Algorithm for Optimal Receiving Signals of Jump-Like Change of Carrying Frequency with Rapid Fluctuations of Time Lag

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Abstract – **The paper presents an algorithm for optimal receiving signals with jump-like change of frequency with random lag. The direct examination on the random lag allows obtaining an algorithm involving a wide range of tasks. For example, such tasks are receiving signals for which the random lag occurs not only in moving away the limits of the time tact intervals but also in receiving under the conditions of changing lag.**

Keywords – **Signals of jump-like change of carrying frequency, random time lag**

I. Problem Setting

As it is known, with radio signals of a jump-like change of carrying frequency [1-3] the value of the carrier in the tact intervals is given in a code sequence. Without considering the information modulation, such signals are described analytically, using a complex presentation as follows:

$$s(t) = \mathrm{Re}\{U_0 c(t) \exp[j(\omega_0 t + \varphi)]\} \tag{1}$$

where:

$$c(t) = \sum U_0[t-(i-1)T] \exp[j(\omega_i t - \varphi_i)] \tag{2}$$

is the modulating function where ω_i is the value of the frequency jump in the i-th interval and φ_i is the value of the phase in the same interval.

Let one of the n-number element signals, which are due for those identically equal to zero out of the interval, is marked with $s_k\ [0,T]$. Then the signal transmitted can be described with the expression:

$$s(t) = \sum_{k=0}^{n} s_k(t-iT) \tag{3}$$

With the presence of random time lag, the useful signal received is:

$$s(t,\tau(t)) = s(t-\tau(t)) = \sum_{i=0}^{n} s_i(t-iT-\tau(t)) \tag{4}$$

Let present it in the kind of a known function of information discrete parameter $\theta(t)$ and random time lag $\tau(t)$, i.e.:

$$s(t,\theta,\tau) = s[t-\tau(t), \theta(t-\tau(t))] \tag{5}$$

The discrete parameter takes constant values on tact intervals $\theta(t) = \theta_i$, $t \in [t_i, t_{i+1}]$. The values of the information parameter on the various tact intervals form a simple Markov's chain θ_i, $i = 0,1,...n$ with n states and a known matrix of the transitions from the i-th into the j-th state $\Pi = \pi_{i,j}$, and vector of the initial states $p = p_{\text{ı}}$. The limits of the tact intervals are determined by random time lag $\tau(t)$, i.e. $t_i = t_i(\tau)$. With the time lag realization specified, the limits of tact interval are:

$$t_i = iT + \tau(t_i)$$

On tact interval t_i, t_{i+1}, signal $s(t,\theta,\tau)$ coincides with elementary signal $s_i(t-iT-\tau(t))$ if $\theta(t) = \theta i$. Random time lag $\tau(t)$ corresponds to the signal lag caused by the relative motion of the receiver and the transmitter and in the common case it can be examined as a component of the diffusion Markov's process:

$$\lambda(t), \tau(t) = \lambda_1(t)$$

Process $\lambda(t)$ satisfies the system of stochastic differential equations:

$$\frac{\partial \lambda_i(t)}{\partial t} = f_i(t,\lambda) + n_i(t) \tag{6}$$

Here $f_i(t,\lambda)$ are functions satisfying the condition of Lipshits [4] and $n_i(t)$ are Gauss noises with intensity $b_{ij}(t,\lambda)$. The a priori probable features of process $\lambda(t)$ are determined by the equation of Kolmogorov–Foker–Plank [4]:

$$\frac{\partial W}{\partial t} = -\sum_{\alpha=1}^{n} \frac{\partial}{\partial \lambda_\alpha}[a_\alpha(t,\lambda)W] + \\ + \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\gamma=1}^{n} \frac{\partial^2 b_{\alpha\gamma}(t,\lambda)W}{\partial\lambda_\alpha \Lambda_\gamma} \equiv L[W] \tag{7}$$

where $W = W(t,\lambda)$ is the a priori probable density of process $\lambda(t)$; $a_\alpha(t,\lambda)$ are the diffusion coefficients of Markov's process $\lambda(t)$.

The observation on signal $s(t,\theta,\tau)$ is realized on the background of noise, i.e. it has the kind of

$$\xi(t) = s(t,\theta,\tau) + n(t) \tag{8}$$

where $n(t)$ is non-correlated with $\theta(t)$ and $\tau(t)$ is white noise of feature $M\{n(t)\} = 0$.

II. Optimal Assessment of the Information Parameter

With a certain realization of lag $\tau(t)$, the optimal (according to the criterion of the error probability minimum) assess-

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ment of the information discrete parameter constant in interval $[t_i, t_{i+1}]$ is determined with expression [5]:

$$\theta_i^* = \max{}^{-1}\{P(\theta_i = i)\} = \\ = \max{}_i^{-1}\{P[(i+1)T + \tau, i]\} \quad (9)$$

With random values of tact interval t_{i+1}, when posteriori probabilities $P(\theta_i = i)$ have to be compared, in the moments of time the conditional probability has to be examined with fixed τ, i.e.

$$P(\theta_i = i) = P[t_{i+1}(\tau), i/\tau] \quad (10)$$

The optimal assessment of the discrete parameter can be examined as a non-conditional discrete parameter posteriori probability at the end of the interval under observation. The probability can be determined by the expression:

$$P[t_{i+1}(\tau), i/\tau] = P[i/\tau, \xi^{t_{i+1}(\tau)}] \quad (11)$$

made average with weight corresponding to posteriori probable density of random lag $P(t, \tau)$. Then in the i-th tact interval the algorithm of assessment of the information discrete parameters takes the kind of:

$$\theta_i^* = \max{}_i^{-1}\{P(\theta_i = i)\} = \\ = \max{}_i^{-1}\left\{\int P[t_{i+1}(\tau), i/\tau]P[t_{i+1}(\tau), \tau]d\tau\right\} \quad (12)$$

III. Conclusions

The paper presents an algorithm for optimal receiving signals with jump-like change of frequency with random lag. The direct examination on the random lag allows obtaining an algorithm involving a wide range of tasks. For example, such tasks are receiving signals for which the random lag occurs not only in moving away the limits of the time tact intervals but also in receiving under the conditions of changing lag.

The algorithm obtained (12) presents a summary of an algorithm for assessment of a constant parameter in a certain interval with the presence of indeterminacy at the moment of signal appearing.

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