3D Numerical Analysis of Electromagnetic Systems with Magnetic Circuit Using Impedance Boundary Condition

Marin Dimitrov, Stoimen Balinov and Pavel Mintchev[1]

Abstract – **The rates of convergence of the edge element method (EEM) coupled with impedance boundary condition (IBC) and of the finite element method based on a gauged system of differential equations for the magnetic vector potential and coupled with IBC are compared in this paper in the case of numerical analysis of magnetic systems with concentrating and shielding magnetic cores. It is established that EEM coupled with IBC has a substantial advantage.**

Keywords – **crucible furnaces, edge element method, impedance boundary condition, magnetically shielded systems.**

I. Introduction

The use of impedance boundary condition (IBC) for 3D numerical analysis of the electromagnetic field in systems with eddy currents results in substantial acceleration of the calculations. A numerical model coupling the finite-element method (FEM) with IBC and based on a gauged system of differential equations for the magnetic vector potential (MVP) is proposed in [1]. The gauging of the equations for the MVP improves the convergence but can result in violation of the continuity conditions on the boundaries between regions with different permeabilities [2,3]. A coupling of the IBC and the edge-element method (EEM), for which the continuity conditions can't be violated from a theoretical point of view, is proposed in [4] for the study of an induction heater with a magnetic core for concentrating the magnetic flux. It is established in [4] that the use of EEM coupled with IBC results in a substantial improvement of the convergence rate, providing at the same time good accuracy. The non-linear properties of the ferromagnetic materials are not taken into account in this study.

The purpose of the present paper is a comparison of the rates of convergence of both formulations: EEM coupled with IBC [4] and FEM coupled with IBC [1,5,6] in the case of 3D numerical analysis of systems with concentrating and shielding magnetic cores, taking into account the non-linear properties of the materials.

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II. Formulation of the Problem for 3-D Numerical Analysis of the Electromagnetic Field by Means of EEM Coupled with ibc

The following equation for the magnetic vector potential (MVP) is used:

$$rot\left(\frac{1}{\mu} rot\, A\right) + j\omega\sigma\, A = J \;, \qquad (1)$$

where μ is permeability, $\omega = 2\pi f$, f is frequency, σ is conductivity, J is the source current density and $j = \sqrt{-1}$.

The following relationships can be written in the case of strong skin-effect in the electroconductive details [1]:

$$K = H \times n = \frac{1}{Z_S}(n \times E) \times n \qquad (2)$$

$$Z_S = \frac{1+j}{\sigma\delta} \;. \qquad (3)$$

Here K is the density of the current on the surface Γ_C of the details, n is the outer normal to Γ_c, Z_S is the surface impedance and δ is the electromagnetic penetration depth. Eq. 2 is IBC.

After applying EEM and taking into account Eq. 2 and the corresponding boundary condition (for the distant points and for the planes of symmetry [1,7]), Eq. 1 is transformed into the following functional:

$$\int_\Omega \left(\mu^{-1} rot A\right)\cdot(rot W)\, d\Omega + \int_\Omega j\omega\sigma A\cdot W d\Omega =$$
$$= \int_{\Gamma_c} W.\; (H\times n) d\Gamma + \int_{\Gamma_S} J.W d\Gamma \;, \qquad (4)$$

where W is the weight function, Ω is the region of integration and Γ_S is the surface on which is distributed the exciting current.

The region Ω is divided into tetrahedral finite elements while the surface Γ_c is divided into triangular ones.

The weight function for an edge connecting the nodes with numbers i and j is defined as [6]:

$$W = \varsigma_i \nabla\varsigma_j - \varsigma_j \nabla\varsigma_i \;, \qquad (5)$$

where ς is the shape function.

The following relations are used to determine A and J within the boundaries of a finite element:

$$A = \sum_{e=1}^{m} A_e W_e \quad J = \sum_{e=1}^{m} J_e W_e \;, \qquad (6)$$

where the index e is the local number of the edge, A_e and J_e are circulations of A and J on the edge e, $m = 6$ for a tetrahedral element and $m = 3$ for a triangular element.

The following formulae are used to determine $rotA$ within the boundaries of a tetrahedral element:

$$rot\,A = \sum_{e=1}^{6} A_e rot\,W_e \quad rot\,W_e = 2\nabla\varsigma_i \times \nabla\varsigma_j\,. \tag{7}$$

Taking into account Eqs. 5, 6 and 7, the functional (Eq. 4) can be transformed into a system of algebraic equations in which the unknowns are the circulations of the MVP A along the finite element edges. The Incomplete Cholesky-Conjugate Gradient (ICCG) Method [10] is used to solve this system. The accuracy of the solution of the system is evaluated by means of the relative Euclidean norm of the residuals ε_i [7].

III. Numerical Algorithm

The numerical algorithm contains an iteration cycle, each step of which includes: calculation of the coefficients of the system of algebraic equations; solution of the system by means of a second iteration cycle which terminates when ε_i reaches a preliminary given value; calculation of the values of the magnetic field strength B for each volume and surface element; correction of the values of the permeability μ for each element using the $B(H)$ curve for the corresponding material.

At the end of the iteration cycle the values of the reactive and active power are calculated.

IV. Numerical Results

Numerical analysis of the field in two electromagnetic systems has been carried out.

The first one is an induction heater for flat surfaces. The heater has a U-shaped magnetic core and two inductors (see Fig. 1). The heated surface is ferrous steel. The working frequency is 4000 *Hz* and the surface density of the inductor current is 2.10^6 A/m. The same system is studied in [4] but the non-linear properties of the ferromagnetic materials are not considered. The studied region, encircled in the figure with a dash line, contains 1/4 of the volume of the whole device.

The second system is an induction crucible furnace-mixer for copper alloys with short inductor, large capacity - above 100 t and working frequency of 50 Hz (see Fig. 2). The crucible of the furnace is filled with melt up to the upper end of the inductor. 30 shielding magnetic cores are installed in the space between the inductor and the furnace mantle, which is made of ferrous steel. The surface density of the inductor current is 2.10^5 A/m. The region of integration, which is dashed on Fig. 2, contains 1/60 of the volume of the furnace.

In the case of FEM, the region of integration is divided into hexahedra, while the surfaces on which IBC is applied are divided into quadrangles. In the case of EEM, each hexahedron is divided into 6 tetrahedrons and each quadrangle – into 2 triangles. On Fig. 3 is shown the division of the area

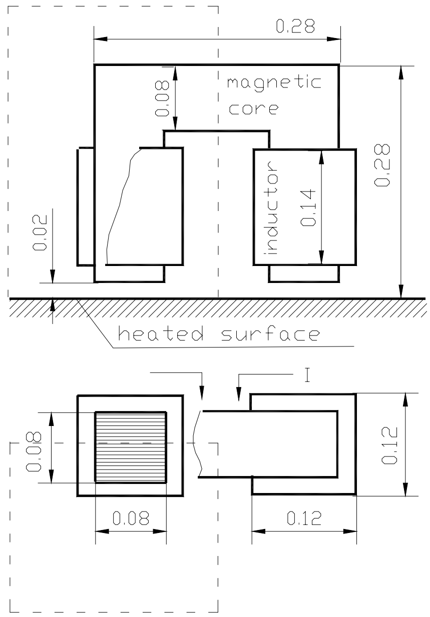

Fig. 1. Diagram of the induction heater for flat surfaces. All dimensions are in m.

of integration into tetrahedrons and triangles for the second studied system – the induction crucible furnace.

Two algorithms are applied for the solution of the problems: EEM coupled with IBC [6] and FEM with gauged MVP coupled with IBC [1] in a non-orthogonal coordinate system [5,6]. The rates of convergence of both algorithms are compared. The EEM coupled with IBC is used to verify the accuracy of the FEM coupled with IBC.

The geometry of the studied devices allows the region of integration to be divided into high-quality hexahedral elements with a shape close to cubical and into almost equilateral tetrahedral elements. That is why high accuracy of the solution of the systems of algebraic equations approximating the differential equations can be expected.

In Table 1 are shown the results from the calculation of the complex power in the region of integration, the necessary number of iterations and the achieved value of the Euclidean norm of the residuals ε_i for the first studied system. As can be seen from the table, the necessary number of iterations for EEM coupled with IBC is nearly 10 times smaller than the corresponding number of iterations for FEM coupled with IBC. Also, the achieved accuracy of the solution of the system of algebraic equations is low: $\varepsilon_i < 0.12$, while for EEM coupled with IBC $\varepsilon_i < 0.02$. The minimum value of ε_i which can be achieved is an important indicator for the rate of convergence of the numerical process.

Despite the relative high value of ε_i in the case of FEM coupled with IBC, the accuracy of the SOLUTION of the problem as a whole is good: the values for the complex power obtained by means of both algorithms are almost identical.

In Table 2 are shown the results from the 3D numerical analysis of the electromagnetic field for the second studied system. The accuracy of the calculation of the value of the complex power by means of FEM coupled with IBC and gauged MVP is high: the results are almost identical with

Table 1. Results of the numerical modelling of the system with concentrating magnetic core

	EEM coupled with IBC [3]	FEM coupled with IBC and gauged MVP [1,4]
Number of iterations	265	2521
Value of ε_i	<0,02	<0,12
Complex power, VA	4139+ j55122	4300+ j56143

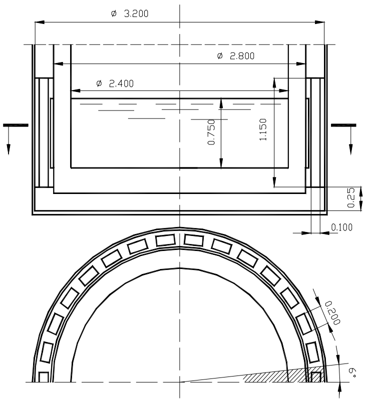

Fig. 2. Diagram of the induction crucible furnace for copper alloys. All dimensions are in m.

Table 2. Results of the numerical modelling of the system with shielding magnetic cores

	EEM coupled with IBC [3]	FEM coupled with IBC and gauged MVP [1,4]
Number of iterations	190	491
Value of ε_i	<0,01	<0,01
Complex power, VA	15568+ j256830	15314+ j241563
Power losses in the mantle's bottom, W	281	300
Power losses in the cylindrical part of the mantle, W	591	620

those obtained by means of the EEM coupled with IBC. The accuracy of the calculation of the power losses in the mantle is satisfactory. Better convergence is observed in the case of EEM coupled with IBC: the number of iterations is 2.5 times smaller than those for FEM coupled with IBC and gauged MVP.

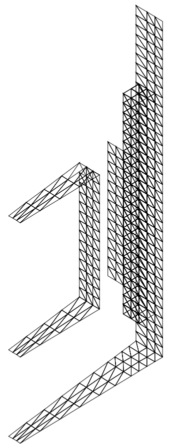

Fig. 3. Division of the area of integration for the induction crucible furnace.

V. Conclusion

It is established that the use of EEM coupled with IBC instead of the gauged formulation of FEM coupled with IBC improves the convergence of the numerical process in the case of 3D numerical analysis of the electromagnetic field, taking into account the non-linear properties of the magnetic materials. In the case of the studied system with a concentrating magnetic circuit, the number of iterations decreases almost tenfold and the accuracy of the solution of the system of algebraic equations approximating the differential equations of the electromagnetic field increases substantially. In the case of the second studied system, which contains shielding magnetic cores, the convergence improves to a lesser degree – the number of iterations decreases 2.5 times while the accuracy of the solution of the system of algebraic equations is the same for both algorithms.

References

[1] Louai, F.-Z., D. Benzerga, M. FelIachi , F. Bouillault, "A 3D finite element analysis coupled to the impedance boundary condition for the magnetodynamic problem in radio frequency plasma devices", *IEEE Trans. on Magn.*, May 1996, pp. 812-815.

[2] Mimoune S.M., J.Fouladgar, G.Develey, "Modeling of 3D Electromagnetic and Heat Transfer Phenomena for Materials with Poor Conductivity", *IEEE Trans. on Magn.*, November 1995, pp. 3578-3580.

[3] Preis, K., I.Bardi, O.Biro, C. Magele, G. Vrisk, K.R. Richter, "Differrent Finite Element Formulations of 3D Magnetostaic Fields", *IEEE Trans. on Mag.*, 28, 1992, No 2, 1056-1059.

[4] Mintchev, P., M. Dimitrov, S. Balinov, "Impedance boundary condition coupled with edge-element method to solve 3-D electro magnetic fields in induction heaters", *XII-th International Symposium on Electrical Apparatus and Technologies*, 31 May – 1 June 2001, Plovdiv. Proceedings Volume I, 237-244.

[5] Димитров М., П. Минчев, "Тримерен числен анализ на електромагнитното поле в индукционни нагреватели с използване на импедансни гранични условия и неортогонална координатна система", *Техническа мисъл*, 1999, є 1-2, 13-20.

[6] Dimitrov M., P. Mintchev, S. Balinov, A. Krusteva, "Choice of Coordinate System in Eddy Current Problems Using 3D Finite Element Numerical Analysis Coupled to the Impedance Boundary Condition", Applied electromagnetics. *Proceedings of the 1st Japanese-Bulgarian-Macedonian Joint Seminar on Applied Electromagnetics*, September 14-15, 1998, Sofia. 2000 Heron Press Ltd., 80-88.

[7] Biro O., K. Preis, "On the Use of the Magnetic Vector Potential in the Finite Element Analysis of the Tree-Dimentional Eddy Currents", *IEEE Trans. on Magn.* Vol. 25, No 4, July 1989, pp. 3145-3159.

[8] Biro O., K. Preis, "On the Use of the Magnetic Vector Potential in the Finite Element Analysis of the Tree-Dimentional Eddy Currents", *IEEE Trans. on Magn.* Vol. 25, No 4, July 1989, pp. 3145-3159.

[9] Golias, N., T.Tsiboukis, "3-D Eddy-Current Computation with a Self-Adaptive Refinement Technique", *IEEE Trans. on Magn.*, 31, 1995, No 3, 2261-2268.

[10] Kershaw, D, "The incomplete Cholesky-conjugate gradient method for the iterative solution of systems of linear equations", *Journal of Computational Physics*, No 26, 1978, pp. 43-65.