pi}$ and $n_2 = \frac{1}{2}$:

$$P_{d(AV)} = \frac{V_{ss}.V_O}{\pi.R_L} - \frac{V_O^2}{4.R_L} = \frac{2}{\pi}.m - \frac{1}{2}.m^2, \tag{8}$$

In class A and in class AB, but at $m = V_0/V_{ss} < v = V_{Rt}/V_{ss}$, $n_0 = 2.v$, $n_{1a} - n_{1b} = 0$ and $n_2 = 1$:

$$P_{d(AV)} = \frac{1}{2.\pi}.\int_0^{2.\pi} P_{d(inst)} d\varpi.t = 2.v - m^2, \tag{9}$$

III. CLASS AB OUTPUT STAGE DISSIPATION OUTPUT DUE TO A REACTIVE LOAD

As the load becomes more reactive (See Fig. 5), the load current leads or lags (by phase ϕ) the load voltage and the peak dissipation increases.

$$P_{d(inst)} = V_{CE}.i_E \text{ , } V_{CE} = V_{ss} - V_O.\sin(\varpi.t) \text{ ,} \qquad (10)$$

$$i_E = \frac{V_O.\sin(\varpi.t+\phi) + V_{Rt} + \left|V_O.\sin(\varpi.t+\phi) + V_{Rt}\right|}{2.\left|Z_{sp}\right|}$$

If $V_{Rt} = 0$ (class B amplifier) and when wt varies $\varpi.t$ from 0 to 180°, the half of the sinusoid will be:

$$i_E = \frac{V_O.\sin(\varpi.t+\phi)}{\left|Z_L\right|} \text{ [1].} \qquad (11)$$

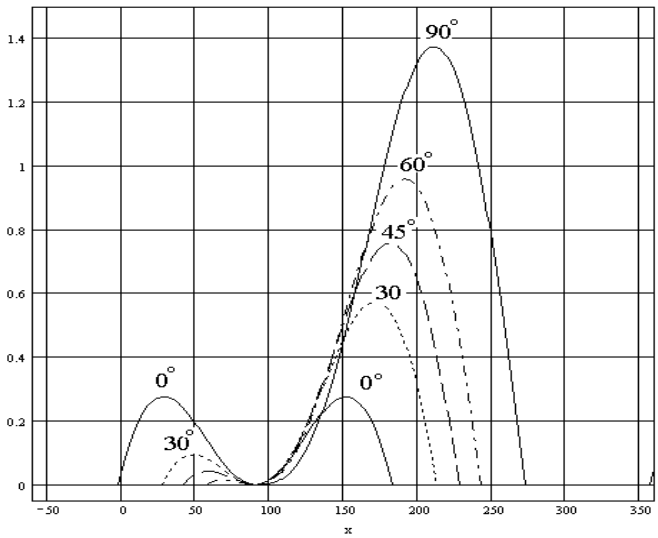

Fig.4. The instantaneous dissipation output $P_{d(inst)}$ as a function of the load phase in class AB.

Fig.4. shows the instantaneous dissipation output $P_{d(inst)}$, normalized to the peak output power $P_{0(\max)}$ during the whole cycle $\alpha = \varpi.t = 0 \div 360^o$ as a function of the load phase $\phi = 0 \div 90^o$ and at $V_{Rt}/V_{SS} = 0{,}05$ - class AB. Peaks can easily be seen on the graphical presentation of $P_{d(inst)_{norm}}$ under the mathematical relation Eq. (10).

The derivative of relation Eq. (10) with respect to m is:

$$\frac{d}{dm} P_{d(inst)_{norm}} = 0$$

$$= \frac{-1}{2}.\sin(x).\left[m.\sin(x-\phi) + \left|m.\sin(x-\phi)\right|\right] + \qquad (12)$$

$$+ \frac{1}{2}.\left[1 - m.\sin(x)\right]....... = 0$$

The result is: $m = 0{,}5/\sin(x)$ (13)

Then the peak dissipation output is:

$$P_{d(inst)_{norm\max}} = 0{,}25.\frac{1}{2}.\left[\frac{\sin(x-\phi)}{\sin(x)} + \left|\frac{\sin(x-\phi)}{\sin(x)}\right|\right] \qquad (14)$$

In class B ($V_{Rt}/V_{SS} = 0$) when the signal is maximum m=1 and the load is active, $\phi = 0^0$, there are two peaks: $P_{d(inst)_{norm\max}} = 0{,}25$ at $x = 30^0$ and $x = 150^0$ (See Fig. 1).

Under the same conditions, but when the load is of a reactive type, $\phi = -90^0 \div +90^0$, the following cases can be considered:

When the reactive load is: $\phi = 30^0$,

$P_{d(inst)_{norm\max}} = 0{,}53$, at $x = 170^0$

$P_{d(inst)_{norm\min}} = 0$, at $x = 90^0$ and from 210^0 to 30^0.

When the reactive load is $\phi = 90^0$,

$P_{d(inst)_{norm\max}} = 1{,}3$, at $x = 210^0$

$P_{d(inst)_{norm\min}} = 0$, when x varies from 270^0 to 30^0.

The ratio of the peak instantaneous output at $\phi = 90^0$ and $\phi = 0^0$ is (see Table 2):

$$P_{d(inst)\phi=90^0} / P_{d(inst)\phi=0^0} = 1{,}3/0{,}25 = 5{,}2 \qquad (15)$$

In class AB $V_{Rt}/V_{SS} = 0{,}05$ and at maximum signal, $m = 1$, Fig. 4.

$$P_{d(inst)\phi=90^0} / P_{d(inst)\phi=0^0} = 1{,}38/0{,}28 = 4{,}9 \qquad (16)$$

When the output stage operates very closely to class A $V_{Rt}/V_{SS} = 0{,}5$ and at maximum signal $m = 1$:

$$P_{d(inst)\phi=90^0} / P_{d(inst)\phi=0^0} = 2{,}08/0{,}57 = 3{,}6$$

When the output stage operates in class A $V_{Rt}/V_{SS} = 1$ and at maximum signal $m = 1$:

$$P_{d(inst)\phi=90^0} / P_{d(inst)\phi=0^0} = 2{,}93/1 = 2{,}93 \qquad (17)$$

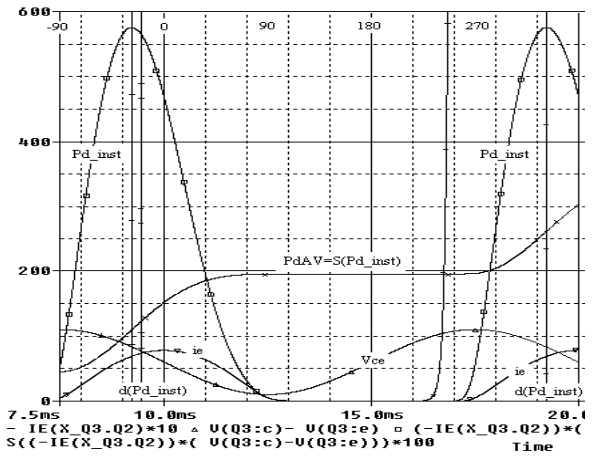

Fig. 5. Computer simulation of an output stage operating in class AB at an inductive load 250 μ F $\Rightarrow$ j.6 Ω.

Fig. 5 shows the computer simulation of the transistor voltage $V_{CE} \Rightarrow$ V(Q3:c-V(Q3:e), the transistor current

$i_E \Rightarrow$ IE(X_Q3.Q2), the instantaneous dissipation output $P_{d(inst)} = V_{CE}.i_E \Rightarrow$ -IE(X_Q3.Q2)*(Q3:c-V(Q3:e), and its first derivative (extremums) $d/dm\left(P_{d(inst)}\right) \Rightarrow$ D(-IE(X_Q3.Q2)*(Q3:c-V(Q3:e)), as well as the average dissipation output $\int P_{d(inst)} \Rightarrow$ S(IE(X_Q3.Q2)* (Q3:c-V(Q3:e)).

As the load becomes more reactive (See Fig. 5), the load current leads or lags (by phase ϕ) the load voltage and the peak dissipation increases $P_{d(inst)} = V_{CE}.i_E \Rightarrow$ IE(X_Q3.Q2)*(Q3:c-V(Q3:e).

The average dissipation output can be derived by integrating the instantaneous dissipation output:

$$P_{d(AV)} = \overline{P_{d(inst)}} = \frac{1}{T}.\int_{t_0}^{t_1} P_{d(inst)}dt = \frac{\varpi}{2.\pi}.\int_{\frac{\pm\phi}{\varpi}}^{\frac{\pi\pm\phi}{\varpi}} P_{d(inst)}dt \qquad (18)$$

When $T = 2.\pi/\varpi = 1/f$ the integration is from $t_0 = \pm\phi/\varpi$ to $t_1 = (\pi \pm \phi)/\varpi$.

In class AB and input level $m = V_0/V_{ss} = 0 \div 1$, and $v = V_{Rt}/V_{ss}$, but when $m > v$ integration is:

from $-\arcsin(v/m) - \phi$ $\quad$ ($t_0 = -\phi_{V_{Rt}} - \phi$)

to $\pi + \arcsin(v/m) - \phi$ $\quad$ ($t_1 = \pi + \phi_{V_{Rt}} - \phi$):

$$P_{d(AV)} = \frac{1}{2.\pi}.\int_{-\phi_{V_{Rt}}-\phi}^{\pi+\phi_{V_{Rt}}-\phi} P_{d(inst)}d\varpi.t =$$

$$P_{d(AV)} = n_0 + [n_{1a} - n_{1b}.\cos(\phi)].m - n_2.\cos(\phi).m^2 \qquad (19)$$

For n_0, n_{1a}, n_{1b} and n_2 see Eqs. 7a.

In class B ($v = V_{Rt}/V_{SS} = 0$) the result of integrating $P_{d(inst)}$ is an average output dissipated by the end transistors of one arm of the output stage ($n_0 = 0$, $n_{1a} - n_{1b} = \frac{2}{\pi}$ and $n_2 = \frac{1}{2}$):

$$P_{d(AV)} = \frac{V_{ss}.V_O}{\pi.R_L} - \frac{V_O^2}{4.R_L}.\cos(\phi) = \frac{2}{\pi}.m - \frac{1}{2}.m^2.\cos(\phi) \qquad (20)$$

The same relation for $P_{d(AV)}$ is shown in the publication of Mr. Eric Benjamin – relation of Widlar and Yamatake [1, p. 671, Eq. (5)]

In class A and in class AB, but when $m < v$, $n_0 = 2.v$, $n_{1a} - n_{1b} = 0$ and $n_2 = 1$:

$$P_{d(AV)} = \frac{1}{2.\pi}.\int_0^{2.\pi} P_{d(inst)}d\varpi.t = 2.v - m^2.\cos(\phi) \qquad (21)$$

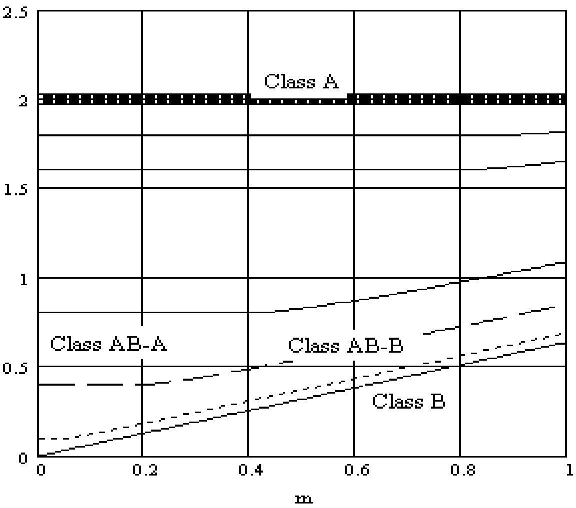

Fig.6. Average dissipation output $P_{d(AV)}$, normalized for maximum output in class A, AB and B at a reactive load, as a function of output level.

IV. Conclusions

The peak instantaneous dissipation output in class B output stage with a reactive load is 5.2 times bigger than with a rated active load, Eq. (15). This increase is less, Eq. (16), in class AB amplifiers and in class A it is 2.93 times, Eq. (17).

TABLE 2

Class	A	AB	B
$v = V_{Rt}/V_{SS} = 1 \div 0$	1	0,05	0
$P_{d(inst)\phi=90^0} / P_{d(inst)\phi=0^0}$	2,93	4,9	5,2

The mathematical relations which determine the average dissipation output Eqs. (6), (7), (8), (19), (20) and (21) are for the corresponding operation class of the output stage and/or depend on the relation of the output level and the voltage which determines the class.

The general relation which describes the average dissipation output can be derived from relations Eqs. (6) and (19), especially from their real portion, as modeled in Figs. 3 and 6.

References

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